Mortgage Credit, Aggregate Demand, and Unconventional Monetary Policy *

Vadim Elenev†
NYU Stern

June 20, 2017

Abstract

I develop a quantitative model of the mortgage market operating in an economy with financial frictions and nominal rigidities. I use this model to study the effectiveness of large-scale asset purchases (LSAPs) by a central bank as a tool of monetary policy. When negative shocks hit, homeowner and financial sector balance sheets are impaired, borrowing constraints bind, asset prices and aggregate demand drop, hampering the transmission of conventional monetary policy. LSAPs boost aggregate demand in a crisis by directing additional lending to homeowners, raising house prices, and establishing expectations of future financial stability. However, subsequent policy normalization requires the financial sector to absorb heightened levels of borrower debt, depressing output and consumption in recovery. In the long run, a commitment to ongoing use of LSAPs in crises reduces credit and business cycle volatility and redistributes resources from borrowers and intermediaries to savers.

JEL: G12, E44, E52, H81, R30.

Keywords: financial intermediation, unconventional monetary policy, large-scale asset purchases, housing policy, credit spread, intermediary-based asset pricing

*First draft: November 1, 2016.

†I am truly grateful to my thesis advisers Stijn van Nieuwerburgh, Viral Acharya, and Johannes Stroebel for their invaluable support and guidance. I thank Eduardo Dávila, Cecilia Parlatore, Miguel Faria-e-Castro, Mohsan Bilal, Aaditya Iyer, and seminar participants at NYU Stern and Columbia Business School for their feedback, which greatly improved this paper. All errors are my own. I am grateful for the generous support of the NYU Center for Real Estate Finance Research.

1 Introduction

During the global financial crisis and ensuing recession, the Federal Reserve (Fed) expanded its monetary policy toolkit. In January 2009, the central bank began conducting large-scale asset purchases (LSAPs), buying not only U.S. Treasuries – the traditional securities in the Fed's portfolio – but also mortgage-backed securities (MBS) issued by Fannie Mae, Freddie Mac, and Ginnie Mae. As of August 2016, the Fed owns \$1.8 trillion worth of MBS and continues to reinvest the proceeds. Including \$623 billion of MBS owned by the U.S. Treasury through its conservatorship of Fannie and Freddie, the public sector owns \$2.4 trillion residential mortgage debt, or 24% of the U.S. total.

As the economic situation improves, the Fed expects to cease new purchases and allow its holdings to decrease. But so far, as Chair Yellen has argued, the Fed "has decided against this approach because [its] ability to predict the effects of changes in the balance sheet on the economy is less than that associated with changes in the federal funds rate." The goal of this paper is to expand that ability.

To understand how central bank ownership of mortgages affects the economy, one needs a general equilibrium model. Large scale credit market interventions change equilibrium prices. And they affect the tightness of borrowing constraints, providing an important additional transmission channel. This model must capture not only the role of mortgage markets in monetary policy transmission, but the intermediating role of financial institutions, which borrow short-term from savers to lend long-term to homeowners, and who are on the other side of the Fed's large-scale asset purchases.

I construct a new, rich model of the macroeconomy, which combines New Keynesian nominal frictions with financial frictions facing both levered homeowners and levered financial intermediaries. Nominal rigidities cause changes in aggregate demand to influence output in the short run, and provide a role for monetary policy. Financial frictions hamper the transmission of conventional monetary policy in bad times. Homeowners borrow from the financial sector to finance their housing and non-housing consumption subject to a constraint. Mortgages are nominal long-term contracts with options to prepay and to default. The financial sector finances its portfolio of mortgage bonds by issuing government-guaranteed deposits to savers, retaining at least as much equity as capital requirements mandate. Monopolistic wholesale firms employ households to produce differentiated intermediate goods, which retail firms then package into a final consumption good. The government consists of a fiscal authority and a monetary authority. The fiscal authority taxes labor income and profits to spend on government consumption, social transfers to agents, and occasional bailouts of the financial sector. It can run deficits by issuing government debt. The monetary authority sets nominal interest rates and carries out LSAPs.

A decline in aggregate productivity and an increase in cross-sectional dispersion of home values lead to lower output and lower house prices. With less equity in their homes, homeowners are more likely to default and less able to finance their consumption with new borrowing. Producers only partially lower prices in response to lower demand because of nominal price stickiness. As a result, consumption and output fall further. Yet the partial price adjustment results in lower inflation and raises the real cost of the borrowers' remaining mortgage obligations, additionally contributing to defaults. Moreover, defaults beget deadweight losses (costly processing and costly upkeep of foreclosed properties), meaning a smaller fraction of already smaller output is available for consumption. In response to lower output and inflation and to stimulate consumption, the central bank lowers the short-term nominal rate. This leads savers to consume more. Depending on the severity of the shock, losses suffered by financial intermediaries impair their balance sheets and occasionally lead to financial sector insolvency and government bailouts. Impaired financial intermediaries face binding constraints during a crisis and expect to continue to face them. This reduces their ability to smooth consumption, raising the conditional volatility of their stochastic discount factor. Becoming effectively more risk averse, they demand a higher spread to continue holding mortgages and do not pass through their cheaper cost of borrowing to borrower households. Therefore, the ability of conventional monetary policy to ameliorate mortgage crises by boosting short-term demand is limited when borrowing constraints facing homeowners and intermediaries bind.

These are the periods in which unconventional monetary policy can be beneficial. I study the effect resulting from an unexpected commencement of LSAPs in response to the onset of a mortgage crisis. I model the share of mortgage originations that the Fed purchases as an increasing function of the mortgage spread, an indicator of stress in the mortgage market and financial sector. I identify three channels through which this policy operates.

First, when constrained financial intermediaries are unable to lend, the monetary authority lends instead. This additional lending to borrowers leaves them with more resources out of which to consume, and its value in consumption units directly boosts demand in partial equilibrium. This is the consumption channel of unconventional monetary policy.

Second, when non-housing consumption becomes less scarce, real house prices go up. With more equity in their homes, homeowners default less and use the additional equity to borrow more, further increasing their consumption. This is the collateral channel. With more demand for their product, wholesale firms refrain from lowering prices as much, and a smaller drop in inflation keeps the real cost of outstanding mortgages more manageable for borrowers. Fewer defaults result in fewer foreclosures, saving extra resources from being spent on upkeep of intermediary-owned (REO) homes. Smaller portfolio losses for financial intermediaries make financial sector insolvency less likely. With more equity, intermediaries borrow more in deposits and lend more in the mortgage markets, such that their overall size is bigger, even though their intermediating function is now partially performed by the central bank.

Third, if in addition to announcing LSAPs today, the central bank provides LSAP guidance by committing to state-contingent purchases in the future whenever credit markets tighten again, the policy affects the economy through a third, expectations, channel. The commitment to intervene in future crises makes future defaults less likely. Expecting to keep their houses for longer, households anticipate a larger stream of discounted housing services in the future. The covariance between house prices and borrower consumption becomes lower, lowering the housing risk premium. Both effects lead to higher house prices. As in the collateral channel, homeowners consume out of the additional housing wealth, and the additional home equity discourages defaults.

As the economy recovers, the central bank reduces its balance sheet. The private sector must replace $2.4 trillion of public mortgage lending. I consider the kind of policy normalization anticipated by the Federal Reserve – a cessation of reinvestment, which leads to a gradual decline in Fed holdings as mortgages are amortized or prepaid. Because the central bank intervened in a crisis, less deleveraging took place in the downturn, and so the economy recovers with more debt. Higher mortgage debt makes borrowers less willing to consume. A bigger financial sector consumes more. So do savers, who hold more safe assets, and whose assets yield a higher return than if the LSAP intervention had not taken place. As the Fed ceases new purchases, newly originated mortgages are purchased by the financial sector, which, on the one hand, has a larger balance sheet immediately after the crisis, but, on the other hand, has to pay a higher rate on its deposits. The net effect on the quantity of originations is minimal, yet there are effects on prices. Some of the higher nominal rate is passed through to households in the form of a higher mortgage rate, while the mortgage spread that the financial sector gets to earn is lower. As a result, as the recovery continues, financial intermediaries become smaller.

As Chair Yellen indicated, even as the Fed normalizes its balance sheet, it may not eliminate its holdings of MBS altogether. When the next downturn hits, it “may need to purchase assets during future recessions to supplement conventional interest rate reductions.” The era of persistent and substantial non-private ownership of household debt may be here to stay. My paper sheds light on what this “new normal” may look like.

To understand the long-term effects of LSAPs, I compare an economy with no LSAPs to one where a state-contingent LSAP policy, procyclical in the credit spread, is known to households. Consistent with the expectations channel of LSAP transmission, the economy with procyclical purchases features higher house prices and lower defaults. Output and consumption are unchanged on average, but considerably less volatile. Smoother business and credit cycles reduce the savers' precautionary savings motive, leading them to demand a higher return on their assets. Financial intermediaries in part pass on the higher cost of deposit financing to borrowers, whose mortgage rates remain unchanged despite lower credit risk. The higher effective cost of mortgages and the lower spread earned by intermediaries results in redistribution from the two types of agents who borrow to the one type who exclusively lends.

The richness of the model allows for quantitative evaluation of the effects of unconventional monetary policy. In the main experiment of this paper, I calibrate the economy to match recent U.S. data. Given the sequence of shocks observed during the boom-bust-recovery period and the unanticipated commencement of LSAPs in 2008, I compare the evolution of house prices, mortgage debt stocks, default rates, and macroeconomic quantities in this period to their path in an alternative economy, in which no LSAPs took place. I find that, had the central bank not intervened during the crisis, house prices would have fallen by an additional 16 percentage points. Absent the intervention, Mortgage spreads would have been 70 basis points larger. Mortgage foreclosures would have been 4.5 times higher. Higher defaults would have caused the stock of mortgages to shrink by an additional 10%. Output would have fallen by an additional 0.1%. With more defaults, bankruptcy costs rise, constituting a higher fraction of output. As a result, aggregate consumption as a share of output would have fallen by an additional 3 percentage points, with almost the entirety of this drop borne by borrowers. Inflation would have been 40 basis points lower.

Without LSAPs, more mortgages get extinguished by default and fewer new ones are taken out, so that that the recovery begins with less household debt. Homeowner loan-to-value (LTV) ratio in the year after the onset of a crisis is 4 percentage points lower than in the economy with LSAPs, and LTV ratios remain lower for 4 years after the crisis. Borrowers have more debt not just relative to their home equity but also relative to their income. The debt-to-income (DTI) ratio is lower by 13 percentage points initially and it remains lower for 6 years. Less indebted households consume more, so without LSAPs, borrowers' consumption in recovery is 4.4% higher and remains higher for 5 years. The effect on savers' and intermediaries' consumption is the opposite but is smaller in magnitude, so that in an economy without LSAPs, aggregate consumption falls more in a crisis but grows faster in recovery. Boosted by demand, output in a no-LSAP recovery is 0.6% higher at first and remains higher for 5 years.

Differential dynamics of the two economies in recovery are partly symptomatic of the long-run differences between an economy in which the central bank exclusively targets nominal rates versus an economy in which the central bank responds to deterioriation in credit markets by directly purchasing assets. With LSAPs as the new normal, the financial system is less fragile, and negative shocks translate into smaller price drops, fewer defaults, and less frequently binding constraints. House prices are 4% higher and mortgage debt is 1% higher, while both are 40% less volatile. Stronger household balance sheets lead to safer financial sector balance sheets. Greater financial stability translates to smoother business cycles, as output, hours worked, and consumption are all 30% less volatile. Individual consumption is also less volatile, with borrowers especially benefiting. Lower risk in the economy weakens the precautionary savings motive of savers and raises the real rate they want to earn on deposits and government debt (17 bps). This pushes up inflation (+11 bps) and the nominal rate (+21 bps) unconditionally, and especially in crises (+1% and +2.5% respectively). Higher rates translate into a higher cost of borrowing for the banks, which take 2% fewer deposits, hold 1.3% less equity, and earn a smaller credit spread (-0.2%). The higher short rate and lower spread net out to leave the mortgage rate unchanged, despite borrowers defaulting less often. This creates long-run redistribution from borrowers and intermediaries, whose consumption is 0.9% and 5.8% lower on average, to savers, whose consumption is 0.4% higher.

The rest of the paper is structured as follows. Section 2 reviews related literature. Section 3 describes the model in detail. Section 4 explains the choices for parameter values. Section 5 illustrates the interaction between nominal and financial frictions in a model with only conventional monetary policy. Section 6 studies the effects of unanticipated introduction of LSAPs during a crisis. Section 7 compares economies with and without LSAPs over the long run. Section 8 concludes.

2 Review of Literature

Recent empirical studies have shown that housing wealth is an important driver of household consumption, and thus dynamics in housing prices and mortgage credit availability and affordability have causal effects on macroeconomic quantities like aggregate demand and employment.[1] By comparing rates and issuance volumes of similar securities around announcement times and across eligibility thresholds, new work finds that LSAPs had locally significant effects, lowering rates and expanding credit.[2] Other studies look at the time series of LSAPs and mortgage spreads, and find relatively small effects.[3] This paper proposes a general equilibrium framework to study LSAPs, identifying the channels through which it works and suggesting aggregate magnitudes of their effect.

My paper contributes to several strands of literature on housing finance, macroeconomic role of financial frictions, and the effects, both aggregate and distributional, of government policies aimed at smoothing business cycles.

The boom and bust episode in house prices and mortgage debt during the 2000s has prompted

[1] A partial list of this large literature includes Mian and Sufi (2011), Mian, Rao, and Sufi (2013), Mian, Sufi, and Verner (2015), Adelino, Schoar, and Severino (2015a), Adelino, Schoar, and Severino (2015b), Adelino, Schoar, and Severino (2016), Ramcharan, Kermani, and Maggio (2015), Aladangady (2014), Favara and Imbs (2015). Agarwal, Chomsisengphet, Mahoney, and Stroebel (2016) study the pass-through of credit expansions to households in the context of credit cards.

[2] Gagnon, Raskin, Remache, and Sack (2010), Di Maggio, Kermani, and Palmer (2016).

[3] E.g. Stroebel and Taylor (2012).

considerable interest from quantitative theorists.[4] One strand of the literature identifies channels which can generate the dynamics witnessed over this period.[5] More closely related to the current paper are studies of government policy in mortgage markets.[6] For tractability, these works typically abstract either from features of the mortgage contract, such as long-term duration and options to default and prepay, or treat output as exogenous. They are also written in real terms and make no predictions about inflation and aggregate demand. Greenwald (2016) studies the consequences of monetary and macroprudential policies for housing markets, working through credit constraints. My paper is unique among these to study nominal, long-term, defaultable, and prepayable mortgages in a production economy.

The importance of financial constraints in amplifying macroeconomic shocks is well-established.[7] Recent contributions to this literature have focused on non-linear price dynamics in crises,[8] identifying financial sector wealth as a key state variable in the amplification of shocks and their effect on financial prices and macroeconomic quantities. This paper also features a levered financial sector, but here it has the option to default and be bailed out by the government. Furthermore, it is not only their balance sheets that matter. Borrower leverage, saver wealth, and government debt also play a role. Moreover, this paper focuses on aggregate demand effects of constrained consumption in a nominal model as opposed to aggregate supply effects of constrained production in a real model.

Lastly, this paper contributes to the growing literature on the effects of government policies in heterogeneous-agent economies. First, heterogeneity recognizes the limits on risk-sharing that exist in the data, and thus leaves room for government policy to improve risk sharing. Second, it allows one to analyze the policy's redistributionary effects. Auclert (2016) studies the redistributionary channel of conventional monetary policy. This paper shows short-run and long-run redistribution resulting from *un*conventional monetary policy. McKay and Reis (2016) measure the ability of countercyclical fiscal policy to smooth business cycle fluctuations. To the extent LSAPs distribute resources to one set of households, financing them with government debt

[4]For a review, see Davis and Van Nieuwerburgh (2014).

[5]Works include Favilukis, Ludvigson, and Van Nieuwerburgh (2010), Landvoigt (2014), Chu (2014), Landvoigt, Piazzesi, and Schneider (2015), and Kaplan, Mitman, and Violante (2015).

[6]Examples include Elenev, Landvoigt, and Van Nieuwerburgh (2016), Jeske, Krueger, and Mitman (2013), and Floetotto, Kirker, and Stroebel (2016)

[7]See Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999) [BGG]

[8]See Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), Adrian and Boyarchenko (2012), and Drechsler, Savov, and Schnabl (2014).

purchased by a different set of households, they share some of the properties of fiscal stabilizers. This paper does not have as rich of a framework of idiosyncratic risk, but features an explicit financial sector and a rich model of the main debt (mortgage) contract.

Most closely related to my paper are the work of Gertler and Karadi (2011) and Curdia and Woodford (2011), who also study unconventional monetary policy in an economy with financial frictions. Gertler and Karadi (2011)'s framework differs from my work along several important dimensions, which are relevant to the U.S. experience with LSAPs. First, the financial sector is not the ultimate borrower, but rather an intermediary between end borrowers and savers. Central bank intervention thus occurs not in the riskless short-term borrowing market but in the risky long-term market. Second, the financial sector finances household consumption, not firm investment; firms are unlevered in my model. Hence, the central bank purchases household debt, not firm debt. Third, borrowing constraints bind only occasionally, and the transition from slack to binding constraints produces strongly amplifying nonlinearities. The economy in Curdia and Woodford (2011) also features intermediaries lending to households, but household debt is short-term and unsecured, defaults are exogenous, and intermediaries' constraints always bind. The channels through which LSAPs work identified in my paper rely on these additional modeling features.

3 Model

I construct a New Keynesian-style production economy populated by three types of agents – homeowners/borrowers, investors/savers, and financial intermediaries/bankers – two types of firms, and a government. Borrowers take out mortgages from bankers to finance housing and non-housing consumption. Bankers finance their mortgage lending with own wealth and short-term bonds ("deposits") owned by savers. In addition to holding deposits, savers own wholesale and retail firms, much like representative agents in standard New Keynesian Models. That is, monopolistic wholesale firms employ workers and set prices subject to menu costs to produce differentiated intermediate goods, and competitive retail firms package intermediate goods into the final consumption good. The government performs its traditional fiscal function of taxing income and spending on social transfers and government consumption. In addition, it guarantees bank deposits and conducts monetary policy by changing the short-term nominal rate ("conventional monetary policy") and by purchasing mortgages ("large-scale asset purchases" (LSAPs) or "unconventional monetary policy").

Time is discrete and is indexed by $t = 0, 1, 2, \ldots$ There are three types of agents $j \in \{B, I, S\}$ with recursive multiplier preferences over a numeraire consumption good, housing, and labor.[9]

$$\begin{aligned}
V_t^j &= (1-\beta_j)u_t^j + \beta CE_t^j \\
CE_t^j &= -\frac{1}{\sigma_j - 1}\log \mathrm{E}_t\left[\exp\left(-(\sigma_j-1)V_{t+1}^j\right)\right] \\
u_t^S &= (1-\theta)\log C_t^S + \theta \log h_t(H_t^S) - \chi_0 \frac{(L_t^S)^{1+\chi}}{1+\chi} \\
u_t^j &= (1-\theta)\log C_t^j + \theta \log h_t(H_t^j) \qquad \text{for } j = B, I
\end{aligned}$$

where $h_t(H_t^j)$ is the service flow from H_t^j units of housing, θ is the aggregation parameter for bundles of housing and non-housing consumption, χ determines the elasticity of savers' labor supply, and χ_0 weighs consumption against leisure. Borrowers and intermediaries supply labor inelastically. β_j is the impatience parameter, σ_j is a measure of risk aversion.[10]

All agents use their stock of houses to produce a housing service flow, which grows over time with Z_t.

$$h_t(H_t^j) = A_H Z_t H_t^j$$

3.1 Borrowers

I begin by describing the intuition behind the borrowers' problem before stating it formally.

Borrowers are impatient (low β^B) and moderately risk-averse (moderately high σ^B). They work for wholesale firms, buy houses, and finance their purchases with labor earnings and long-term, defaultable, and prepayable mortgages. Between themselves, borrowers can trade the full menu of state-contingent contracts with one exception – they cannot pay off each others' mortgages. As a result, borrowers begin the period with individual stocks of houses and mortgages,

[9] The log specification of period utility implies a value of 1 for the elasticity of intertemporal substitution (EIS) and renders the preferences consistent with a balanced growth path (Rudebusch and Swanson (2012)).

[10] When labor supply is elastic, households have an extra margin along which to smooth their consumption, and so their actual coefficient of relative risk aversion (CRRA) will be lower than σ_j by a wedge that depends on the elasticity of labor supply.

make the individual choice to default or not default, but subsequently pool their resources together as a family, deciding how much to work, how many new houses to buy, and how many mortgages to issue. This setup implies perfect consumption risk-sharing within the borrower family and greatly simplifies the analysis.

Each borrower i begins the period with housing and an outstanding mortgage balance. She draws a housing maintenance shock ω_i from the distribution F_ω with mean μ_ω and cross-sectional dispersion $\sigma_{\omega,t}$, which determines what fraction $1-\omega_i$ of the home value she needs to pay to maintain her house. If the required maintenance payment is too large relative to the value of their house and the indebtedness of the borrower, she defaults and is foreclosed on, shedding the mortgage but also losing the house.

Given the consumption insurance available to the borrower family, the borrowers' problem can be stated in aggregate ("borrower") terms. Let H_t^B be the quantity of housing shares worth q_t^H consumption units per share and A_t^B the quantity of mortgage bonds worth $q_t^{\$,B}$ dollars per bond, taken out by the borrower family as a whole.[11]

Mortgage contract In this model, a mortgage bond is a geometrically decaying perpetuity parameterized by duration parameter δ_B and "principal" $F^\$$, committing the mortgagor to make annual nominal coupon payments of $1, \delta_B, \delta_B^2, \ldots$. By comparing the principal $F^\$$ to the sum of all coupon payments $\frac{1}{1-\delta_B}$, the current coupon payment of 1 can be broken into a principal component $F^\$(1-\delta_B)$ and a tax-deductible interest component $1-F^\$(1-\delta_B)$. The borrower has the option to default, foregoing current and future coupon payments. The borrower also has the option to extinguish the mortgage by paying the principal $F^\$$ instead of the market price $q^{B,\$}$.

Maintenance Shocks and Default Given the distribution of after-maintenance payment home values $\omega_{i,t} q_t^H H_t^B$, the borrower decides which houses to default on. The relative benefit vs. cost of default decreases in ω so the optimal default policy for the borrower is to pick a threshold ω_t^* and to default on all houses, which cost more than $1-\omega_t^*$ to maintain. Define the fraction of

[11] I use \$ to denote nominal prices in dollars and thus distinguish them from real prices in units of the consumption good.

mortgages repaid as $Z_A(\omega_t^*)$ and the real value of homes retained as $Z_H(\omega_t^*)q_t^H H_t^B$:

$$Z_A(\omega_t^*) = \int_{\omega_t^*}^{\infty} dF(\omega)$$
$$Z_H(\omega_t^*) = \int_{\omega_t^*}^{\infty} \omega dF(\omega)$$

Prepayment Next, the borrower chooses to prepay some quantity R_t^B of the remaining (undefaulted) $Z_A(\omega_t^*)\delta_B A_t^B$ mortgages, either to reduce her leverage or to take advantage of lower rates (higher prices) by refinancing. Prepayment is costly, and the cost of prepayment $\Psi(R_t^B, A_t^B)$ is convex in the share of mortgages prepaid R_t^B/A_t^B to capture the congestion in the mortgage processing industry when too many prepayments need to be processed. Prepayments cannot be negative nor can they exceed the remaining mortgage balance:

$$0 \leq R_t^B \leq \delta_B Z_A(\omega_t^*) A_t^B$$

Having made default and prepayment decisions, the borrower has a quantity of houses $Z_K(\omega_t^*)H_t^B$ and mortgages $Z_A(\omega_t^*)\delta_B A_t^B - R_t^B$. Given real house prices q_t^H, nominal mortgage prices $q_t^{\$,B}$, real wages w_t^B, and price level P_t, she now chooses how many houses to buy H_{t+1}^B, how many new mortgages to take out B_t^B, and thus how much to consume C_t^B.

As the nominal budget constraint shows, borrowers use after-tax labor income $(1-\tau_t^B)P_t w_t^B L_t^B$, social security transfers from the government $P_t T_t^B$, and new borrowings $q_t^B B_t^B$ to finance consumption expenditures $P_t C_t^B$, after-tax coupon payments $(1-\tau_t^M)Z_A(\omega_t^*)A_t^B$, prepayments $FR_t^B + \Psi(R_t^B, A_t^B)$, and new house purchases $q_t^H(H_{t+1}^B - Z_H(\omega_t^*)H_t^B)$.

$$(1-\tau_t^B)P_t w_t^B L_t^B + P_t T_t^B + q_t^B B_t^B =$$
$$P_t C_t^B + (1-\tau_t^M)Z_A(\omega_t^*)A_t^B + F^\$ R_t^B + \Psi(R_t^B, A_t^B) + q_t^H P_t(H_{t+1}^B - Z_K(\omega_t^*)H_t^B)$$

End-of-period borrower mortgage debt reflects default, prepayment, and new borrowing decisions:

$$A_{t+1}^B = \delta_B Z_A(\omega_t^*)A_t^B - R_t^B + B_t^B$$

It is subject to a maximum loan-to-value constraint, which restricts the book value of mortgages to be no greater than a fraction ϕ_B of the market value of houses:

$$\phi_B P_t q_t^H H_{t+1}^B \geq F^{\$} A_{t+1}^B$$

Like in the real world, changes in the value of a homeowner's mortgage due to changes in mortgage rates do not affect her ability to tap into home equity, but changes in house prices do.

3.2 Financial Intermediaries

Financial **I**ntermediaries (or "bankers") are patient (high β^I) and relatively risk-tolerant (low σ^I). Because they are patient, they lend to impatient borrowers in the mortgage market. Because they are relatively risk-tolerant, when lending they supplement own funds with funds raised from the savers in the deposit market. As a result, they provide levered intermediation between borrowers and savers.

Bankers begin the period with a portfolio of nominal mortgage bonds, nominal government bonds, and nominal deposits. As I explain later, government-provided liability insurance for banks mean that deposits are indistinguishable from short-term government debt. The bankers' total position in these two assets is B_t^I, where a net short position $B_t^I < 0$ means banks issue deposits.

Wealth The realization of TFP and uncertainty shocks determines prices, borrower default, and borrower prepayment decisions and hence the payoff on the bankers' mortgage bond portfolio A_t^I. Mortgage bonds provide three sources of cash flows: coupons, prepayments, and foreclosure sales of seized collateral. Borrowers who do not default pay a nominal coupon payment of 1, meaning that bankers' total coupon revenue is $Z_A(\omega_t^*)A_t^I$. Prepaying borrowers pay principal $F^{\$}$, so the total prepayment revenue is $F^{\$} Z_t^R A_t^I$. Bankers also foreclose on the homes of defaulting borrowers, performing the required maintenance, incurring an additional bankruptcy maintenance cost ζ proportional to the value of the seized house, and then selling them back. The nominal proceeds from the collateral sales are $(1-\zeta)(\mu_\omega - Z_H(\omega_t^*))P_t q_t^H H_t^B$, where $\mu_H - Z_H(\omega_t^*) < 1 - Z_A(\omega_t^*)$ due to selection effects: it is precisely the houses requiring the greatest maintenance that end up in default. After cash flows, remaining mortgage bonds $(\delta_B Z_A(\omega_t^*) - Z_t^R)A_t^I$ are identical to new issuances, and so their ex-coupon price is also $q_t^{\$,B}$. The bankers' total start-of-period wealth is the value of the mortgage portfolio plus the nominal amount 1 per each risk-free bond B_t^I:

$$W_t^I = Z_A(\omega_t^*)A_t^I + F^{\$}Z_t^R A_t^I + (1-\zeta)(\mu_\omega - Z_H(\omega_t^*))P_t q_t^H H_t^B + (\delta_B Z_A(\omega_t^*) - Z_t^R)q_t^{\$,B}A_t^I + B_t^I$$

It is more convenient to rewrite this as

$$W_t^I = \left[M_t^{\$} + Z_t^R(q_t^{\$,B} - F) + \delta_B Z_A(\omega_t^*)q_t^{\$,B}\right]A_t^I + B_t^I$$

where I define the per-bond nominal payoff as:

$$M_t = Z_A(\omega_t^*)A_t^I + (1-\zeta)(\mu_\omega - Z_H(\omega_t^*))\frac{P_t q_t^H H_t^B}{A_t^I}$$

Default Like the borrowers, bankers have an option to default and take advantage of a government bailout. If a banker chooses to default, both his assets and his liabilities are assumed by the government resetting his wealth to 0, while he pays a random utility penalty of default. The penalty in value function units is $\rho_t \sim F_\rho$ and is i.i.d. At the time of decision, its realization is unknown. Instead, bankers commit to a rule mapping the support of F_ρ into a binary decision whether to default. I guess here and verify in the appendix that the optimal default rule for bankers is also a threshold rule, committing to default if the penalty is lower than threshold level ρ_t^*. Since all bankers have the same portfolio and realize the same utility penalty, they make the same default decision, meaning financial sector bankruptcy is a systemic event.

Subsequently, bankers choose how much labor to supply, how much to consume, and what positions to take in mortgage bonds, trading at nominal price $q_t^{\$,B}$, and risk-free bonds, trading at a nominal price $q_t^{\$}$ i.e. a nominal short rate of $\frac{1}{q_t^{\$}}$. They spend their wealth, transfers from the government, and after-tax labor income on consumption, housing maintenance, and the new portfolio of assets. Their budget constraint is

$$W_t^I + P_t T_t^I + (1-\tau_t^I)P_t w_t^I L_t^I = P_t C_t^I + (1-\mu_\omega)P_t q_t^H H_t^I + q_t^{\$,B}A_{t+1}^I + q_t^{\$}B_{t+1}^I$$

Bankers can choose a negative position in risk-free bonds, but cannot short mortgages:

$$A_t^I \geq 0$$

If bankers choose to take a negative position in risk-free bonds i.e. issue deposits, they are subject to a capital requirement that at least $(1 - \phi_R)$ of their assets need to be backed by own capital. In other words,

$$\phi_R q_t^{\$,B} A_t^I \geq q_t^{\$} B_t^I$$

3.3 Savers

Savers are as patient as bankers (high $\beta^S = \beta^I$) but more risk-averse (high $\sigma^S > \sigma^R$). They invest in risk-free bonds, issued by the government or by banks, and in the market for shares of monopolistic wholesale firms, but do not buy mortgages.

Savers begin the period with a portfolio of stocks and bonds.[12] Given stock holdings A_t^S and bond holdings B_t^S, savers receive dividends per share $A_t^S D_t$ from firms and bond payments B_t^S from the government and bankers. Together with the ex-dividend value of their shares, this constitutes their nominal wealth:

$$W_t^S = (P_t D_t + P_t q_t^S) A_t^S + B_t^S$$

Savers choose how much labor to supply, how much to consume, and what positions to take in shares, trading at an ex-dividend real price of q_t^S, and risk-free bonds, trading at a nominal price $q_t^{\$}$. Like bankers, they spend their wealth, transfers from the government, and after-tax labor income on consumption, housing maintenance, and the new portfolio of assets. Their budget constraint is

$$W_t^S + P_t T_t^S + (1 - \tau_t^S) P_t w_t^S L_t^S = P_t C_t^S + (1 - \mu_\omega) P_t q_t^H H_t^S + P_t q_t^S A_{t+1}^S + q_t^{\$} B_{t+1}^S$$

[12] Savers own shares in all wholesale firms, indexed by i. As I show later, despite these firms having market power, they face identical problems, make identical choices, and pay identical dividends, so in this section I suppress the i notation and consider the savers' total position in all firms. The precise derivation of the savers' problem is in the Appendix.

and they face no-shorting constraints for all assets:

$$A^S_{t+1} \geq 0$$

$$B^S_{t+1} \geq 0$$

3.4 Firms

The production sector is a typical one in New Keynesian models. Wholesale firms, indexed by i, use capital and labor to produce differentiated intermediate goods, which retail firms then package into the final consumption good. Differentiation of intermediate goods gives wholesale firms market power over perfectly competitive retail firms.

3.4.1 Retail firms

Final goods are produced by a representative firm using intermediate goods as inputs. Its production function is

$$Y_t = \left(\int_0^1 Y^R_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

where is ϵ is the elasticity of substitution. Retail firms maximize their nominal profit

$$P_t Y_t - \int_0^1 P_t(i) Y^R_t(i) di$$

which free entry ensures to be zero. Profit maximization yields a demand for each good i given by

$$Y^R_t(i) = \left(\frac{P_t}{P_t(i)} \right)^{\epsilon} Y_t$$

3.4.2 Wholesale firms

Intermediate goods are differentiated and produced by a continuum of monopolistic firms indexed by $i \in [0,1]$.Each firm i uses own firm-specific capital $K_t(i)$ and hires $N^B_t(i)$, $N^I_t(i)$, and $N^S_t(i)$

units of labor of each agent's type to produce

$$Y_t(i) = Z_t^Y K_t(i)^\alpha \left(Z_t (N_t^B(i))^{\gamma_B} (N_t^I(i))^{\gamma_I} (N_t^S(i))^{1-\gamma_B-\gamma_I}\right)^{1-\alpha}$$

units of intermediate good i subject to technology shocks. Total factor productivity Z_t^Y follows an AR(1) process with a transitory shock:

$$\log Z_t^Y = \rho_{z,Y} Z_{t-1}^Y + \sigma_{z,Y} \epsilon_t^{z,Y}$$

Labor-augmenting productivity Z_t follows a deterministic trend g.

$$\log Z_t = \log Z_{t-1} + g$$

Being a monopolist in good i, the firm chooses the price $P_t(i)$ at which to sell its output to the final goods producer, given its demand curve and subject to a menu cost $Z_t \Xi \left(P_t(i)/P_{t-1}(i)\right)$, which grows along with the economy. It uses revenues net of menu costs to pay wages and invest in new capital I_t. Investment is subject to an adjustment cost $\Xi_K(I_t/K_t)$, and capital efficiency grows with Z_t. The law of motion for capital efficiency units is:

$$K_{t+1} Z_{t+1} = (1 - \delta_K) Z_t K_t + Z_t I_t$$

After menu, labor, and investment costs, the firm is left with a nominal profit

$$P_t D_t(i) = P_t(i) Y_t(i) - P_t Z_t \Xi(P_t(i)/P_{t-1}(i)) - P_t \sum_{j \in \{B,I,S\}} w_t^j N_t^j(i) - I_t^K - \Xi_K(I_t/K_t) K_t$$

which it pays out to stockholders as a dividend.

Menu cost Ξ is nonnegative and convex, and the cost-minimizing policy for the firm is to let prices grow at the government's target inflation rate i.e. $\Xi(\bar{\Pi}) = 0$. Investment cost Ξ_K is also nonnegative and convex, and the cost-minimizing policy for the firm is to choose replacement-level investment i.e. $\Xi_K(\delta_K) = 0$.

The firms' objective is to maximize their stream of dividends discounted at their owners' –

the savers – stochastic discount factor $\mathcal{M}^S_{t+1}$:

$$V^F(P_{t-1}(i), K_t(i)) = \max_{P_t(i), Y_t(i), N^B_t(i), N^R_t(i), N^S_t(i)} \left\{ D_t(i) + \mathrm{E}_t \left[\mathcal{M}^S_{t+1} V^F_{t+1}(P_t(i), K_{t+1}(i)) \right] \right\}$$

Particularly, their choice of prices trades off monopoly rents today against price adjustment costs in the current and next periods. I show in the Appendix that the problems faced by all firms are identical and their choice of price $P_t(i) = P_t(j) = P_t$ is governed by a standard New Keynesian Phillips Curve.

3.5 Government

The government sets fiscal and monetary policy.

Fiscal policy Fiscal policy consists of tax rates, government spending, and debt issuance. The government taxes labor and dividend income:

$$T_t = \sum_j \tau^j_t w^j_t L^j_t + \tau_D D_t$$

and uses the proceeds to finance government consumption and lump-sum transfers to agents:

$$G_t = G^o_t + \sum_j \ell_j G^{T,j}_t$$

where ℓ_B, ℓ_I, and ℓ_S are population shares. The government finances primary deficits and repayments of maturing debt with new short-term nominal debt. Denote its net (long - short) position in risk-free debt to be B^G_t. Then the budget constraint of the government arising purely from fiscal policy is

$$P_t T_t + B^G_t = G_t + q^\$_t B^G_{t+1}$$

Conventional monetary policy The government sets the nominal short rate following a version of the Taylor rule:

$$\frac{1}{q_t^{\$}} = \left[\frac{1}{q_{t-1}^{\$}}\right]^{\rho_q} \left[\frac{1}{\bar{q}^{\$}} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\pi} \left(\frac{Y_t/Z_t^Y}{\bar{Y}}\right)^{\gamma_y}\right]^{1-\rho_q} \exp(\sigma_q \epsilon_t^q)$$

The nominal rate depends on the previous period's nominal rate, the target rate in the current period, and a random disturbance. The target rate today depends on the target nominal rate (i.e. deterministic steady state real risk-free rate + target inflation), the deviation of inflation from target, and the deviation of production from its steady-state level. Conventional monetary policy is fully parametrized by $(\bar{q}^{\$}, \bar{\Pi}, \rho_q, \gamma_\pi, \gamma_y)$ and the process for ϵ_t^q.

Unconventional monetary policy Government may also pursue unconventional monetary policy by directly purchasing and holding mortgage bonds A_t^G. The goal of these large-scale asset purchases (LSAPs) is to reduce mortgage spreads and promote mortgage lending. Therefore, I parametrize LSAPs to be an increasing function of the mortgage spread i.e. difference between the long-term mortgage rate and the nominal short rate. To define a mortgage spread, I first introduce the notion of a gross mortgage rate $R_t^{b,\$}$ as the price-implied yield on the geometrically decaying mortgage bond cash flows absent defaults and prepayments:

$$q_t^{\$,B} = \frac{1 + \delta_B q_t^{\$,B}}{R_t^{b,\$}} \Leftrightarrow R_t^{b,\$} = \frac{1}{q_t^{\$,B}} + \delta_B$$

Then the log mortgage spread is given by

$$s_t = \log R_t^{b,\$} + \log q_t^{\$}$$

The government buys a fraction X_t of new mortgage originations B_t^B, where X_t is given by

$$X_t = \frac{\exp(x_t)}{1 + \exp(x_t)} X_{\max}$$
$$x_t = \bar{x} + \gamma_x (s_t - \bar{s})$$

A positive value for γ_x indicates that when the mortgage spread is high, the government purchases more mortgages than when it is low, with the magnitude of γ_x governing the intensity of government response. $\bar{x}$ governs the (low) steady-state level of government mortgage holdings. The total share of government purchases stays strictly bounded by 0 and $X_{\max}$ for all levels of the mortgage spread. In other words, the government never sells off its current portfolio. Even a sharp decrease in mortgage spreads precipitates a gradual roll off of the government's mortgage position as borrowers amortize and prepay their loans. On the other hand, even at large levels, the government leaves more than $1 - X_{\max}$ of mortgages outstanding in private hands.

A government which started the period with mortgage holdings A_t^G, ends the period with

$$A_{t+1}^G = \max\{\delta_B[Z_A(\omega_t^*) - Z_t^R]A_t^G + X_t B_t^B, 0\}$$

Parameters $(\bar{x}, \gamma_x, X_{\max})$ fully describe unconventional monetary policy.

Consolidated Government Sector Unconventional monetary policy augments the government's budget constraint with payoffs and prices of mortgage holdings. Defining government's wealth consistent with the definition of intermediary wealth,

$$W_t^G = \left[M_t^\$ + Z_t^R(q_t^{\$,B} - F^\$) + \delta_B Z_A(\omega_t^*)q_t^{\$,B}\right] A_t^G + B_t^G$$

Then the government budget constraint is

$$W_t^G + T_t = G_t + q_t^\$ B_{t+1}^G + q_t^{\$,B} A_{t+1}^G$$

The government is unconstrained, so, unlike for bankers, the space (B_{t+1}^G, A_{t+1}^G) is not restricted by a capital requirement.

In this model, both fiscal and monetary policies are performed by the same government entity. In the data, fiscal policy is implemented by the Treasury, while monetary policy is the purview of the Federal Reserve System. Large-scale asset purchases were conducted by the Fed, consisted of both Treasuries and MBS purchases, and were funded by an expansion in the Fed's balance sheet and the growth in bank reserves. In this model, LSAPs are conducted by the government, consist only of MBS, and are funded by issuance of government debt. Yet an alternative model with disaggregated Fed and Treasury balance sheets would produce the same prices and consumption allocations. The Appendix lays out the argument.

3.6 Equilibrium

Given a sequence of technology shocks $\{Z_t^Y\}$, credit shocks $\{\sigma_{\omega,t}\}$, utility cost of default shocks $\{\rho_t\}$ monetary policy shocks $\{\epsilon_t^q\}$, and a government policy defined by fiscal policy parameters $\left\{\{\tau_t^j\}_{j\in\{B,S,I\}}, \tau_D, G_t^o, \{T_t^j\}_{j\in\{B,S,I\}}\right\}$, conventional monetary policy parameters $\{\bar{q}^{\$}, \gamma_\pi, \gamma_y, \rho_q, \bar{\Pi}\}$, and unconventional monetary policy parameters $\{\bar{x}, \gamma_x, X_{\max}\}$, a competitive equilibrium is a sequence of borrower allocations $\{C_t^B, H_t^B, L_t^B, B_t^B, R_t^B\}$, borrower default policies $\{\omega_t^*\}$, banker allocations $\{C_t^I, L_t^I, A_t^I, B_t^I\}$, banker bankruptcy policies $\{\rho_t^*\}$, saver allocations $\{C_t^S, L_t^S, B_t^S, \{A_t^S(i)\}_{i\in[0,1]}\}$, wholesale firm allocations $\{\{Y_t(i)\}_{i\in[0,1]}, I_t^K(i), \{N_t^j(i)\}_{j\in\{B,S,I\}}\}_{i\in[0,1]}\}$, wholesale firm pricing decisions $\{\{P_t(i)\}_{i\in[0,1]}\}$, retail firm allocations $\{\{Y_t^R(i)\}_{i\in[0,1]}\}$ and prices $\{P_t, q_t^H, q_t^{\$,B}, q_t^{\$}, \{w_t^j\}_{j\in\{B,S,I\}}$ such that the allocations are optimal and the following markets clear:

1. Mortgages: $A_{t+1}^B = A_{t+1}^I + A_{t+1}^G$
2. Risk-free debt: $0 = B_{t+1}^I + B_{t+1}^S + B_{t+1}^G$
3. Houses: $\bar{H} = H_{t+1}^B + H_{t+1}^S + H_{t+1}^I$
4. Labor: $L_t^j = N_t^j$ for $j = B, S, I$
5. Intermediate goods: $Y_t^R(i) = Y_t(i)$ for $i \in [0,1]$
6. Consumption goods:

$$\underbrace{Y_t - \Xi(P_t, P_{t-1})Z_t - \zeta(\mu_H + Z_H(\omega_t^*))q_t^H H_t^B - \Psi(R_t^B, A_t^B)}_{GDP_t} =$$
$$\underbrace{C_t^B + C_t^S + C_t^I}_{C_t} + G_t^o + \underbrace{\int (I_t^K(i) + \Xi_K(I_t^K(i), K_t(i))K_t(i))di + q_t^H(1-\mu_\omega)H_t}_{I_t}$$

3.7 Computation

To make the problem more tractable by reducing the number of state variables, I make several assumptions, simplifying those features of the model, whose effects are well-known and are not the focus of this paper. First, I assume infinite capital adjustment costs, when non-residential investment deviates from its replacement level. This ensures $I_t = \delta_K \bar{K}$, where $\bar{K}$ is the steady state level of capital. Second, I assume no explicit Taylor rule persistence ($\rho_q = 0$). Third, I assume no uncertainty about monetary policy ($\sigma_q = 0$). These changes eliminate 1 shock and 2 state variables.

Yet solving this model is still computationally challenging. Because markets are incomplete, the wealth distribution affects equilibrium outcomes. In addition to the exogenous state variables Z_t^Y, and $\sigma_{\omega,t}$, there are 5 endogenous state variables: A_t^B, W_t^I, W_t^S, B_t^G, and A_t^G.

The presence of occasionally binding borrowing constraints for borrowers and bankers and the presence of prepayment constraints for borrowers makes the dynamics of the system non-linear and calls for a global solution as opposed to a Taylor expansion around a steady state. I define quantities, prices, value functions, and Lagrange multipliers as functions of the state variables, and approximate these functions on a fine rectangular grid with higher density of points in regions where non-linearities are most pronounced. Between grid points, the functions are approximated using linear interpolation. This interpolation results in superior accuracy.

At any point on the grid, the system is described by the agents' Euler Equations and market-clearing conditions. To solve the model, I iterate on this set of equations until equilibrium quantities converge. At every point in every iteration, the system of equilibrium conditions must be solved numerically. I implement this iteration scheme in such a way as to admit the use of an analytically computed Jacobian matrix, which significantly speeds up the calculations, making it feasible to use a very large ($> 100,000$) number of grid points.

To produce the results described below, once the model is solved, I compute the ergodic distributions of relevant quantities by simulating for 10,000 periods starting at the steady state of an equivalent deterministic (all variances set to zero) model and discarding the first 500 observations.

4 Calibration

I solve the model at an annual frequency. The parameters of the model and their targets or sources are summarized in Table 1.

Technology Labor-augmenting productivity follows a deterministic trend $Z_t = Z_{t-1}\exp(g)$. I set the value of g equal to mean real per capita U.S. GDP growth over the period 1985-2013,

Table 1: Calibration

Parameter	Description	Value	Target
Exogenous Shocks			
$\bar{g}$	mean income growth	1.47%	Mean rpc GDP gr 85-13
$\sigma_{Z,Y}$	vol. income growth	1.5%	Vol rpc HP GDP 85-13
$\rho_{Z,Y}$	persistence income growth	0.62	AC(1) rpc HP GDP 85-13
μ_ω	mean idio. depr. shock	2.5%	Housing depreciation Census
σ_ω	vol. idio. depr. shock	{0.10,0.14}	Mortgage default rates
p^ω_{LL}, p^ω_{HH}	transition prob	0.8,0.8	Frequency and duration of mortgage crises
Population, Income, and Housing Shares			
ℓ^j	pop. shares $j \in \{B, S, I\}$	{52.4,45.9,1.5}%	Population shares SCF 95-13
γ_j	inc. shares $j \in \{B, S, I\}$	{38,52,10}%	Labor inc shares SCF 95-13
H^j	housing shares $j \in \{B, S, I\}$	{40,47.7,12.3}%	Housing wealth shares SCF 95-13
Mortgages			
ζ	DWL of foreclosure	{0.25,0.425}	Mortgage severities
δ	average life mortgage pool	0.97	Duration Fcn.
κ	prepayment strike price	0.27	Duration Fcn.
ϕ_B	maximum LTV ratio	0.60	Borrowers' mortg. debt-to-inc. SCF 95-13
ψ	refinancing cost parameter	8	Mean Conditional Prepayment Rate
Preferences			
σ^B	risk aversion B	8	Vol househ. mortgage debt to GDP 85-13
β^B	time discount factor B	0.94	Mean housing wealth to GDP 85-13
θ	housing expenditure share	0.20	Housing expend. share NIPA
σ^S	risk aversion D	25	Vol. real rate 85-13
$\beta^S = \beta^I$	time discount factor S, I	0.99	Mean real rate 85-13
χ_0	disutility of labor	1.224	Rudebusch and Swanson (2012)
$1/\chi$	Frisch elast. of lab supply	3.6	Gertler and Karadi (2011)
σ^I	risk aversion I	1	Standard Value
ν	intertemp. elasticity of subst.	1	Standard Value
Production			
ϵ	elasticity of substitution	6	Standard value
ξ	Rotemberg adjustment cost	5%	Qtly Calvo 0.77 (standard)
δ_K	capital depr	7.2%	NIPA fixed asset depr 85-13
$\bar{K}/\bar{Y}$	captal / output	2.5	Standard value
α	capital share of input costs	0.2	Labor share of output 2/3
Government Policy			
τ	income tax rate	27%	BEA personal tax rev. to GDP 53-13
τ_D	income tax rate	23%	BEA corp tax rev. to GDP 53-13
G^o	exogenous govmt spending	17.58%	BEA govt. spending to GDP 53-13
G^T	govmt transfers to agents	3.19%	BEA govt. transfers to GDP 53-13
$\log \bar{\Pi}$	s.s. inflation	2%	Fed inflation target
γ_Π	Taylor rule, inflation	2.1	Vol inflation 85-13
γ_Y	Taylor rule, output	0.15	Vol nominal rate 85-13
$\bar{x}$	s.s. LSAP	-4.85	Avg Fed mtge holdings 85-13
γ_x	LSAP rule, mortgage spread	60	Fed mtge holdings, 08-10
ϕ_I	Capital req, mtge	96%	Basel reg. capital charge, mtges

which is 1.47%. This number is low compared to the average of a longer-time series. However, LSAPs interest monetary policymakers precisely because they may be a useful tool in a new era of lower growth.

The TFP process follows a mean-zero AR(1) process. I calibrate its standard deviation $\sigma_{z,Y} = 0.015$ to have detrended output volatility in the model match the Hodrick Prescott filter of real per capita U.S. GDP growth over the same period, which is 1.72%. Without endogenous capital accumulation and wage ridigities, it is difficult for the model to produce endogenous persistence of output commensurate with that in the data. Instead, I set the persistence of the TFP process to equal the persistence of de-trended GDP in the data: 0.62.

I use the Rouwenhorst (1995) method to discretize the process into a 5-state Markov Chain. The discretization produces both grid nodes and a Markov transition matrix, and the ergodic distribution of a simulated series exactly matches mean, standard deviation, and autocorrelation.

Credit shocks The second source of uncertainty stems from stochastic housing maintenance shocks. These shocks $\omega_{i,t}$, are drawn from a Gamma distribution characterized by shape and a scale parameters $(\chi_{t,0}, \chi_{t,1})$. The choice of distribution implies a closed-form solution for the optimal default threshold. I choose $\{\chi_{t,0}, \chi_{t,1}\}$ to keep the mean housing depreciation to be $1 - \mu_\omega = 0.025$, consistent with prior work (Tuzel (2009)). In a given period, draws from this distribution are i.i.d. and represent a source of idiosyncratic uncertainty. The cross-sectional standard deviation $\sigma_{t,\omega}$ follows a 2-state Markov chain, and its fluctuations represent the second source of aggregate risk. When $\sigma_{t,\omega}$ is high, the tail of the distribution beyond the default threshold is thicker, leading to more defaults. Hence, high $\sigma_{t,\omega}$ states indicate mortgage crises.

I set the two values $(\sigma_{H,\omega}, \sigma_{L,\omega}) = (0.10, 0.14)$ and the bankruptcy costs associated with foreclosure $(\zeta_H, \zeta_L) = (0.25, 0.425)$ in order to match the mortgage default rates and losses given default (LGD) in normal times and in mortgage crises. Given that I study a time period during which policy changed, I compare data moments to a weighted average of moments from the models with and without LSAPs, where the weights are determined by the fraction of the period 1985-2013 after LSAPs have been introduced. This procedure delivers an average default rate 0.66%, which rises to 1.1% in regular recessions and 13.6% in crises. The unconditional default rates are somewhat lower than in the data, and the crises default rates are somewhat higher than the data, partly because the model does not allow for a lag in processing mortgage delinquencies. Reassigning lingering default rates in the recent recovery to the height of the crisis renders these estimates consistent with the data.[13] The values for ζ imply equilibrium LGD of 30.5% in expansions, 28.3% in recessions, and 45.2% in crises. The unconditional LGD of 30.4% is consistent with typical values in the literature (Campbell, Giglio, and Pathak (2011)).

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, I assume that when aggregate TFP is at or above trend (the top 3 of 5 TFP states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. When TFP is below trend, I set transition probability parameters p^{ω}_{LL} and p^{ω}_{HH}[14] to match the frequency and length of mortgage crises, given the evidence from Jordà, Schularick, and Taylor (2015) and Reinhart and Rogoff (2009). 10% of all simulation states are mortgage crises. On average, crises last two years.

Production The elasticity of substitution between intermediate goods determines the degree of market power a monopolistic wholesale firm has, and thus the markup it can charge over marginal cost. I target the average markup of 1.2, standard in the literature, which implies an elasticity $\epsilon = 6$. Given the firm profits implied by this markup, I set the Cobb-Douglas weight on capital $\alpha = 0.2$ in the production function to generate a labor share of output of 2/3, standard in the literature. I set capital depreciation to 7.2% to match the fixed asset depreciation in the data, and target a capital-output ratio of 2.5, a standard value. Together, these two parameters imply annual investment fixed at 18% of trend GDP.

Adjusting prices is costly. For tractability, I assume a Rotemberg (1982) quadratic price adjustment cost:

$$\Xi\left(P_t, P_{t-1}\right) = \frac{\xi}{2}\left(\frac{P_t/P_{t-1}}{\bar{\Pi}} - 1\right)^2$$

and choose $\xi = 5$. In a standard DSGE model and given parameters β_S, g, and ϵ, this produces inflation dynamics equivalent to the alternative Calvo specification of price stickiness, where 61.6% of firms adjust prices every year. The equivalent quarterly number – 21.3% – is within the standard range in the DSGE literature.[15] Empirical evidence suggests that aggregate effects

[13]See Appendix C.2 in Elenev, Landvoigt, and Van Nieuwerburgh (2016).

[14]These are the probabilities of staying in the low or high state, respectively. Switching probabilties are one minus the staying probability.

[15]For instance, Gertler and Karadi (2011) use a value of 22.1%.

of infrequently changing retail prices are on the order of the estimates typically used in the New Keynesian literature.[16] In the simulation, the DWL of price adjustment never exceeds 0.7% of trend GDP, and 95% of the time is less than 0.1%.

Population and wealth shares To pin down the labor income and housing shares for borrowers, savers, and bankers, I calculate a net fixed-income position for each household in the Survey of Consumer Finances (SCF).[17] Net fixed income equals total bond and bond-equivalent holdings minus total debt. If this position is positive, the household is a saver, otherwise it is a borrower. For savers, I calculate the share of their total wealth comprised of risky assets, that is, holdings of stocks, business wealth, and real estate wealth. Since in the model intermediaries have a higher tolerance for risk than savers, I define intermediaries as households that are within the top 5% of risky asset holdings and have a risky asset share of at least 75%. This delivers population shares of $\ell^B = 52.4\%$, $\ell^S = 45.9\%$, and $\ell^I = 1.7\%$. Based on this classification, I set the housing wealth distribution (constant in equilibrium) $\{H^B, H^S, H^R\}$ and the relative efficiency of labor $\{\gamma_B, \gamma_S, \gamma_R\}$. Borrowers receive 45.6% of aggregate labor income and own 40% of residential real estate. Savers receive 48.9% of income and 47.7% of housing wealth. Finally, intermediaries receive 5.5% of income and 12.3% of housing wealth. The calibration implies consumption and wealth inequality, with borrows receiving the smallest per-capita labor income while bankers receive the highest.

Mortgage Contract In my model, the mortgage contract represents the aggregate mortgage debt owed by borrowers. The issuer of one bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower refinances the mortgage, she prepays a "principal repayment" $F = \frac{\kappa}{1-\delta}$, a constant parameter that does not depend on prevailing mortgage rates or house prices. I estimate values for δ and F such that the duration of the geometric mortgage in the model matches the duration of the portfolio of outstanding mortgage-backed securities, as measured by the Barclays MBS Index, across a range of historically observed nominal mortgage rates. This procedure recognizes that the mortgage in the model represents the pool of all outstanding mortgages of all vintages. I find that values of $\delta = 0.97$ and $\kappa = 0.27$ imply a relationship between price and mortgage rate for the geometric mortgage that closely

[16] Midrigan (2011)

[17] I average across all survey waves from 1995 until 2013.

matches the price-rate relationship for a real-life MBS pool of fixed-rate mortgages issues across a history of vintages.

Borrowers cannot obtain a mortgage with a face value in excess of a fraction ϕ_B of the market value of their house. I set the parameter $\phi = 0.60$ to match the average mortgage debt-to-income ratio for borrowers in the SCF of 133%. The calibration produces an unconditional mortgage debt-to-income ratio among borrowers of 148%, which is somewhat higher than the target. The model produces borrowers' mean loan-to-value ratios of 48.5% in book value and 57.8% in market value terms.

Lastly, I set the marginal prepayment cost parameter ψ to generate conditional prepayment rates (CPR) of 15% per year, the historical average. The model generates a CPR of 15.5%.

Fiscal Policy Fiscal policy consists of tax revenues – corporate profit taxes and labor income taxes net of a mortgage interest deduction – and fiscal spending – government consumption and social transfers.

I set the labor tax rate to $\tau = 27\%$ in order to match personal tax revenue to GDP in post-war U.S. data of 17.3%.[18] Net of the mortgage interest deduction, the model produces labor tax revenues of 18.7%. Both hours worked and wages are procyclical, yielding procyclical labor tax revenue. I set the corporate profit tax rate to $\tau^D = 23\%$ to match corporate tax revenue to GDP of 3.4%. The model produces 3.5%. [19] Dividends respond only mildly to the business cycle, and so total tax revenue is procyclical.

I set government consumption equal to $G^o = 17.58\%$ of trend GDP in order to exactly match

[18]For taxes and spending, I use the period 1953-2013. I do not use the shorter sample like I do for other moments, because near-constant deficit spending in the U.S. over the past several decades yields non-stationary dynamics for government debt.

[19]In a long enough simulation, there will be sequences of shocks, which could lead the government to run deficits or surpluses for many consecutive periods. To keep the ratio of government debt to GDP stationary, I decrease corporate tax rates τ_t^D when debt-to-GDP threatens falls below $\underline{b^G} = 15\%$ and increasing corporate tax rates when debt-to-GDP exceed $\overline{b^G} = 120\%$. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -5% (In the model, negative debt-to-GDP implies that the government is investing in bank deposits along with the savers). Tax rates are gradually and convexly increased until they hit 50% at a debt-to-GDP ratio of 160%. The simulations never reach the extreme -5% and +160% debt/GDP states. These tax policies do not affect the amount of resources that are available for private consumption in the economy. Furthermore, firms do not take these rates into account when setting prices. Because the adjustment is done only for corporate tax rates, the labor tax wedge remains constant. Savers anticipate fluctuations in their consumption resulting for changing after-tax dividends, but since the saver also owns all of the government debt in equilibrium, a government that raises corporate taxes to pay off its debt is only mildly contributing to redistribution.

average post-war government consumption to GDP in the data.[20] Agents derive no utility from government consumption. The government also distributes 3.19% of trend GDP as lump-sum per capita transfers to agents, which equals the net transfer spending in the 1953-2013 data. Spending remains constant across the business cycle i.e. goes up as percentage of GDP in recessions.

Conventional Monetary Policy The monetary policy is described by target trend inflation $\bar{\Pi}$, target natural rate of interest $\frac{1}{\bar{q}}$, and parameters γ_π and γ_y, which govern the strength of central bank response to deviations of inflation Π_t and production inputs Y_t/A_t from trend. I set $\bar{\Pi} = \exp(0.02)$, the Federal Reserve's inflation target. I set the target natural rate of interest to deterministic steady state real rate, given by the saver's impatience parameter β_S and trend productivity growth. The sensitivity parameters are chosen to match the volatility of inflation and nominal rates. $\gamma_\pi = 2.1$ and $\gamma_y = 0.15$ produce 2.34% volatility in the nominal rate and 1% volatility in inflation, close to the corresponding data value of 2.79% and 0.89%.

Large-Scale Asset Purchases LSAPs policy consists of a sequence of fractions X_t of new originations purchased by the government. In the benchmark economy, $X_t = 0$ for all t i.e. no LSAPs take place. In the economy with LSAPs, X_t is governed by $\bar{x}, \gamma_x$ and $X_{\max}$. I set $X_{\max}$ to $0.9 < 1$ as a technical assumption to ensure that the financial sector's Euler Equation (including collateral value when constraints bind) continues to price mortgages. I set $\bar{x} = -4.85$, which implies purchases of 0.7% of new originations when the mortgage spread is at its deterministic steady-state level. 0.7% is the average Federal Reserve MBS holdings over the calibration period – consisting of 0 holdings before the crisis, and substantial holdings thereafter. γ_x determines how many more mortgages the central bank buys in response to widening spreads. I set $\gamma_x = 60$ to target the total quantity of purchases observed in 2008-2010. In the data, Fed holdings of MBS peak at 25.1%. In the unanticipated introduction of LSAPs experiment below, Fed holdings peak at 17.5% of the total mortgage stock, somewhat lower than the target.

Financial Sector Solvency The market value of deposits, issued by the financial sector, must be no greater than a fraction ϕ_I of the market value of intermediary's mortgage portfolio. Under Basel II and III, "first liens on a single-family home that are prudently underwritten and

[20]The data are from Table 3.1 from the BEA (line 21).

performing" enjoy a 50% risk weight on a capital requirement of 8%. Accordingly, I set $\phi_I = 0.96$.

The model features a random utility penalty that intermediaries suffer when they default. Because random default is mostly a technical assumption to preserve differentiability and concavity of the problem, a small penalty is sufficient as long as it is volatile enough to accomplish its technical purpose. I assume ρ_t is normally distributed with a mean of $\mu_\rho = 0$ and a small standard deviation of $\sigma_\rho = 0.05$. The lower the mean penalty, the more likely a financial sector is to declare default.

Preferences Preference parameters affect most equilibrium quantities and are difficult to target. However, through agents' first-order conditions they have first-order effects on means and volatilities of prices, and I use these moments to set parameters.

The coefficients of risk aversion are $\sigma_I = 1$, $\sigma_B = 8$, and $\sigma_S = 25$. The high coefficient of the saver recognizes not just his high risk aversion but also his ability to smooth consumption by varying labor supply. The annual subjective time discount factors are $\beta_I = \beta_S = 0.99$ and $\beta_B = 0.94$.

Preferences of saver disproportionately affect the mean and volatility of the real rate. The model generates a one-year real rate of 2.79%, overshooting the one-year realized real rate in the data of 1.97% (computed as nominal rate less realized inflation), but its volatility of 3.06% is closer to the data value of 2.35%.

The borrower's discount factor governs house prices. In the model, housing wealth to trend GDP is 2.24, while in the Financial Accounts (1985-2013) it is 2.4. I set borrower risk aversion to target the volatility of the annual change in household mortgage debt to GDP (Financial Accounts and NIPA), which is 4.1% in the 1985-2013 data. The model overshoots this target, producing a volatility of 6.1%, but the volatility in just the economy with LSAPs matches the data exactly. This is the more relevant target because the unconditional volatility of mortgage debt to GDP growth is primarily driven by the recent crisis.

The financial intermediaries have log period utility and their subjective discount factor is set equal to that of the savers.

I set the elasticity of inter-temporal substitution equal to 1 for all agents, which is a common value in the asset pricing literature. It also allows my preference specification for savers to be

Table 2: Macro moments: Model vs. Data

	mean	stdev	corr(,GDP)
	Data (1985-2013)		
GDP		1.72%	1.000
Consumption		1.18%	0.877
Hours		2.74%	0.813
Nominal Rate	3.94%	2.79%	0.393
Inflation	2.49%	0.89%	0.627
	Benchmark Model		
Output		2.02%	0.969
GDP		2.76%	1.000
Consumption		2.79%	0.987
Hours		1.68%	0.718
Nominal Rate	4.66%	2.34%	0.587
Inflation	2.02%	1.01%	0.583

consistent with a balanced growth path, and implies a simple closed-form threshold rule for the intermediary's utility penalty of bankruptcy, making the model more tractable.

5 Results: Economy without LSAPs

I first present the moments of a 10,000 period simulation of the economy with LSAPs turned off.

Macro In Table 2, I compare macroeconomic moments to their data equivalents. As discussed in the Calibration section above, the model comes close to matching targets. Significantly, the model generates procyclical hours worked, whose correlation with GDP is 0.718, only slightly lower than 0.813 in the data. This illustrates the aggregate demand effect on output, generated by models with nominal frictions. Absent nominal rigidities, households with elastic labor supply respond to negative productivity shocks by working harder. But when prices do not fully adjust, aggregate demand drops, and at lower wages, the extra labor income isn't worth the disutility of labor. The model also matches the procyclicality of inflation (0.583 correlation with GDP vs. 0.627 in the data), but overshoots on the procyclicality of the nominal rate, the natural result of a simplified Taylor rule with no room for rate persistence or monetary policy shocks.

In addition to unconditional moments, I partition the set of simulation periods into expansions, recessions, and crises. Expansions are defined as periods when the total productivity (deterministic trend + persistent TFP) has increased or stayed the same – 75% of all states. Most of these states also have low dispersion of housing shocks $\sigma_{\omega,L}$. Some of these states do have a high dispersion $\sigma_{\omega,H}$, but positive TFP growth minimizes its impact. Effectively, when TFP is growing, credit shocks don't matter. But when TFP falls, they matter a great deal. Recessions are periods when dispersion of housing shocks is low but total productivity has declined – 22% of all states. Finally, crises are periods when the dispersion of housing shocks is high, this time coupled with a decline in TFP – the remaining 3.1% of all states. Conditional macro moments are presented in Table 3.

The economy exhibits limited risk-sharing. The fluctuations in aggregate consumption are smaller than the fluctuations in the consumptions of the two levered agents – borrowers and bankers. Savers do not face leverage constraints and hold safe assets, and thus experience smaller fluctuations in their consumption.

While real rates go negative on average in crises, nominal rates do not. Therefore, the limited effectiveness of conventional monetary policy, described below, does not rely on a binding zero-lower-bound (ZLB) on nominal rates. Unlike most of the literature that studies the role of unconventional monetary policy at the ZLB, this paper grounds the rationale for LSAP interventions in financial frictions and incomplete markets.

All crises are below-trend TFP states, whereas some recessions are not, explaining the larger negative deviation of crisis TFP (-2.0%) from trend than recession TFP (-0.7%). But the difference in the deviation of hours is even greater: -2.6% vs. -0.5%. Moreover, holding both TFP growth and current TFP constant, hours worked are 1.4% lower when σ_ω is high. This discrepancy identifies the credit channel effect on aggregate demand. To examine the mechanism, consider how crises affect individual balance sheets. Macrofinancial quantities are presented in Table 3.

Borrowers House prices are procyclical even absent credit shocks, but when the dispersion of housing shocks goes up, more homeowners find themselves in the left tail of the distribution of after-maintenance home values. As a result, they default. Defaults inundate the market with houses, that in equilibrium the borrowers must buy back. House prices fall by 18% to

Table 3: No LSAPs: Conditional Moments

	Unconditional	Expansions	Recessions	Crises
	mean	mean	mean	mean
		Macro		
TFP	1.000	+0.30%	-0.70%	-2.04%
Output	1.009	+0.46%	-1.14%	-4.13%
Hours	0.804	+0.24%	-0.53%	-2.62%
Consumption	0.590	+1.05%	-2.15%	-12.70%
- Borrower	0.225	+2.58%	-4.87%	-30.02%
- Banker	0.023	+0.88%	-6.59%	-37.58%
- Saver	0.342	+0.06%	-0.05%	+0.37%
Nominal Rate	4.66%	4.92%	4.25%	1.19%
Inflation	2.02%	2.13%	1.86%	0.52%
Real Rate	2.75%	3.08%	2.23%	-1.64%
		Borrower		
House prices	2.220	+1.71%	-3.33%	-17.91%
Market LTV	58.29%	56.89%	60.96%	73.22%
Book LTV	48.95%	47.46%	51.97%	63.66%
Debt-to-Income	147.47%	146.95%	150.24%	140.54%
Default rate	0.82%	0.02%	1.29%	17.03%
Mortgage rate	6.55%	6.52%	6.62%	6.89%
Fraction LTV constraint binds	3.63%	0.00%	9.45%	50.00%
Originations	0.122	+2.49%	-7.90%	-7.45%
Inflation	2.02%	2.13%	1.86%	0.52%
		Banker		
Loss-given-default rate	30.63%	30.69%	28.36%	45.43%
Loss Rate	0.34%	0.01%	0.39%	8.00%
Mkt fin leverage	93.13%	92.87%	93.76%	94.93%
Banker wealth	0.037	+5.84%	-10.00%	-52.79%
Bankruptcies	3.59%	0.00%	9.27%	50.00%
Fraction leverage constr binds	7.52%	3.66%	14.55%	50.97%
Short-term debt	0.503	+0.59%	+0.04%	-14.64%
mortgage spread	1.89%	1.60%	2.37%	5.71%
Expected Default rate	1.27%	1.25%	1.38%	0.81%
		Saver		
Bond holdings	1.890	+0.32%	-0.61%	-3.99%

accommodate the extra supply. As a result, loan-to-value ratios go up by 16 percentage points in book value terms. Higher LTVs push more borrowers into default and leave others constrained. Borrowers are never constrained in expansions, but during crises, they are constrained half the time. As a result, new originations drop. They drop more in recessions (-7.9%) than in crises (-7.5%) only because higher defaults wiped out a larger fraction of the existing mortgage stock, and defaulting borrowers seek new mortgages to finance the purchase of a new house to live in.

In sum, in crises borrowers have less resources to spend on consumption, and more of the resources they do have available must be spent on housing, rather than non-housing, consumption. Their demand for non-housing consumption good plummets by 30%.

Though firms do not fully adjust prices to avoid high menu costs, they increase them by less (0.5%) than they normally would (2%). With lower inflation, the real cost of household debt $q_t^B A_t^B / P_t$ is higher than expected. A lower ability to inflate away their debt lowers the option value of not defaulting, causing more borrowers to default, further pushing down house prices. The nexus of financial and nominal frictions generates a substantial downturn.

Financial Sector and Savers The wave of defaults caused by the credit shock causes the financial sector to bear 8% credit losses. Given their high leverage (93.1% on average), these losses wipe out half the financial sector wealth on average, and in half of the crises they are large enough to trigger financial sector insolvency and bailouts by the government. Bankruptcy or not, low equity leaves financial intermediaries unable to choose their optimal portfolio, because they don't have enough equity to back the deposits they wish to issue. Their borrowing constraints bind, and they are forced to reduce the amount of deposits they issue by 14.6% and slash their consumption by 38%. Lower demand for mortgages, which they must own in equilibrium, causes spreads to widen 410 basis points relative to expansions, despite expected defaults next period falling from 1.3% to 0.8%.[21] Higher spreads translate into a higher cost of borrowing for homeowners.

The contraction of deposits is partially offset by counter-cyclical budget deficits causing the government to issue more debt in crises. But the total amount of safe assets still falls by 4%. Together with an aggressive response by the central bank (nominal rate is lowered from 4.9%

[21] A wave of defaults this year implies less mortgage debt and lower LTVs at the start of next year, making the remaining mortgage portfolio safer.

in expansions to 1.2% in crises), this pushes the real rate down to -1.6%, and stimulates saver consumption (+0.37% relative to its unconditional mean). However, savers are patient and unconstrained. It is not their demand that is most in need of a boost from the central bank, but rather that of the borrowers.[22] The hopes of the financial sector passing on their lower cost of financing do not materialize because the financial sector is itself constrained.

The impaired balance sheets of households and financial intermediaries mean that conventional monetary policy is ineffective at preventing a large drop in aggregate demand, even without nominal rates hitting the zero lower bound. There is room for an alternative policy that will specifically target the market in which borrowers and financial intermediaries trade – the mortgage market – instead of relying on pass-through from the savers. One such policy is large-scale purchases of mortgages by the central bank.

6 Unanticipated Introduction of LSAPs in a Crisis

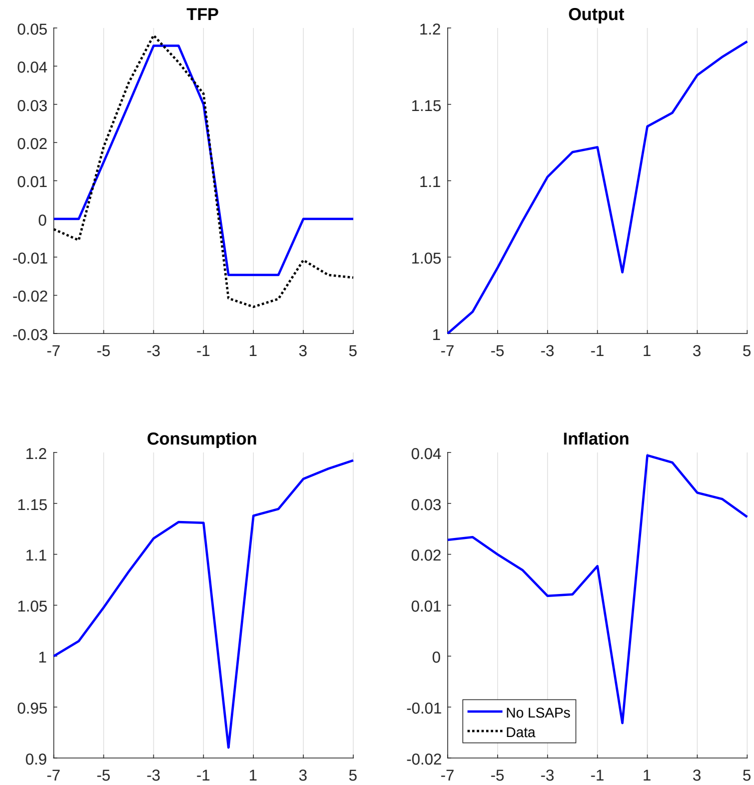
In this section, I provide answers to the first two questions motivating this paper. First, how would the macroeconomy have performed during the crisis in the absence of LSAPs? Second, what effect does policy normalization in the aftermath of a crisis have on the recovery?

To do that, I use the stochastic steady state of the long-run simulation of the no-LSAP economy from the previous section to initialize a separate simulation, into which I feed a set of TFP and credit shocks that reflect the dynamics of the U.S. GDP from 2001 to 2013. Year 0 denotes the onset of a mortgage crisis (high σ_ω), broadly corresponding to 2008 in the data. The TFP shocks used for this experiment are shown in Figure 1, along with the macroeconomic dynamics they generate *in the absence of LSAPs.* Compared to the subsample comparisons of the previous section, the crisis in period 0 is more severe. A TFP drop of 3% relative to the previous year, coupled with the onset of a mortgage crisis, causes a severe consumption drop of 20%. Large deadweight losses from foreclosure and unchanged investment and government consumption prevent a similarly sized drop in output, which drops “only” by 8.6%.

House prices drop by 40%. Loan-to-value ratios jump from 57% to 90%. Nearly a third of

[22] The drop in financial intermediaries' demand is also large and indeed greater in percentage terms than the drop in borrowers' demand. However, financial intermediaries constitute a small fraction of the population, so the drop in aggregate demand is mainly driven by borrowers.

Figure 1: Dynamics: No LSAPs: Macro



homeowners default, and aggregate debt to income drops by 20 percentage points (Figure 3). More than 15% losses wipe out financial sector equity triggering bankruptcy. Afterwards,the bailed-out banks face binding capital requirements, which force them to lower their deposit base (Figure 2). As a result, the aggressive conventional monetary policy response (nominal rates are lowered by 7 percentage points) does not get passed through to the borrower, offset by spiking mortgage spreads.

These dynamics are extreme, far greater than what was observed in the data. The next set of results suggests that one reason we did not observe this level of distress is timely intervention by the central bank directly into mortgage markets.

I take the aggregate state of the no-LSAP economy at the end of period -1, pass in the same severe TFP and credit shock in period 0, but evaluate the transition to an economy where the central bank regularly pursues LSAPs in response to widening mortgage spreads. Large spreads

Figure 2: Dynamics: No LSAPs: Markets

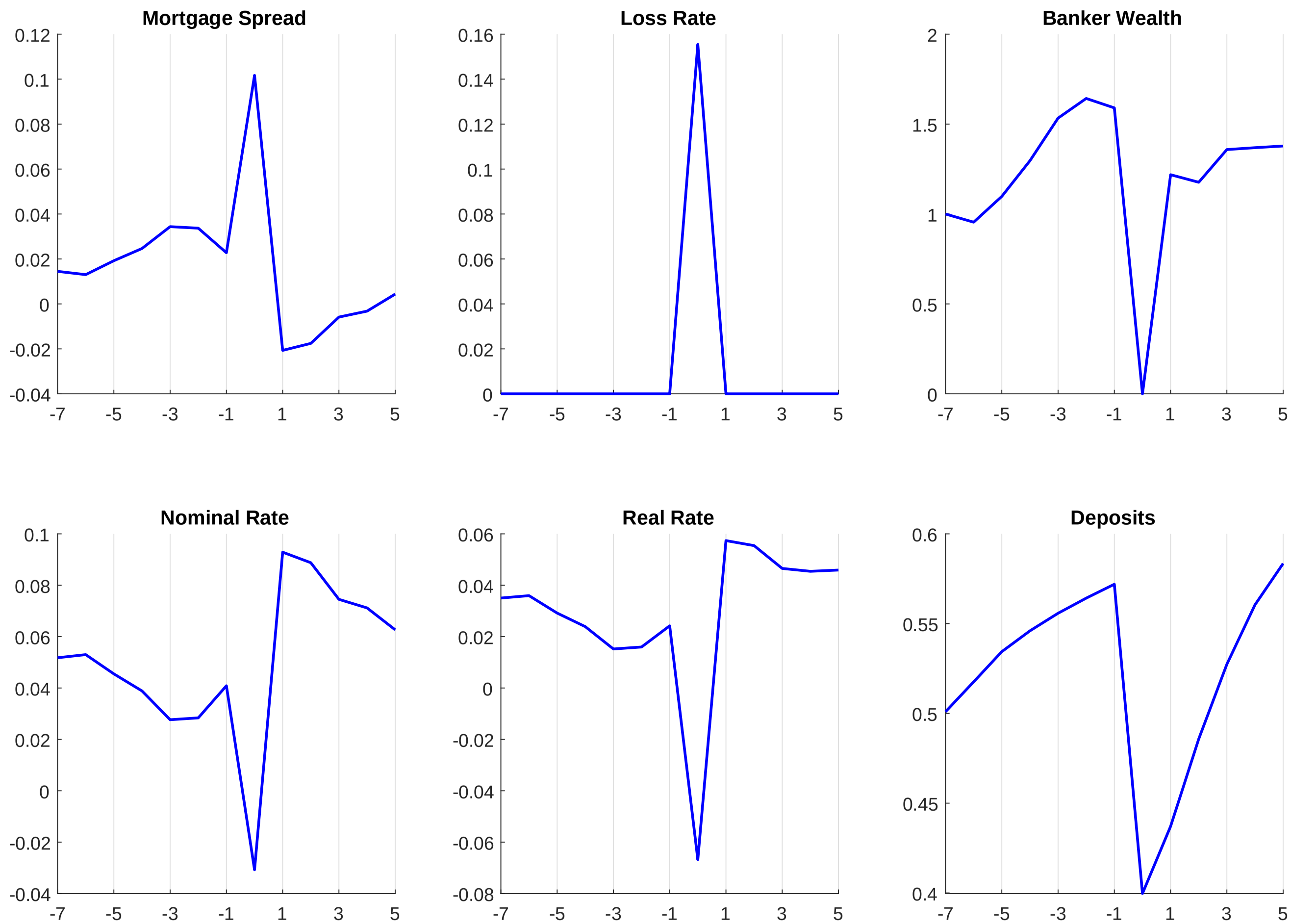
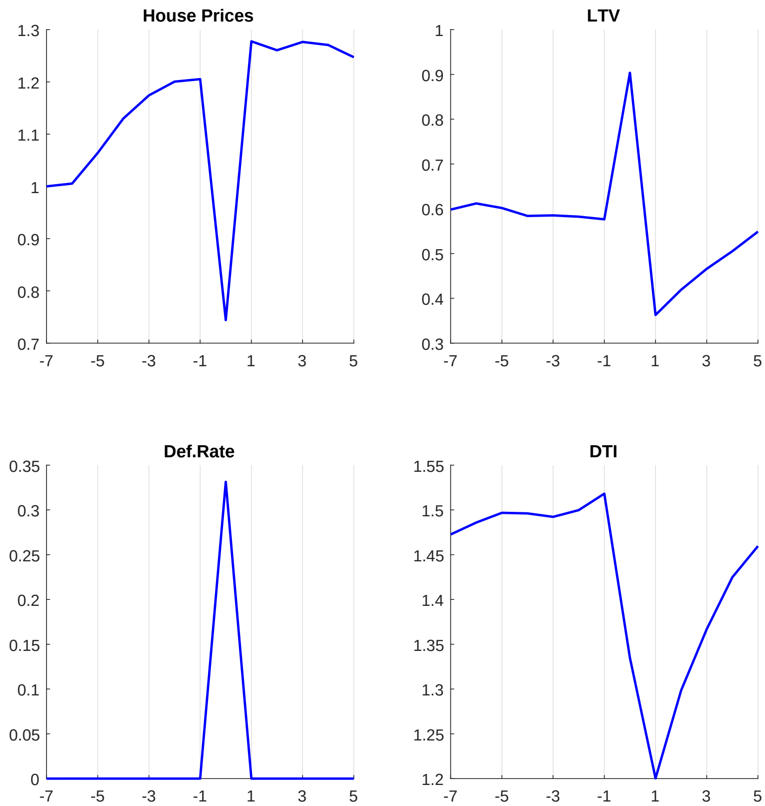


Figure 3: Dynamics: No LSAPs: Borrower



in period 0 imply purchases of 38% of new originations, in addition to policy guidance, which commits the central bank to make additional purchases the next time spreads are large. The dynamics of purchases and subsequent holdings are shown in Figure 4.

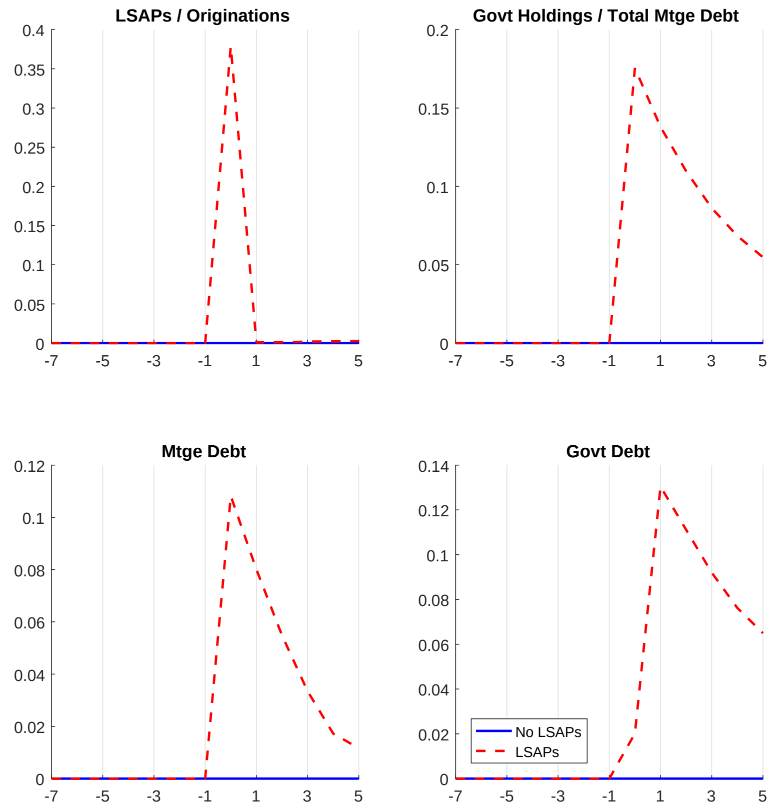
The net lending from the central bank to borrowers is small – 0.6% of trend GDP, largely because the central bank buys mortgages with the same down payment as the mortgages bought by the private sector.[23] The proceeds from this additional lending directly boost demand in partial equilibrium. This is the consumption channel of unconventional monetary policy.

But while the consumption channel has a modest effect, the combined effect of the collateral and expectations channels is great.

When non-housing consumption becomes less scarce, real house prices go up. Figure 5 shows dynamics of the transition to an LSAP economy (red dashed line) relative to the benchmark no-LSAP economy (solid blue line, fixed at 0 for a clear visual illustration of the relative effect).

[23]This is consistent with the data. MBS purchased by the Fed consisted only of conforming loans.

Figure 4: Dynamics: Comparison: Government

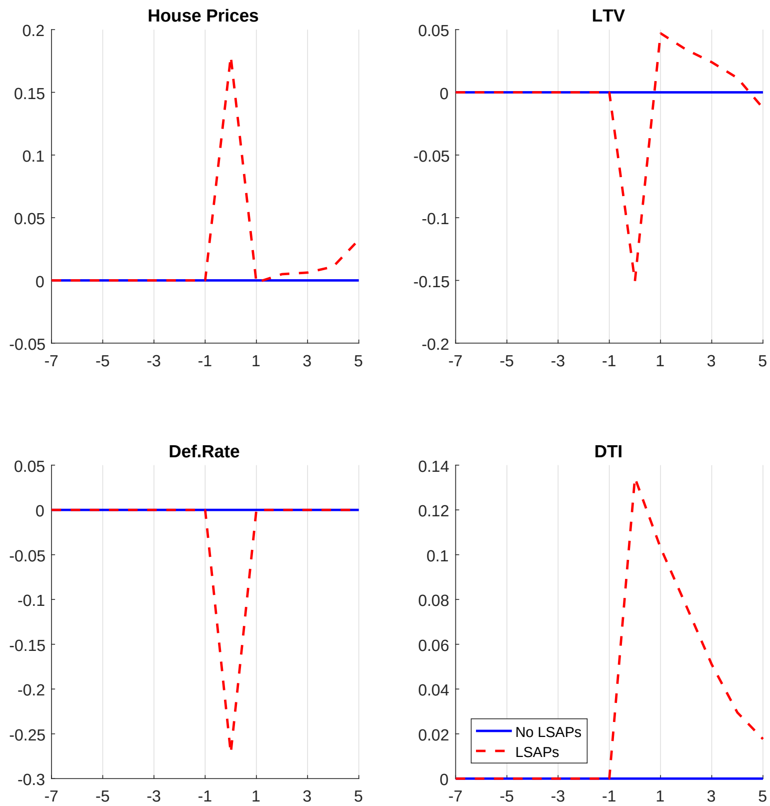


While house prices still fall, LSAPs boost them by nearly 18%. With more equity in their homes (LTV of 75% instead of 90%), fewer homeowners default (6% instead of 33%) and use the additional equity to borrow more (debt-to-income of 147% instead of 133%).

Fewer defaults result in fewer foreclosures, saving extra resources from being spent on upkeep of intermediary-owned (REO) homes (1.1% of trend GDP instead of 6.2%). While intermediary losses are smaller, they are still large enough in this case that the intermediaries still declare bankruptcy – LSAPs reduce the likelihood of bankruptcy but don't eliminate it entirely.

With more demand for their product, wholesale firms refrain from lowering prices as much (deflation of -0.9% instead of -1.3%), keeping the real cost of outstanding mortgages more manageable for borrowers. Less deflation allows the central bank to lower the nominal rate less (-2.2% vs. -3.1%), suggesting a key interaction between conventional and unconventional monetary policy. Intuitively, the goal of lowering rates is to boost consumption. Without LSAPs, impaired balance sheets of both borrowers and the financial sector meant that low rates were not going

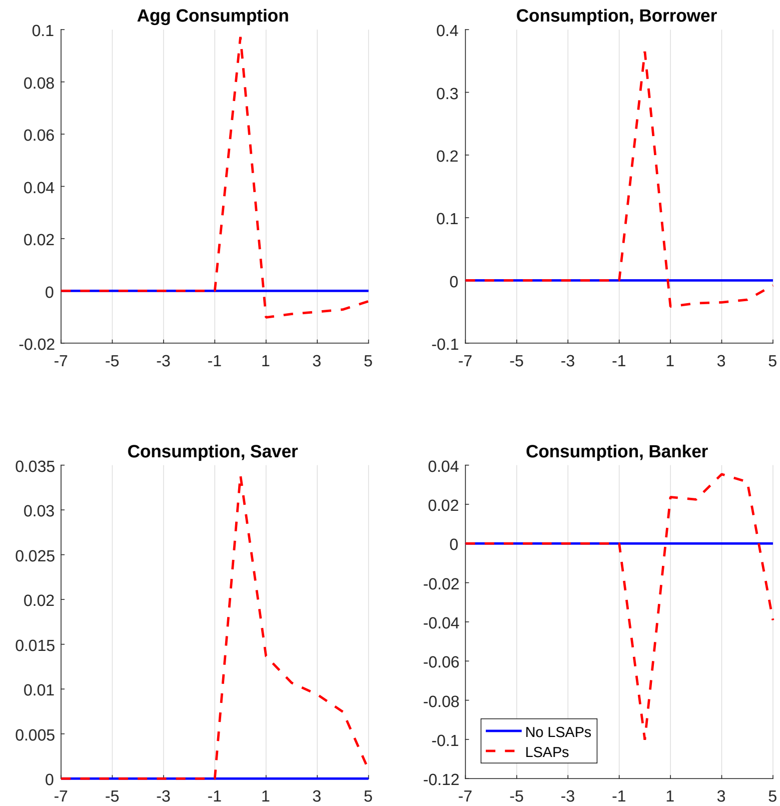
Figure 5: Dynamics: Comparison: Borrower



to result in more mortgage credit for borrowers. The best the central bank could do is cut rates so much that the very patient saver would be prompted to increase her consumption. But with healthier borrower balance sheets, monetary policy pass-through becomes possible even though the financial sector is recovering from bankruptcy. By keeping the nominal short rate higher in the economy with LSAPs than in the economy without LSAPs, the central bank encourages savers to save 6.9% more in deposits. A post-bailout constrained financial sector is willing to borrow at any rate. It accepts the extra deposits and uses them to finance a portfolio of mortgages larger by 7.7% in market value, even though their intermediating function is now partially performed by the central bank.

If in addition to announcing LSAPs today, the central bank provides LSAP guidance by committing to state-contingent purchases in the future whenever credit markets tighten again, the policy affects the economy through a third, expectations, channel. The commitment to intervene in future crises makes future defaults less likely. Expecting to keep their houses for

Figure 6: Dynamics: Comparison: Consumption



longer, households anticipate a larger stream of discounted housing services in the future, which raises house prices today. As in the collateral channel, homeowners consume out of the additional housing wealth, and the additional home equity discourages defaults.

The total effects on demand, shown in Figure 4, are substantial. Borrower consumption is 32% higher than in the economy without LSAPs. Saver consumption is also slightly larger (+3.3%) due to the expectations channel – the income effect of higher future nominal rates dominate the substitution effect from a higher interest rate today. Financial sector consumption is lower because borrowers demand more mortgage financing, and offer the constrained financial sector a higher expected return – 2.9% vs. 0.8%. But overall, LSAPs boost aggregate demand by 9.7%.

In the economy with LSAPs, less deleveraging takes place in the downturn, and so the economy recovers with more debt. Figure 5 shows the evolution of borrower quantities in recovery. Homeowner loan-to-value (LTV) ratio in the year after the onset of a crisis is 4 percentage points higher in the economy with LSAPs, and LTV ratios remain higher for 4 years after the crisis.

Borrowers have more debt not just relative to their home equity but also relative to their income. Debt-to-income (DTI) ratio remains higher for 6 years. Higher mortgage debt makes borrowers less willing to consume. At the same time, financial intermediaries have larger balance sheets and consume more. Savers consume more as well because they hold more safe assets yielding a higher return than if the LSAP intervention had not taken place.

I consider the kind of policy normalization anticipated by the Federal Reserve – a cessation of reinvestment, which leads to a gradual decline in Fed holdings as mortgages are amortized or prepaid, shown in Figure 4. Five years out, the mortgage portfolio of the central bank in the model declines by half. This is a faster decline than what has been observed in the data. Because this model abstracts away from endogenous investment and wage rigidities, it fails to generate persistence in quantities such as the mortgage spread. Low spreads in recovery produce few ongoing purchases and a fast roll-off of the central bank's mortgage portfolio. But by the time the portfolio is mostly rolled off, output and consumption have also recovered, so the joint dynamics of LSAPs and macro quantities are consistent.

New originations are purchased by the financial sector, which, on the one hand, has a bigger balance sheet after the crisis, but, on the other hand, has to pay a higher rate on its deposits. The net effect on the quantity of originations is minimal, yet there are effects on prices. Some of the higher nominal rate is passed through to households in the form of a higher mortgage rate, while the mortgage spread that the financial sector gets to earn is lower. As a result, as the recovery continues, financial intermediaries become smaller.

7 Long-Run Comparison

Differential dynamics of the two economies in recovery are partly symptomatic of the long-run differences between an economy in which the central bank exclusively targets nominal rates versus an economy in which the central bank responds to deterioriation in credit markets by directly purchasing assets. Table 4 presents a side-by-side comparison of the two economies.

With LSAPs as "the new normal," the financial system is less fragile, and negative shocks translate into smaller price drops, fewer defaults, and less binding constraints. House prices are 4% higher and mortgage debt is 1% higher, while both are 40% less volatile. LTV ratios are

Table 4: Long-run comparison

	Benchmark			LSAPs		
	Unconditional		**Crisis**	**Unconditional**		**Crisis**
	mean	stdev	mean	mean	stdev	mean
			Macro			
Output	1.007	0.027	0.933	+0.00%	-31.30%	+4.38%
Hours	0.804	0.013	0.783	-0.17%	-28.06%	+1.09%
Consumption	0.590	0.028	0.515	-0.33%	-34.87%	+7.25%
- Borrower	0.225	0.028	0.157	-0.88%	-40.08%	+25.04%
- Banker	0.023	0.004	0.014	-5.18%	-11.59%	+2.02%
- Saver	0.342	0.009	0.343	+0.35%	-15.14%	-0.70%
Nominal Rate	4.66%	2.34%	1.19%	+0.21%	-0.59%	+2.26%
Inflation	2.02%	1.01%	0.52%	+0.11%	-0.25%	+1.02%
Real Rate	2.75%	2.37%	-1.64%	+0.17%	-0.69%	+2.53%
			Borrower			
House prices	2.220	0.179	1.822	+4.59%	-39.39%	+17.98%
Mortgage rate	6.55%	0.22%	6.89%	+0.00%	+0.01%	+0.13%
Book val mtge debt	0.453	0.026	0.405	+0.98%	-39.73%	+10.38%
Market LTV	58.29%	7.99%	73.22%	-2.38%	-4.34%	-14.92%
Book LTV	48.95%	7.25%	63.66%	-2.06%	-4.03%	-12.37%
Default rate	0.82%	4.65%	17.03%	-0.76%	-2.99%	-16.74%
Loss-given-default rate	30.63%	9.83%	45.43%	-0.97%	-5.02%	-1.38%
Loss Rate	0.34%	2.08%	8.00%	-0.32%	-1.53%	-7.88%
Fraction LTV constraint binds	3.63%	18.71%	50.00%	-1.14%	-3.12%	-14.84%
Mortgage debt growth	3.40%	6.64%	-13.48%	+3.19%	-38.87%	+9.35%
			Banker			
Mortgage spread	1.89%	2.33%	5.71%	-0.20%	-0.59%	-2.13%
Banker wealth	0.037	0.012	0.017	-1.33%	+4.51%	+16.11%
Bankruptcies	3.59%	18.61%	50.00%	-0.80%	-2.14%	-13.23%
Bailout size	0.005	0.001	0.005	-2.64%	+56.20%	-3.02%
Fraction leverage constr binds	7.52%	26.37%	50.97%	-0.97%	-1.63%	-6.77%
Deposits	0.503	0.041	0.429	-2.04%	-0.50%	+4.66%
Mkt fin leverage (end-of-period)	93.13%	2.13%	94.93%	-0.32%	+0.09%	-0.38%

lower by 2.4 percentage points in market value terms and 2.1 percentage points in book value terms, and do not go up nearly as much in a crisis (-15%). As a result, defaults becomes unlikely even in a crisis.

Stronger household balance sheets lead to safer financial sector balance sheets, with intermediaries enjoying 16.1% higher wealth during crises. Greater financial stability translates to smoother business cycles. Output, hours worked, and consumption are all 30% less volatile. Individual consumption is also less volatile, with borrowers particularly benefiting from a 40% decline in volatility. Lower risk in the economy weakens the precautionary savings motive of savers and raises the real rate they want to earn on deposits and government debt (+0.17%). This translates into a higher cost of borrowing for the banks, which take 2% fewer deposits, hold 1.3% less equity, and earn a smaller mortgage spread (-0.2%), lowering their consumption by 5.8%. The higher short rate and lower spread net out to leave the mortgage rate unchanged, despite borrowers defaulting less often. Because borrowers are not rewarded for lower propensity to default with lower rates, their expected discounted sum of all future mortgage payments increases, lowering their consumption by 0.9%. Savers consumption goes up by 0.4%.

In sum, an economy with LSAPs is safer, with business cycles and credit cycles significantly dampened and defaults substantially reduced. While all agents benefit from lower consumption volatility, the benefits from the safer economy accrue mainly to the saver, who earns a higher rate of return on her investment.

8 Conclusion

When indebted homeowners are exposed to declines in house prices, they are less able to finance their consumption by borrowing against their homes, leading to defaults and a decline in consumption. Tighter constraints and lower inflation lead to a financial-nominal spiral, and the resulting mortgage losses impair bank balance sheets and lower aggregate demand. A conventional monetary policy of lowering the short-term nominal rate is ineffective at undoing these effects. The lower rate encourages low-MPC savers to consume more, but financial intermediaries do not pass on their cheaper cost of borrowing to borrowers because both they and the borrowers are constrained.

In these periods, large-scale purchases of mortgages (LSAPs) by the central bank can be effective at preventing a sharp drop in aggregate demand. LSAPs work through three channels. First, the market value of additional lending by the central bank directly finances borrower consumption. This effect is small. Second, the additional consumption by borrowers raises house prices. Borrowers with more home equity default less and borrow more. Healthier financial intermediaries take in more deposits and lend more in mortgages, helping to restore the transmission of conventional monetary policy. Third, when LSAPs are unexpected and when the central bank provides guidance about their ongoing use, they update agents' expectations about future aggregate risk. Anticipating higher and less volatile prices tomorrow, households are willing to pay higher prices today, further raising home equity and relaxing constraints.

Defaults are a way for households to deleverage. By discouraging defaults during the crisis, LSAPs leave borrowers more indebted in recovery, and higher mortgage payments cause them to consume less. The economy with LSAPs is safer, so risk-averse savers do not want to save as much and demand a higher rate on deposits. This prompts redistribution from the agents who borrow – borrowers and financial intermediaries – to savers.

This work opens up several avenues for future work. First, model extensions could build in additional channels of propagation and persistence beyond intermediary capital. Endogenous capital accumulation, wage rigidities, and monetary policy persistence could all slow down the recovery. Second, it would be interesting to study how an effective lower bound on interest rates affects the recapitalization of the intermediary sector after a crisis, and the effectiveness of unconventional monetary policy. Third, the paper is well suited to study the interactions of unconventional monetary policy with fiscal policy, bailout policy, and conventional monetary policy. These policies all affect the allocation of risk in a nominal economy, financial sector fragility, asset prices, and macro-economic volatility.

References

Adelino, M., A. Schoar, and F. Severino (2015a): "Changes in buyer composition and the expansion of credit during the boom," *NBER Working Paper Series*, p. 20848.

——— (2015b): "Loan originations and defaults in the mortgage crisis: Further evidence," Discussion paper, National Bureau of Economic Research.

——— (2016): "Loan originations and defaults in the mortgage crisis: The role of the middle class," *Review of Financial Studies*, p. hhw018.

Adrian, T., and N. Boyarchenko (2012): "Intermediary leverage cycles and financial stability," *Working paper.*

Agarwal, S., S. Chomsisengphet, N. Mahoney, and J. Stroebel (2016): "Do Banks Pass Through Credit Expansions to Consumers Who Want To Borrow? Evidence from Credit Cards," .

Aladangady, A. (2014): "Homeowner balance sheets and monetary policy," .

Auclert, A. (2016): "Monetary Policy and the Redistribution Channel (PDF)," *Description: Leaving the Board unpublished paper, Stanford University, Department of Economics, January.*

Bernanke, B., and M. Gertler (1989): "Agency costs, net worth, and business fluctuations," *The American Economic Review*, pp. 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1, 1341–1393.

Brunnermeier, M. K., and Y. Sannikov (2014): "A macroeconomic model with a financial sector," *The American Economic Review*, 104(2), 379–421.

Campbell, J. Y., S. Giglio, and P. Pathak (2011): "Forced Sales and House Prices," *American Economic Review*, 101, 2108–31.

Chu, Y. (2014): "Credit constraints, inelastic supply, and the housing boom," *Review of Economic Dynamics*, 17(1), 52–69.

Curdia, V., and M. Woodford (2011): "The central-bank balance sheet as an instrument of monetary policy," *Journal of Monetary Economics*, 58(1), 54–79.

Davis, M. A., and S. Van Nieuwerburgh (2014): "Housing, finance and the macroeconomy," Discussion paper, National Bureau of Economic Research.

Di Maggio, M., A. Kermani, and C. Palmer (2016): "Unconventional Monetary Policy and the Allocation of Credit," *Columbia Business School Research Paper*, (16-1).

Drechsler, I., A. Savov, and P. Schnabl (2014): "A Model of Monetary Policy and Risk Premia," *Working paper.*

Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2016): "Phasing out the GSEs," *Journal of Monetary Economics*, 81, 111 – 132.

FAVARA, G., AND J. IMBS (2015): "Credit supply and the price of housing," *The American Economic Review*, 105(3), 958–992.

FAVILUKIS, J., S. C. LUDVIGSON, AND S. VAN NIEUWERBURGH (2010): "The macroeconomic effects of housing wealth, housing finance, and limited risk-sharing in general equilibrium," Discussion paper, National Bureau of Economic Research.

FLOETOTTO, M., M. KIRKER, AND J. STROEBEL (2016): "Government Intervention in the Housing Market: Who Wins, Who Loses?," *Journal of Monetary Economics*.

GAGNON, J., M. RASKIN, J. REMACHE, AND B. P. SACK (2010): "Large-scale asset purchases by the Federal Reserve: did they work?," *FRB of New York Staff Report*, (441).

GERTLER, M., AND P. KARADI (2011): "A model of unconventional monetary policy," *Journal of monetary Economics*, 58(1), 17–34.

GREENWALD, D. L. (2016): "The Mortgage Credit Channel of Macroeconomic Transmission," *Available at SSRN 2735491.*

HE, Z., AND A. KRISHNAMURTHY (2012): "Intermediary Asset Pricing," *American Economic Review*, Forthcoming.

JESKE, K., D. KRUEGER, AND K. MITMAN (2013): "Housing, mortgage bailout guarantees and the macro economy," *Journal of Monetary Economics*, 60(8), 917–935.

JORDÀ, Ò., M. SCHULARICK, AND A. M. TAYLOR (2015): "Betting the house," *Journal of International Economics*, 96, S2–S18.

KAPLAN, G., K. MITMAN, AND G. VIOLANTE (2015): "Consumption and house prices in the Great Recession: Model meets evidence," *Manuscript, New York University.*

KIYOTAKI, N., AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

LANDVOIGT, T. (2014): "The Allocation and Pricing of Mortgage Risk," .

LANDVOIGT, T., M. PIAZZESI, AND M. SCHNEIDER (2015): "The housing market (s) of San Diego," *The American Economic Review*, 105(4), 1371–1407.

MCKAY, A., AND R. REIS (2016): "The role of automatic stabilizers in the US business cycle," *Econometrica*, 84(1), 141–194.

MIAN, A., AND A. SUFI (2011): "House prices, home equity–based borrowing, and the us household leverage crisis," *The American Economic Review*, 101(5), 2132–2156.

MIAN, A. R., K. RAO, AND A. SUFI (2013): "Household balance sheets, consumption, and the economic slump," *Chicago Booth Research Paper*, (13-42).

MIAN, A. R., A. SUFI, AND E. VERNER (2015): "Household debt and business cycles worldwide," Discussion paper, National Bureau of Economic Research.

Midrigan, V. (2011): "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," 79(4), 1139–1180.

Ramcharan, R., A. Kermani, and M. D. Maggio (2015): "Monetary Policy Pass-Through: Household Consumption and Voluntary Deleveraging," Discussion paper, Society for Economic Dynamics.

Reinhart, C. M., and K. Rogoff (2009): *This Time is Different.* Princeton University press.

Rotemberg, J. J. (1982): "Sticky prices in the United States," *The Journal of Political Economy*, pp. 1187–1211.

Rouwenhorst, K. G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *rontiers of Business Cycle Research*, ed. by T. Cooley. Princeton University Press.

Rudebusch, G. D., and E. T. Swanson (2012): "The bond premium in a dsge model with long-run real and nominal," *American Economic Journal: Macroeconomics*, 4(1), 105–143.

Stroebel, J., and J. B. Taylor (2012): "Estimated Impact of the Federal Reserves Mortgage-Backed Securities Purchase Program," *International Journal of Central Banking*, 8(2), 1–42.

Tuzel, S. (2009): "Corporate Real Estate Holdings and the Cross Section of Stock Returns," Unpublished paper, Marshall School, University of Southern California.

A Equilibrium Conditions

I reformulate the problem to ensure stationarity by scaling all variables by the productivity trend Z_t. Additionally, I stationarize nominal quantities by the price level P_t to allow for positive trend inflation.

A.1 Functional Forms

A.1.1 Distribution of Depreciation Shocks

Let idiosyncratic uninsurable housing depreciation shocks be distributed with a Gamma distribution with parameters (χ_0, χ_1). In the calibration, I set mean and variance of the distribution, which imply values for these parameters:

$$\mu_\omega \equiv \mathrm{E}_j[\omega_t^j; \chi_0, \chi_1] = \chi_0 \chi_1$$
$$\sigma_\omega^2 \equiv \mathrm{Var}_j[\omega_t^j; \chi_0, \chi_1] = \chi_0 \chi_1^2$$

Let $F_\omega(\omega) = F(\omega; \chi_0, \chi_1)$ and $f_\omega(\omega) = f(\omega; \chi_0, \chi_1)$ denote the CDF and PDF, respectively of the Gamma distribution with the given parameters. Then the function $Z_A(\omega)$ and $Z_H(\omega)$ defined in Section 3 are given by

$$Z_A(\omega) = 1 - F_\omega(\omega)$$
$$Z_H(\omega) \equiv \mathrm{E}_j[\tilde{\omega} | \tilde{\omega} \geq \omega] = \mu_w \frac{1 - F(\omega; \chi_0 + 1, \chi_1)}{1 - F_\omega(\omega)}$$

For the optimal default condition, it will be useful to differentiate these functions:

$$\frac{\partial Z_A}{\partial \omega} = -f_\omega(\omega)$$
$$\frac{\partial Z_H}{\partial \omega} = -\omega f_\omega(\omega)$$

A.1.2 Prepayment Congestion Cost

Let the convex prepayment cost be

$$\Psi(R, A) = \frac{\psi}{2} \left(\frac{R}{A} \right)^2 A$$

A.2 Borrower Problem

I consider the borrower family's problem after the depreciation shocks have been drawn, after intermediary agents have decided on a default policy, and after their random utility penalty has been realized. I define the following stationarized variables:

$$\{\hat{C}_t^B, \hat{w}_t^B, \hat{q}_t^H\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t}$$

To also allow for non-zero trend inflation, I define the following stationarized variables for quantities of nominal securities:

$$\{\hat{A}_t^B, \hat{R}_t^B, \hat{B}_t^B\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t P_t}$$

This implies, for example, that the value of A_t^B nominal mortgage bonds at time t in time t consumption units is $q_t^{\$,B} A_t^B / P_t = q_t^{\$,B} \hat{A}_t^B Z_t$, and ensures that nominal prices of nominal securities are stationary, just like real prices of real securities are.

Finally, I define stationarized next-period mortgage bond holdings

$$\hat{\hat{A}}_{t+1}^B \equiv \frac{\hat{A}_{t+1}^B}{Z_t P_t}$$

This implies the following law of motion for mortgage bonds:

$$\hat{A}_{t+1}^B = \hat{\hat{A}}_{t+1}^B \frac{Z_t P_t}{Z_{t+1} P_{t+1}} = \hat{\hat{A}}_{t+1}^B \frac{e^{-g_{t+1}}}{\Pi_{t+1}}$$

Let $\mathcal{S}_t^B = \{a_t, g_t, \sigma_{\omega,t}^2, W_t^I, W_t^S, W_t^G, q_{t-1}\}$ denote a vector of state variables exogenous to the borrower.

Then the borrower problem is

$$\begin{aligned}\hat{V}(H_t^B, \hat{A}_t^B, \hat{\mathcal{S}}_t^B) &\equiv V(H_t^B, A_t^B, \mathcal{S}_t^B) - \log Z_t \\ &= \max_{\{\omega_t^*, \hat{\hat{A}}_{t+1}, H_{t+1}^B, L_t^B, \hat{C}_t^B\}} \left\{ (1-\beta_B)\hat{u}_t^B - \frac{\beta_B}{\sigma_B - 1} \log \mathrm{E}_t \left[e^{-(\sigma_B - 1)(\hat{\hat{V}}_{t+1}^B + g_{t+1})} \right] \right\}\end{aligned}$$

where

$$u_t^B = (1-\theta)\log \hat{C}_t^B + \theta \log A_H H_t^B - \chi_0 \frac{\left(L_t^B\right)^{1+\chi}}{1+\chi} \tag{1}$$

$$\begin{aligned}\hat{C}_t^B = &(1-\tau_t^B)\hat{w}_t^B L_t^B + Z_H(\omega_t^*)\hat{q}_t^H H_t^B + q_t^{\$,B} \hat{\hat{A}}_{t+1}^B - (1 - \tau_t^m + \delta_B q_t^{\$,B}) Z_A(\omega_t^*)\hat{A}_t^B \\ &- \hat{q}_t^H H_{t+1}^B - (F^{\$} - q_t^{\$,B})\hat{R}_t^B - \frac{\psi}{2}\left(\frac{\hat{R}_t^B}{\hat{A}_t^B}\right)^2 \hat{A}_t^B\end{aligned} \tag{2}$$

$$\phi_B \hat{q}_t^H H_t^B \geq F^{\$} \hat{\hat{A}}_{t+1}^B \tag{3}$$

$$\hat{R}_t^B \geq 0 \tag{4}$$

$$\delta_B Z_A(\omega_t^*)\hat{A}_t^B \geq \hat{R}_t^B \tag{5}$$

The continuation value $\hat{\hat{V}}_{t+1}^B$ depends through $\mathcal{S}_{t+1}^B$ (specifically, through the wealth distribution W_t^I, W_t^S, and W_t^G) on the intermediary's bankruptcy decision, which is itself a function of the random utility penalty ρ_{t+1}.

$$\hat{\hat{V}}_t^B = \mathrm{E}_\rho \left[\hat{V}_{t+1}^B \right]$$

I guess here and verify in the next section that the intermediary's default decision follows a cut-off rule summarized by threshold utility penalty ρ_t^*. Hence, the continuation value $\hat{\hat{V}}_t$ can be written as

$$\hat{\hat{V}}_t^B = (1 - F_\rho(\rho_t^*))\mathrm{E}_\rho \left[\hat{V}_{t+1}^B | \rho \geq \rho_t^* \right] + F_\rho(\rho_t^*)\mathrm{E}_\rho \left[\hat{V}_{t+1}^B | \rho < \rho_t^* \right]$$

Furthermore, I guess here and verify in the next section that W_t^I and W_t^G depend on ρ_t only through the intermediary's bankruptcy decision. Then the expectations with respect to ρ in the continuation function are in fact scalars:

$$\hat{\hat{V}}_t^B = (1 - F_\rho(\rho_t^*))\hat{V}^B(H_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho \geq \rho_t^*)) + F_\rho(\rho_t^*)\hat{V}^B(H_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho < \rho_t^*))$$

To simplify notation, define the borrower's certainty equivalent CE_t^B

$$CE_t^B = -\frac{1}{\sigma_B - 1} \log \mathrm{E}_t \left[e^{-(\sigma_B - 1)\left(\hat{\hat{V}}_{t+1}^B + g_{t+1}\right)} \right]$$

and note that this implies

$$e^{-(\sigma_B-1)CE_t^B} = \mathrm{E}_t\left[e^{-(\sigma_B-1)\left(V_{t+1}^B+g_{t+1}\right)}\right]$$

A.2.1 First Order Condition: Labor Supply

The choice of labor is static, and thus reduces to maximizing u_t^B:

$$\frac{(1-\theta)(1-\tau_t^B)\hat{w}_t^B}{\hat{C}_t^B} = \chi_0\left(\mathrm{L}_t^B\right)^{\chi}$$

It is useful to write down an expression for L_t^B as a function of pre-tax labor income $\hat{Y}_t^B \equiv L_t^B \hat{w}_t^B$.

$$\frac{(1-\theta)(1-\tau_t^B)\hat{Y}_t^B}{\hat{C}_t^B} = \chi_0\left(\mathrm{L}_t^B\right)^{1+\chi}$$

$$L_t^B = \left(\frac{(1-\theta)}{\chi_0}\frac{(1-\tau_t^B)\hat{Y}_t^B}{\hat{C}_t^B}\right)^{\frac{1}{1+\chi}}$$

A.2.2 First Order Condition: Default

The borrower family chooses a threshold depreciation shock ω_t^* such that for all $\omega < \omega_t^*$, individuals borrowers default:

The FOC after dividing both sides by the marginal utility of consumption is

$$(1-\tau_t^m+\delta_B q_t^{\$,B}-\lambda_t^{RB}\delta_B)\hat{A}_t^B\frac{\partial Z_A}{\partial\omega}(\omega_t^*) = \hat{q}_t^H H_t^B\frac{\partial Z_H}{\partial\omega}(\omega_t^*)$$

where

$$\lambda_t^{RB} = \tilde{\lambda}_t^{RB}\left(\frac{\partial\hat{u}_t^B}{\partial\hat{C}_t^B}\right)^{-1}$$

is the Lagrange multiplier on the maximum prepayment constraint (5) divided by the marginal utility of consumption.

Differentiating Z_A and Z_H and solving, we get

$$\omega_t^* = \frac{(1-\tau_t^m+\delta_B q_t^{\$,B}-\lambda_t^{RB}\delta_B)\hat{A}_t^B}{\hat{q}_t^{\$,h}H_t^B}$$

A.2.3 First Order Condition: Prepayment

The FOC after dividing both sides by the marginal utility of consumption is

$$0 = -(F^{\$}-q_t^{\$,B})-\psi\frac{\hat{R}_t^B}{\hat{A}_t^B}+\mu_t^{RB}-\lambda_t^{RB}$$

where

$$\lambda_t^{RB} = \tilde{\mu}_t^{RB}\left(\frac{\partial\hat{u}_t^B}{\partial\hat{C}_t^B}\right)^{-1}$$

is the Lagrange multiplier on the non-negative prepayment constraint (4) divided by the marginal utility of consumption.

Let $Z_t^B = \hat{R}_t^B / \hat{A}_t^B$ be the fraction of starting-period bonds to prepay. Then

$$Z_t^B = \frac{q_t^{\$,B} - F^{\$} + \mu_t^{RB} - \lambda_t^{RB}}{\psi}$$

A.2.4 First Order Condition: Mortgages

The FOC for mortgage bonds $\hat{\hat{A}}_{t+1}^B$ is

$$\frac{(1-\beta_S)(1-\theta)}{\hat{C}_t^B} q_t^{\$,B} = \tilde{\lambda}_t^B F^{\$} - \beta_B \mathrm{E}_t \left[e^{-g_{t+1}} \Pi_{t+1}^{-1} \frac{\partial \hat{V}_{t+1}^B}{\partial \hat{A}_{t+1}^B} e^{-(\sigma_B - 1)(\hat{V}_{t+1}^B + g_{t+1} - CE_t^B)} \right]$$

The marginal value of mortgages $\frac{\partial \hat{V}_{t+1}^B}{\partial \hat{A}_{t+1}^B}$ depends on whether or not the intermediary declares bankruptcy:

$$\frac{\partial \hat{\hat{V}}_t^B}{\partial \hat{A}_t^B} = (1 - F_\rho(\rho_t^*)) \frac{\partial \hat{V}_t^B}{\partial \hat{A}_t^B}(H_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho \geq \rho_t^*)) + F_\rho(\rho_t^*) \frac{\partial \hat{V}_t^B}{\partial \hat{A}_t^B}(H_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho < \rho_t^*))$$

where the marginal value conditional on the intermediary's bankruptcy decision is:

$$\begin{aligned}
\frac{\partial \hat{V}_t^B}{\partial \hat{A}_t^B} &= \frac{\partial \hat{V}_t^B}{\partial \hat{C}_t^B} \frac{\partial \hat{C}_t^B}{\partial \hat{A}_t^B} + \tilde{\lambda}_t^{RB} \delta Z_A(\omega_t^*) \\
&= -\frac{(1-\beta_S)(1-\theta)}{\hat{C}_t^B} \left(\left[1 - \tau_t^m + \delta_B q_t^{\$,B} \right] Z_A(\omega_t^*) - \frac{\psi}{2} (Z_t^R)^2 \right) + \tilde{\lambda}_t^{RB} \delta Z_A(\omega_t^*) \\
&= -\frac{(1-\beta_S)(1-\theta)}{\hat{C}_t^B} \left(\left[1 - \tau_t^m + \delta_B q_t^{\$,B} - \delta_B \lambda_t^{RB} \right] Z_A(\omega_t^*) - \frac{\psi}{2} (Z_t^R)^2 \right)
\end{aligned}$$

Define the stochastic discount factor:

$$\mathcal{M}_{t+1}^B(\rho) = \beta^B e^{-\sigma_B g_{t+1}} \frac{\hat{C}_t^B}{\hat{C}_{t+1}^B(\rho)} e^{-(\sigma_B - 1)(\hat{V}_{t+1}^B - CE_t^B)}$$

where I make the dependence of $\hat{C}_{t+1}^B$ on the random utility penalty ρ. Next, write the Euler equation integrand is:

$$\mathcal{E}_{t+1}^{B,\$,b}(\rho) = \mathcal{M}_{t+1}^B(\rho) \left(\Pi_{t+1}(\rho) \right)^{-1} \left(\left[1 - \tau_{t+1}^m(\rho) + \delta_B q_{t+1}^{\$,b}(\rho) - \delta_B \lambda_{t+1}^{RB}(\rho) \right] Z_A(\omega_{t+1}^*(\rho)) - \frac{\psi}{2} (Z_{t+1}^R(\rho))^2 \right)$$

Combining, I get the Euler Equation for mortgages:

$$q_t^{\$,B} = \lambda_t^B F^{\$} + \mathrm{E}_t \left[(1 - F_\rho(\rho_t^*)) \mathcal{E}_{t+1}^{B,\$,b}(\rho \geq \rho_t^*) + F_\rho(\rho_t^*) \mathcal{E}_{t+1}^{B,\$,b}(\rho < \rho_t^*) \right]$$

A.2.5 First Order Condition: Housing Shares

The FOC for housing shares H_{t+1}^B is

$$\frac{(1-\beta_B)(1-\theta)}{\hat{C}_t^B} \hat{q}_t^h (1 - \phi_B \tilde{\lambda}_t^B) = \beta_B \mathrm{E}_t \left[\frac{\partial \hat{\hat{V}}_{t+1}^B}{\partial H_{t+1}^B} e^{-(\sigma_B - 1)(V_{t+1}^B + g_{t+1} - CE_t^B)} \right]$$

The marginal value of housing $\frac{\partial \hat{\hat{V}}^B_{t+1}}{\partial \hat{H}^B_{t+1}}$ depends on whether or not the intermediary declares bankruptcy:

$$\frac{\partial \hat{\hat{V}}^B_t}{\partial H^B_t} = (1 - F_\rho(\rho^*_t))\frac{\partial \hat{V}^B_t}{\partial H^B_t}(H^B_t, \hat{A}^B_t, \mathcal{S}^B_t(\rho \geq \rho^*_t)) + F_\rho(\rho^*_t)\frac{\partial \hat{V}^B_t}{\partial H^B_t}(H^B_t, \hat{A}^B_t, \mathcal{S}^B_t(\rho < \rho^*_t))$$

where $\frac{\partial \hat{V}^B_t}{\partial H^B_t}$ is

$$\frac{\partial \hat{V}^B_t}{\partial H^B_t} = \frac{(1-\beta_B)\theta}{H^B_t} + \frac{(1-\beta_B)(1-\theta)}{\hat{C}^B_t} Z_H(\omega^*_t)\hat{q}^h_t$$

Leaving the dependence of quantities on next period's ρ_{t+1} implicit and using the definition of the real SDF from the previous section, I write the Euler Equation integrand as

$$\mathcal{E}^{B,h}_{t+1} = \mathcal{M}^B_{t+1} e^{g_{t+1}} \left(Z_H(\omega^*_t)\hat{q}^h_t + \frac{\theta}{1-\theta}\frac{\hat{C}^B_{t+1}}{H^B_{t+1}} \right)$$

Combining, I get the Euler Equation for housing shares

$$\hat{q}^h_t(1 - \phi_B \lambda^B_t) = \mathrm{E}_t \left[(1 - F_\rho(\rho^*_t))\mathcal{E}^{B,h}_{t+1}(\rho \geq \rho^*_t) + F_\rho(\rho^*_t)\mathcal{E}^{B,h}_{t+1}(\rho < \rho^*_t) \right]$$

A.3 Intermediary Problem

Consider the problem of the intermediary at time t, with wealth W^I_t, before the random utility penalty ρ_t has been realized. If the intermediary declares bankruptcy i.e. if $D_t(\rho_t) = 1$, both her assets and her liabilities are transferred to the government. Hence, I define wealth after the realization of the random utility penalty as

$$\tilde{W}^I_t = (1 - D_t(\rho_t))W^I_t$$

and the effective utility penalty as

$$\tilde{\rho}_t = D_t(\rho_t)\rho_t$$

Using these variables, I now consider the borrower family's problem after the depreciation shocks have been drawn, after intermediary agents have decided on a default policy, and after their random utility penalty has been realized. I define the following stationarized variables:

$$\{\hat{C}^I_t, \hat{w}^I_t\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t}$$

To also allow for non-zero trend inflation, I define the following stationarized variables for quantities of nominal securities:

$$\{\hat{A}^I_t, \hat{B}^I_t\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t P_t}$$

This implies, for example, that the value of A^I_t nominal mortgage bonds at time t in time t consumption units is $q^{\$,B}_t A^I_t / P_t = q^{\$,B}_t \hat{A}^I_t Z_t$, and ensures that nominal prices of nominal securities are stationary, just like real prices of real securities are.

Finally, I define stationarized next-period mortgage bond holdings and next-period deposits:

$$\hat{\hat{A}}^I_{t+1} \equiv \frac{\hat{A}^I_{t+1}}{Z_t P_t}$$
$$\hat{\hat{B}}^I_{t+1} \equiv \frac{\hat{B}^I_{t+1}}{Z_t P_t}$$

For each of these quantities X_t, this implies the following law of motion:

$$\hat{X}_{t+1} = \hat{\hat{X}}_{t+1} \frac{Z_t P_t}{Z_{t+1} P_{t+1}} = \hat{\hat{X}}_{t+1} \frac{e^{-g_{t+1}}}{\Pi_{t+1}}$$

Let $\mathcal{S}^I_t = \{a_t, g_t, \sigma^2_{\omega,t}, A^B_t, W^S_t, W^G_t, q_{t-1}\}$ denote a vector of state variables exogenous to the intermediary. Then the intermediary problem is

$$\hat{V}(\hat{\tilde{W}}^I_t, \tilde{\rho}_t, \hat{\mathcal{S}}^I_t) = \max_{\{\hat{\hat{A}}^I_{t+1}, \hat{\hat{B}}^I_{t+1}, L^I_t, \hat{C}^I_t\}} \left\{ (1-\beta_I)\hat{u}^I_t - \frac{\beta_I}{\sigma_I - 1} \log \mathrm{E}_t \left[e^{-(\sigma_I - 1)(\hat{\hat{V}}^I_{t+1} + g_{t+1})} \right] \right\} \tag{6}$$

where

$$u^I_t = (1-\theta)\log \hat{C}^I_t + \theta \log A_H H^I_t - \chi_0 \frac{\left(L^I_t\right)^{1+\chi}}{1+\chi} - \tilde{\rho}_t \tag{7}$$
$$\hat{C}^I_t = (1-\tau^I_t)\hat{w}^I_t L^I_t + \hat{\tilde{W}}^I_t - (1-\mu_\omega)\hat{q}^H_t H^B_{t+1} - q^{\$,B}_t \hat{\hat{A}}^I_{t+1} - q^{\$}_t \hat{\hat{B}}^I_{t+1} \tag{8}$$
$$\begin{aligned} \hat{W}^I_t &= (M^{\$}_t + \delta_B q^{\$,B}_t - Z^R_t[q^{\$,B}_t - F^{\$}])\hat{A}^I_{t+1} + \hat{B}^I_t = \\ &= \left((M^{\$}_t + \delta_B q^{\$,B}_t - Z^R_t[q^{\$,B}_t - F^{\$}])\hat{\hat{A}}^I_{t+1} + \hat{\hat{B}}^I_t \right) \frac{e^{-g_{t+1}}}{\Pi_{t+1}} \end{aligned} \tag{9}$$
$$M^{\$}_t = Z_A(\omega^*_t) + (1-\zeta)[\mu_\omega - Z_H(\omega^*_t)] \frac{\hat{q}^H_t H^B_t}{\hat{A}^B_t} \tag{10}$$
$$\phi_I q^{\$,B}_t \hat{\hat{A}}^I_{t+1} \geq -q^{\$}_t \hat{\hat{B}}^I_{t+1} \tag{11}$$
$$\hat{\hat{A}}^I_{t+1} \geq 0 \tag{12}$$

The continuation value $\hat{\hat{V}}^I_{t+1} = \hat{\hat{V}}^I(\hat{W}^I_{t+1}, \hat{\mathcal{S}}^I_{t+1})$ is given the optimization problem faced by the intermediary at the start of the next period, when she chooses an optimal default policy $D_{t+1}(\rho_{t+1})$ ahead of the realization of ρ_{t+1}. It is given by

$$\hat{\hat{V}}^I(\hat{W}^I_t, \hat{\mathcal{S}}^I_t) = \max_{D_t(\rho_t)} \mathrm{E}_\rho \left[(1 - D_t(\rho_t))\hat{V}^I(\hat{W}^I_t, 0, \hat{\mathcal{S}}^I_t) + D_t(\rho_t)\hat{V}^I(0, \rho_t, \hat{\mathcal{S}}^I_t) \right]$$

To simplify notation, define the borrower's certainty equivalent CE^I_t

$$CE^I_t = -\frac{1}{\sigma_I - 1} \log \mathrm{E}_t \left[e^{-(\sigma_I - 1)\left(\hat{\hat{V}}^I_{t+1} + g_{t+1}\right)} \right]$$

and note that this implies

$$e^{-(\sigma_I - 1)CE^I_t} = \mathrm{E}_t \left[e^{-(\sigma_I - 1)\left(\hat{\hat{V}}^I_{t+1} + g_{t+1}\right)} \right]$$

A.3.1 Bankruptcy Decision

After the realization of a_t and g_t but before the realization of ρ_t, the intermediary must choose a default policy which maps the support of the ρ_t distribution to a binary default choice $D_t(\rho_t) : \mathbb{R} \to \{0, 1\}$.

$\hat{V}_t^I$ is decreasing in ρ_t. I guess and verify that $\hat{V}_t^I$ is increasing in $\hat{W}_t^I$. These monotonic relationships imply a threshold policy for $D_t(\rho_t)$. In other words, there exist a threshold value ρ_t^* such that for all $\rho < \rho_t^*$, $D_t(\rho) = 1$ and for all $\rho \geq \rho_t^*$, $D_t(\rho) = 0$. Intuitively, the intermediary will choose to default if and only if the utility cost of default is "small enough." The choice of optimal default policy is thus just the choice of the optimal default threshold ρ_t^*:

$$\hat{\hat{V}}^I(\hat{W}_t^I, \hat{\mathcal{S}}_t^I) = \max_{\rho_t^*} \mathrm{E}_\rho \left[(1 - \mathbb{1}_{\rho<\rho_t^*})\hat{V}^I(\hat{W}_t^I, 0, \hat{\mathcal{S}}_t^I) + \mathbb{1}_{\rho<\rho_t^*}\hat{V}^I(0, \rho_t, \hat{\mathcal{S}}_t^I) \right]$$

The value function is continuous, so at ρ_t^* must satisfy

$$\hat{V}^I(\hat{W}_t^I, 0, \hat{\mathcal{S}}_t^I) = \hat{V}^I(0, \rho_t^*, \hat{\mathcal{S}}_t^I)$$

Note that

$$\hat{V}^I(0, \rho_t^*, \hat{\mathcal{S}}_t^I) = \hat{V}^I(0, 0, \hat{\mathcal{S}}_t^I) - (1 - \beta_I)\rho_t^*$$

and hence

$$\rho_t^* = \frac{\hat{V}^I(0, 0, \hat{\mathcal{S}}) - \hat{V}^I(\hat{W}_t^I, 0, \hat{\mathcal{S}}_t^I)}{1 - \beta_I} \tag{13}$$

which implies that $\rho_t^* > 0$ if $\hat{W}_t^I < 0$ and $\rho_t^* < 0$ if $\hat{W}_t^I > 0$. Finally,

$$\hat{\hat{V}}^I(\hat{W}_t^I, \hat{\mathcal{S}}_t^I) = (1 - F_\rho(\rho_t^*))\hat{V}^I(\hat{W}_t^I, 0, \hat{\mathcal{S}}_t^I) + \int_{-\infty}^{\rho_t^*} \hat{V}^I(0, \rho, \hat{\mathcal{S}}_t^I) dF_\rho \rho \tag{14}$$

where ρ $F_\rho = \mathcal{N}(0, \sigma_\rho^2)$.

Thus, equations (6) together with (13) and (14) completely characterize the intermediary's problem.

A.3.2 First Order Condition: Labor Supply

The choice of labor is static, and thus reduces to maximizing u_t^B:

$$\frac{(1-\theta)(1-\tau_t^I)\hat{w}_t^I}{\hat{C}_t^I} = \chi_0 \left(\mathrm{L}_t^I\right)^\chi$$

It is useful to write down an expression for L_t^I as a function of pre-tax labor income $\hat{Y}_t^I \equiv L_t^I \hat{w}_t^I$.

$$\frac{(1-\theta)(1-\tau_t^I)\hat{Y}_t^I}{\hat{C}_t^I} = \chi_0 \left(\mathrm{L}_t^I\right)^{1+\chi}$$

$$L_t^I = \left(\frac{(1-\theta)}{\chi_0} \frac{(1-\tau_t^I)\hat{Y}_t^I}{\hat{C}_t^I} \right)^{\frac{1}{1+\chi}}$$

A.3.3 Marginal Utility of Wealth

Differentiating the continuation value function (14) with respect to $\hat{W}_t^I$,

$$\frac{\partial \hat{\hat{V}}_t^I}{\partial \hat{W}_t^I} = (1 - F_\rho(\rho_t^*)) \frac{\partial \hat{V}^I(\hat{W}_t^I, 0, \hat{\mathcal{S}}_t^I)}{\partial \hat{W}_t^I} = (1 - F_\rho(\rho_t^*)) \frac{(1-\beta_I)(1-\theta)}{\hat{C}^I(\hat{W}_t^I, 0, \hat{\mathcal{S}}_t^I)}$$

where $\hat{C}_t^I$ is explicitly written as a policy function to underscore that it is evaluated at $D_t(\rho_t) = 0$.

Define

$$\mathcal{M}_{t+1}^I = \beta_I e^{-\sigma_I g_{t+1}} \frac{\hat{C}_t^I}{\hat{C}^I(\hat{\hat{W}}_{t+1}^I, \tilde{\rho}_{t+1}, \hat{\mathcal{S}}_{t+1}^I)} e^{-(\sigma_I - 1)(\hat{\hat{V}}(\hat{W}_{t+1}^I, \hat{\mathcal{S}}_{t+1}^I) - CE_t^I)}$$

I show below that the intermediary's stochastic discount factor, taking the bankruptcy option into account, is given by

$$\tilde{\mathcal{M}}_{t+1}^I \equiv \int_{\rho_{t+1}^*}^{\infty} \mathcal{M}_{t+1}^I dF_\rho \rho = (1 - F_\rho(\rho_t^*))\beta_I e^{-\sigma_I g_{t+1}} \frac{\hat{C}_t^I}{\hat{C}^I(\hat{W}_{t+1}^I, 0, \hat{\mathcal{S}}_{t+1}^I)} e^{-(\sigma_I - 1)(\hat{\hat{V}}(\hat{W}_{t+1}^I, \hat{\mathcal{S}}_{t+1}^I) - CE_t^I)}$$

A.3.4 First Order Condition: Mortgage Bonds

The FOC for mortgage bonds $\hat{\hat{A}}_{t+1}^I$ is

$$\frac{(1-\beta_I)(1-\theta)}{\hat{C}_t^I} q_t^{\$,B} = \tilde{\lambda}_t^I \phi_I q_t^{\$,B} + \tilde{\mu}_t^I + \beta_I \mathrm{E}_t \left[\frac{\partial \hat{\hat{V}}_{t+1}^I}{\partial \hat{\hat{A}}_{t+1}^I} e^{-(\sigma_I - 1)(\hat{\hat{V}}_{t+1}^I + g_{t+1} - CE_t^I)} \right]$$

The marginal value of mortgages is

$$\frac{\partial \hat{\hat{V}}_t^I}{\partial \hat{A}_t^I} = \frac{\partial \hat{\hat{V}}_t^I}{\partial \hat{W}_t^I} \frac{\partial \hat{W}_t^I}{\partial \hat{A}_t^I} = \frac{\partial \hat{\hat{V}}_t^I}{\partial \hat{W}_t^I} \frac{M_t^\$ + \delta_B q_t^{\$,B} - Z_t^R [q_t^{\$,B} - F^\$]}{e^{g_{t+1}} \Pi_{t+1}}$$

Combining,

$$\begin{aligned} \frac{(1-\beta_I)(1-\theta)}{\hat{C}_t^I} q_t^{\$,B} &= \tilde{\lambda}_t^I \phi_I q_t^{\$,B} + \tilde{\mu}_t^I + \\ &\mathrm{E}_t \left[\beta_I (1 - F_\rho(\rho_t^*)) \frac{(1-\beta_I)(1-\theta)}{\hat{C}^I(\hat{W}_{t+1}^I, 0, \hat{\mathcal{S}}_{t+1}^I)} \frac{M_t^\$ + \delta_B q_t^{\$,B} - Z_t^R [q_t^{\$,B} - F^\$]}{e^{g_{t+1}} \Pi_{t+1}} e^{-(\sigma_I - 1)(\hat{\hat{V}}_{t+1}^I + g_{t+1} - CE_t^I)} \right] \end{aligned}$$

Define transformed Lagrange multipliers

$$\lambda_t^I = \tilde{\lambda}_t^I \frac{\hat{C}_t^I}{(1-\beta_I)(1-\theta)}$$

$$\mu_t^I = \tilde{\mu}_t^I \frac{\hat{C}_t^I}{(1-\beta_I)(1-\theta)}$$

Plugging in, rearranging, and canceling terms,

$$\begin{aligned} q_t^{\$,B}(1 - \lambda_t^I \phi_I) &= \mu_t^I + \\ &\mathrm{E}_t \left[\underbrace{(1 - F_\rho(\rho_t^*))\beta_I e^{-\sigma_I g_{t+1}} \frac{\hat{C}_t^I}{\hat{C}^I(\hat{W}_{t+1}^I, 0, \hat{\mathcal{S}}_{t+1}^I)} e^{-(\sigma_I - 1)(\hat{\hat{V}}_{t+1}^I - CE_t^I)}}_{\tilde{\mathcal{M}}_{t+1}^I} \frac{M_t^\$ + \delta_B q_t^{\$,B} - Z_t^R [q_t^{\$,B} - F^\$]}{\Pi_{t+1}} \right] \end{aligned}$$

or

$$q_t^{\$,B}(1 - \lambda_t^I \phi_I) = \mu_t^I + \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t+1}^I \Pi_{t+1}^{-1} \left(M_t^\$ + \delta_B q_t^{\$,B} - Z_t^R [q_t^{\$,B} - F^\$] \right) \right]$$

A.3.5 First Order Condition: Deposits

The FOC for mortgage bonds $\hat{\hat{B}}^I_{t+1}$ is

$$\frac{(1-\beta_I)(1-\theta)}{\hat{C}^I_t} q^\$_t = \tilde{\lambda}^I_t q^\$_t + \beta_I \mathrm{E}_t \left[\frac{\partial \hat{\tilde{V}}^I_{t+1}}{\partial \hat{\hat{B}}^I_{t+1}} e^{-(\sigma_I - 1)(\hat{\tilde{V}}^I_{t+1} + g_{t+1} - CE^I_t)} \right]$$

The marginal value of deposits is

$$\frac{\partial \hat{\tilde{V}}^I_t}{\partial \hat{B}^I_t} = \frac{\partial \hat{\tilde{V}}^I_t}{\partial \hat{W}^I_t} \frac{\partial \hat{W}^I_t}{\partial \hat{B}^I_t} = \frac{\partial \hat{\tilde{V}}^I_t}{\partial \hat{W}^I_t} \frac{1}{e^{g_{t+1}} \Pi_{t+1}}$$

Combining and rearranging,

$$q^\$_t (1 - \lambda^I_t) = \mathrm{E}_t \left[\tilde{\mathcal{M}}^I_{t+1} \Pi^{-1}_{t+1} \right]$$

A.4 Saver Problem

I consider the savers' problem after the depreciation shocks have been drawn, after intermediary agents have decided on a default policy, and after their random utility penalty has been realized. I define the following stationarized variables:

$$\{\hat{C}^S_t, \hat{w}^S_t, \hat{q}^S_t\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t}$$

To also allow for non-zero trend inflation, I define the following stationarized variables for quantities of nominal securities:

$$\{\hat{B}^S_t\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t P_t}$$

This implies that the value of B^S_t nominal mortgage bonds at time t in time t consumption units is $B^S_t / P_t = q^\$_t \hat{B}^S_t Z_t$, and ensures that nominal prices of nominal securities are stationary, just like real prices of real securities are.

Finally, I define stationarized next-period mortgage bond holdings

$$\hat{\hat{B}}^B_{t+1} \equiv \frac{\hat{B}^B_{t+1}}{Z_t P_t}$$

This implies the following law of motion for mortgage bonds:

$$\hat{B}^B_{t+1} = \hat{\hat{B}}^B_{t+1} \frac{Z_t P_t}{Z_{t+1} P_{t+1}} = \hat{\hat{B}}^B_{t+1} \frac{e^{-g_{t+1}}}{\Pi_{t+1}}$$

Lastly, savers buy shares in each $i \in [0,1]$ of the continuum of monopolistic retail firms. Because these firms are identical, the problem is symmetric and so I consider the savers' optimization problem with respect to shares in a representative firm, which is also equal to aggregate share holdings $A^S_t = \int_0^1 A^S_t(i) di = A^S_t(i)$ for all i.

Let $\mathcal{S}^S_t = \{a_t, g_t, \sigma^2_{\omega,t}, A^B_t, W^I_t, W^G_t, q_{t-1}\}$ denote a vector of state variables exogenous to the borrower.

Then the saver problem is

$$
\begin{aligned}
\hat{V}(\hat{W}_t^S,\hat{\mathcal{S}}_t^S) &\equiv V(W_t^S,\mathcal{S}_t^S) - \log Z_t \\
&= \max_{\{\hat{\hat{B}}_{t+1},A_{t+1}^S,L_t^S,\hat{C}_t^S\}} \left\{(1-\beta_S)\hat{u}_t^S - \frac{\beta_S}{\sigma_S - 1}\log \mathrm{E}_t\left[e^{-(\sigma_S-1)(\hat{V}_{t+1}^S+g_{t+1})}\right]\right\}
\end{aligned}
$$

where

$$
u_t^S = (1-\theta)log\hat{C}_t^S + \theta A_H H_t^S - \chi_0\frac{(L_t^S)^{1+\chi}}{1+\chi} \tag{15}
$$

$$
\hat{C}_t^S = (1-\tau_t^S)\hat{w}_t^S L_t^S + \hat{W}_t^S - (1-\mu_\omega)\hat{q}_t^H H_{t+1}^S - \hat{q}_t^{\$,s} A_{t+1}^S - q_t^{\$}\hat{\hat{B}}_{t+1}^S \tag{16}
$$

$$
\hat{W}_t^S = (\hat{D}_t + \hat{q}_t^s)A_t^S + \hat{B}_t^S = (\hat{D}_t+\hat{q}_t^s)A_t^S + \hat{\hat{B}}_t^S\frac{e^{-g_{t+1}}}{\Pi_{t+1}} \tag{17}
$$

$$
\hat{\hat{B}}_{t+1}^S \geq 0 \tag{18}
$$

$$
A_{t+1}^S \geq 0 \tag{19}
$$

The continuation value $\hat{\hat{V}}_{t+1}^S$ depends through $\mathcal{S}_{t+1}^S$ (specifically, through the wealth distribution W_t^I and W_t^G) on the intermediary's bankruptcy decision, which is itself a function of the random utility penalty ρ_{t+1}.

$$
\hat{\hat{V}}_t^S = \mathrm{E}_\rho\left[\hat{V}_{t+1}^S\right]
$$

As shown in the previous section, the intermediary's default decision follows a cut-off rule summarized by threshold utility penalty ρ_t^*. Hence, the continuation value $\hat{\hat{V}}_t^S$ can be written as

$$
\hat{\hat{V}}_t^S = (1-F_\rho(\rho_t^*))\mathrm{E}_\rho\left[\hat{V}_{t+1}^S|\rho \geq \rho_t^*\right] + F_\rho(\rho_t^*)\mathrm{E}_\rho\left[\hat{V}_{t+1}^S|\rho < \rho_t^*\right]
$$

Furthermore, W_t^I and W_t^G depend on ρ_t only through the intermediary's bankruptcy decision. Then the expectations with respect to ρ in the continuation function are in fact scalars:

$$
\hat{\hat{V}}_t^S = (1-F_\rho(\rho_t^*))\hat{V}^S(\hat{W}_t^S,\mathcal{S}_t^S(\rho\geq\rho_t^*)) + F_\rho(\rho_t^*)\hat{V}^S(\hat{W}_t^S,\mathcal{S}_t^S(\rho<\rho_t^*)) \tag{20}
$$

To simplify notation, define the borrower's certainty equivalent CE_t^S

$$
CE_t^S = -\frac{1}{\sigma_S-1}\log\mathrm{E}_t\left[e^{-(\sigma_S-1)\left(\hat{\hat{V}}_{t+1}^S+g_{t+1}\right)}\right]
$$

and note that this implies

$$
e^{-(\sigma_S-1)CE_t^S} = \mathrm{E}_t\left[e^{-(\sigma_S-1)\left(V_{t+1}^S+g_{t+1}\right)}\right]
$$

A.4.1 First Order Condition: Labor Supply

The choice of labor is static, and thus reduces to maximizing u_t^S:

$$
\frac{(1-\theta)(1-\tau_t^S)\hat{w}_t^S}{\hat{C}_t^S} = \chi_0\left(\mathrm{L}_t^S\right)^\chi
$$

It is useful to write down an expression for L_t^S as a function of pre-tax labor income $\hat{Y}_t^S \equiv L_t^S \hat{w}_t^S$.

$$\frac{(1-\theta)(1-\tau_t^S)\hat{Y}_t^S}{\hat{C}_t^S} = \chi_0 \left(\text{Ł}_t^S\right)^{1+\chi}$$

$$L_t^S = \left(\frac{(1-\theta)}{\chi_0}\frac{(1-\tau_t^S)\hat{Y}_t^S}{\hat{C}_t^S}\right)^{\frac{1}{1+\chi}}$$

A.4.2 Marginal Utility of Wealth

Differentiating the continuation value function (20) with respect to $\hat{W}_t^S$,

$$\frac{\partial \hat{\hat{V}}_t^S}{\partial \hat{W}_t^S} = (1-F_\rho(\rho_t^*))\frac{\partial \hat{V}^S(\hat{W}_t^S, \hat{\mathcal{S}}_t^S(\rho \geq \rho_t^*)}{\partial \hat{W}_t^S} + F_\rho(\rho_t^*)\frac{\partial \hat{V}^S(\hat{W}_t^S, \hat{\mathcal{S}}_t^S(\rho < \rho_t^*)}{\partial \hat{W}_t^S} =$$
$$= (1-F_\rho(\rho_t^*))\frac{(1-\beta_S)(1-\theta)}{\hat{C}^S(\hat{W}_t^S, \hat{\mathcal{S}}_t^S(\rho \geq \rho_t^*))} + F_\rho(\rho_t^*)\frac{(1-\beta_S)(1-\theta)}{\hat{C}^S(\hat{W}_t^S, \hat{\mathcal{S}}_t^S(\rho < \rho_t^*))}$$

Define

$$\mathcal{M}_{t+1}^S(\rho) = \beta_S e^{-\sigma_S g_{t+1}} \frac{\hat{C}_t^S}{\hat{C}^S(\hat{\hat{W}}_{t+1}^S, \hat{\mathcal{S}}_{t+1}^S(\rho))} e^{-(\sigma_S-1)(\hat{V}(\hat{W}_{t+1}^S, \hat{\mathcal{S}}_{t+1}^S)-CE_t^S)}$$

I show below that $\mathcal{M}_{t+1}^S(\rho)$ is the stochastic discount factor from the time t state to a time $t+1$ state with a random utility penalty ρ.

A.4.3 First Order Condition: Short-term Nominal Bonds

The FOC for mortgage bonds $\hat{\hat{B}}_{t+1}^S$ is

$$\frac{(1-\beta_S)(1-\theta)}{\hat{C}_t^S} q_t^\$ = \tilde{\lambda}_t^S + \beta_S \text{E}_t\left[\frac{\partial \hat{\hat{V}}_{t+1}^S}{\partial \hat{\hat{B}}_{t+1}^S} e^{-(\sigma_S-1)(\hat{\hat{V}}_{t+1}^S + g_{t+1} - CE_t^S)}\right]$$

The marginal value of short-term bonds is

$$\frac{\partial \hat{\hat{V}}_t^S}{\partial \hat{B}_t^S} = \frac{\partial \hat{\hat{V}}_t^S}{\partial \hat{W}_t^S}\frac{\partial \hat{W}_t^S}{\partial \hat{B}_t^S} = \frac{\partial \hat{\hat{V}}_t^S}{\partial \hat{W}_t^S}\frac{1}{e^{g_{t+1}}\Pi_{t+1}}$$

Next, write the Euler equation integrand is:

$$\mathcal{E}_{t+1}^{S,\$}(\rho) = \mathcal{M}_{t+1}^S(\rho)\left(\Pi_{t+1}(\rho)\right)^{-1}$$

Combining, I get the Euler Equation for mortgages:

$$q_t^\$ = \lambda_t^S + \text{E}_t\left[(1-F_\rho(\rho_t^*))\mathcal{E}_{t+1}^{S,\$}(\rho \geq \rho_t^*) + F_\rho(\rho_t^*)\mathcal{E}_{t+1}^{S,\$}(\rho < \rho_t^*)\right]$$

A.4.4 First Order Condition: Retail Firms Shares

The FOC for next period shares of the representative retail firm A_{t+1}^S

$$\frac{(1-\beta_S)(1-\theta)}{\hat{C}_t^S}\hat{q}_t^S = \tilde{\mu}_t^S + \beta_S \text{E}_t\left[\frac{\partial \hat{\hat{V}}_{t+1}^S}{\partial A_{t+1}^S} e^{-(\sigma_S-1)(\hat{\hat{V}}_{t+1}^S + g_{t+1} - CE_t^S)}\right]$$

The marginal value of shares is

$$\frac{\partial \hat{\hat{V}}_t^S}{\partial A_t^S} = \frac{\partial \hat{\hat{V}}_t^S}{\partial \hat{W}_t^S}\frac{\partial \hat{W}_t^S}{\partial A_t^S} = \frac{\partial \hat{\hat{V}}_t^S}{\partial \hat{W}_t^S}(\hat{D}_t^S + \hat{q}_t^S)$$

Next, write the Euler equation integrand is:

$$\mathcal{E}_{t+1}^{S,s}(\rho) = e^{g_{t+1}}\mathcal{M}_{t+1}^S(\rho)(\hat{D}_t^S(\rho) + \hat{q}_t^S(\rho))$$

Combining, I get the Euler Equation for mortgages:

$$q_t^s = \mu_t^S + \mathrm{E}_t\left[(1 - F_\rho(\rho_t^*))\mathcal{E}_{t+1}^{S,s}(\rho \geq \rho_t^*) + F_\rho(\rho_t^*)\mathcal{E}_{t+1}^{S,s}(\rho < \rho_t^*)\right]$$

A.5 Retail Firms

The final consumption good is packaged by a representative retail firm using intermediate/wholesale goods as inputs. The packaging function is standard

$$Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

Retail firms maximize their nominal profit

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

taking all prices $P_t, \{P_t(i)\}_{i\in[0,1]}$ as given. The solution to their optimization problem yields demand for each good i:

$$Y_t(i) = \left(\frac{P_t}{P_t(i)}\right)^{\epsilon} Y_t$$

Because entry is free, in equilibrium retail firms must earn zero profits:

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di$$

implying a price index of

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

It is useful to note that if $P_t(i) = P_t(j)$ for all $i, j \in [0,1]$, then $P_t = P_t(i)$ for all $i \in [0,1]$.

A.6 Wholesale Firms and the New Keynesian Philips Curve

Wholesale firms are owned by the savers, and hence maximize the stream of dividends discounted by the savers' stochastic discount factor. I consider the firms' problem after the depreciation shocks have been drawn, after intermediary agents have decided on a default policy, and after their random utility penalty has been realized. I define the following stationarized variables:

$$\{\hat{D}_t, \hat{w}_t^B, \hat{W}_t^R, \hat{W}_t^S, \hat{K}_t\}$$

where for each non-stationary variable X_t,

$$\hat{X}_t \equiv \frac{X_t}{Z_t}$$

We can stationarize the problem by defining $\hat{V}_t^F = V_t^F / Z_t$. Recall that $Z_t/Z_{t-1} \equiv e^{g_t}$. Then the maximization becomes,

$$\begin{aligned}
\hat{V}^F(P_{t-1}(i), K_t(i), \mathcal{S}_t^F(i)) = & \max_{P_t(i), Y_t(i), N_t^B(i), N_t^R(i), N_t^S(i)} \left\{ \hat{D}_t + \mathrm{E}_t \left[e^{g_{t+1}} \hat{\hat{V}}_{t+1}^F \right] \right\} \\
\hat{\hat{V}}_{t+1}^F = & (1 - F_\rho(\rho^*)) \mathcal{M}_{t+1}^S(\rho \geq \rho^*) \hat{V}^F(P_t(i), K_t(i), \mathcal{S}_{t+1}^F(i; \rho \geq \rho^*)) \\
& + F_\rho(\rho^*) \mathcal{M}_{t+1}^S(\rho < \rho^*) \hat{V}^F(P_t(i), K_t(i), \mathcal{S}_{t+1}^F(i; \rho < \rho^*)) \\
\hat{D}_t(i) = & \frac{P_t(i)}{P_t} \hat{Y}_t(i) - \sum_{j \in \{B,R,S\}} \hat{w}_t^j N_t^j(i) - \frac{\xi}{2} \left(\frac{P_t(i)}{\bar{\Pi} P_{t-1}(i)} - 1 \right)^2 - \hat{I}_t(i) \\
\hat{Y}_t(i) = & \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \hat{Y}_t \\
\hat{Y}_t(i) = & A_t (\hat{K}_t(i))^\alpha (N_t(i))^{1-\alpha} \\
N_t(i) = & (N_t^B(i))^{\gamma_B} (N_t^R(i))^{\gamma_R} (N_t^S(i))^{1-\gamma_B-\gamma_R} \\
\hat{K}_{t+1}(i) = & (1 - \delta_K) \hat{K}_t(i) + \hat{I}_t(i)
\end{aligned}$$

The continuation value takes into account the intermediary's default decision. Dividends are the firm's real profits i.e. revenues less labor costs, price adjustment costs, and investment. As a monopolist, the wholesale firm takes the retail firms' demand function as given, and hires labor to meet that demand given the price it chooses. Thus the problem can be split into a dynamic price-setting problem, and, given that price, a static labor cost minimization problem.

The law of motion for capital indicates that end-of-period capital grows at the economy's growth rate g_{t+1}. This exogenous growth of capital allows me to reduce the dimensionality of the problem by rendering capital accumulation exogenous as well. I fix $K_t(i) = \bar{K} Z_t$. Hence, $\hat{K}_t = \bar{K}$, and investment exactly offsets depreciation i.e. $\hat{I}_t(i) = \delta_K \bar{K}$. The resulting dynamics are equivalent to those in a model with endogenous capital accumulation but infinitely high costs of adjusting investment away from its steady-state level.

A.6.1 Labor Demand

For a given firm i, a price $P_t(i)$ implies sales of quantity $\hat{Y}_t(i)$ which requires a quantity $N_t(i)$ of aggregate labor to produce.The firm minimizes its labor costs subject to the need to hire $N_t(i)$ aggregate labor units:

$$\min_{N_t^B(i), N_t^R(i), N_t^S(i)} \sum_{j \in \{B,R,S\}} \hat{w}_t^j N_t^j(i) - \hat{w}_t(i) [(N_t^B(i))^{\gamma_B} (N_t^R(i))^{\gamma_R} (N_t^S(i))^{\gamma_S} - N_t(i)]$$

where $\gamma_S = 1 - \gamma_B - \gamma_R$ and $\hat{w}_t(i)$ is the Lagrange multiplier on the labor-aggregating technology, with the choice of letter w anticipating the multiplier's interpretation as the shadow cost of labor.

For a given agent's labor $j \in \{B, R, S\}$, the first-order condition is:

$$\hat{w}_t^j(i) = \gamma_j \hat{w}_t(i) \frac{N_t(i)}{N_t^j(i)}$$

where the value of the multiplier is the same for all firms:

$$\hat{w}_t = \prod_{j \in \{B,R,S\}} \left(\frac{\hat{w}_t^j}{\gamma_j} \right)^{\gamma_j}$$

For computing the model, it is useful to define total labor costs $\hat{W}_t(i) = \hat{w}_t N_t(i)$ and labor costs for each type of worker $\hat{W}_t^j(i) = \hat{w}_t^j N_t^j(i)$. Then for all j,

$$\hat{W}_t^j(i) = \gamma_j \hat{W}_t(i)$$

This will allow me to clear the labor market using $\int_0^1 \hat{W}_t^j(i)di = \hat{Y}_t^j$ as opposed to $\int_0^1 N_t^j(i)di = L_t^j$.

A.6.2 Price Setting

Incorporating the solution to the labor cost minimization problem, I rewrite dividends as

$$\hat{D}_t(i) = \frac{P_t(i)}{P_t}\hat{Y}_t(i) - \hat{w}_t N_t(i) - \frac{\xi}{2}\left(\frac{P_t(i)}{\bar{\Pi}P_{t-1}(i)} - 1\right)^2 - \delta_K \bar{K}$$

The FOC of V^F with respect to price $P_t(i)$ is

$$\begin{aligned} 0 &= \frac{\partial \hat{D}_t(i)}{\partial P_t(i)} + \mathrm{E}_t\left[e^{g_{t+1}}\frac{\partial \hat{\hat{V}}_{t+1}^F}{P_t(i)}\right] \\ &= (1-\epsilon)\frac{\hat{Y}_t(i)}{P_t} + \epsilon \hat{w}_t \frac{\partial N_t(i)}{\partial \hat{Y}_t(i)}\frac{\hat{Y}_t(i)}{P_t(i)} - \xi\left(\frac{P_t(i)}{\bar{\Pi}P_{t-1}(i)} - 1\right)\frac{1}{\bar{\Pi}P_{t-1}(i)} + \mathrm{E}_t\left[e^{g_{t+1}}\frac{\partial \hat{\hat{V}}_{t+1}^F}{P_t(i)}\right] \end{aligned}$$

The marginal value of last period's price is

$$\frac{\partial \hat{V}_t(P_{t-1}(i), \mathcal{S}_{t+1}(i,\rho))}{\partial P_{t-1}(i)} = -\xi\left(\frac{P_t(i)}{\bar{\Pi}P_{t-1}(i)} - 1\right)\frac{P_t(i)}{\bar{\Pi}}\frac{1}{(P_{t-1}(i))^2}$$

and

$$\begin{aligned} \frac{\partial \hat{\hat{V}}_{t+1}^F}{\partial P_t(i)} &= (1 - F_\rho(\rho^*))\mathcal{M}_{t+1}^S(\rho \geq \rho^*)\frac{\partial \hat{V}_t(P_{t-1}(i), \mathcal{S}_{t+1}^F(i;\rho \geq \rho^*))}{\partial P_{t-1}(i)} \\ &\quad + F_\rho(\rho^*)\mathcal{M}_{t+1}^S(\rho < \rho^*)\frac{\partial \hat{V}_t(P_{t-1}(i), \mathcal{S}_{t+1}^F(i;\rho < \rho^*))}{\partial P_{t-1}(i)} \end{aligned}$$

And lastly, the marginal cost of labor is

$$\hat{MC}_t \equiv \hat{w}_t \frac{\partial N_t(i)}{\partial \hat{Y}_t(i)} = \frac{\hat{w}_t\left(N_t(i)/\bar{K}\right)^\alpha}{A_t(1-\alpha)}$$

Because the problem is symmetric, $P_t(i) = P_t(j) = P_t$ for all $i, j \in [0,1]$. Define $\Pi_t = \frac{P_t}{P_{t-1}}$. Combining,

$$0 = \left[(\epsilon - 1) - \epsilon \hat{MC}_t\right]\hat{Y}_t + \xi\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)\frac{\Pi_t}{\bar{\Pi}} + \xi \mathrm{E}_t\left[e^{g_{t+1}}\mathcal{E}_{t+1}^F\right] \tag{21}$$

This is the New Keynesian Phillips Curve, where expectations are formed over

$$\begin{aligned} \mathcal{E}_{t+1}^F &= (1 - F_\rho(\rho^*))\mathcal{M}_{t+1}^S(\rho \geq \rho^*)\left(\frac{\Pi_{t+1}(\rho \geq \rho^*)}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}(\rho \geq \rho^*)}{\bar{\Pi}} \\ &\quad + F_\rho(\rho^*)\mathcal{M}_{t+1}^S(\rho < \rho^*)\left(\frac{\Pi_{t+1}(\rho < \rho^*)}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}(\rho < \rho^*)}{\bar{\Pi}} \end{aligned}$$

B Consolidation of the Fiscal and Monetary Authority Balance Sheets

In this model, both fiscal and monetary policies are performed by the same government entity. In the data, fiscal policy is implemented by the Treasury, while monetary policy is the purview of the Federal Reserve System. Large-scale asset purchases were conducted by the Fed, consistd of both Treasuries and MBS, and were funded by an expansion in the Fed's balance sheet and the resulting growth in bank reserves. In this model, Large-scale asset purchases are conducted by the government, consist only of MBS, and are funded by issuance of government debt. Yet an alternative model with disaggregated Fed and Treasury balance sheets would look essentially the same, producing the same prices and consumption allocations.

Replace the government agent above with two entities denoted by T and F. The Treasury T performs fiscal policy as described above, and has positive outstanding debt $-TR_t^T$. In conducting monetary policy, the Fed F buys government debt TR_t^F and mortgages A_t^F, financing its purchases with excess reserves RS_t^F, a liability for the Fed and an asset, RS_t^I, to bankers. The bankers portfolio choice problem now has an additional security – reserves. Their holdings of reserves also serve as collateral for deposits D_t^I, carrying a zero risk weight:

$$q_t^{f,\$} RS_{t+1}^I + \phi_t^I q_t^{\$,B} \geq q_t^{\$} D_t^I$$

Savers hold deposits D_t^S and government debt TR_t^S. The three markets are cleared when

$$\begin{aligned} 0 &= TR_t^F + TR_t^T + TR_t^S \\ 0 &= RS_t^F + RS_t^I \\ 0 &= D_t^I + D_t^S \end{aligned}$$

In this model, all government debt is short-term and risk-free. As such, it is indistinguishable from risk-free bank deposits. Excess reserves are also short-term and risk-free, traded in the federal funds market precisely at the short rate targeted by the monetary authority. Hence, in this alternative model, reserves are indistinguishable from the other kinds of short debt, all trading at a price $q_t^{\$} = q_t^{f,\$}$.

Now consolidate the two government balance sheets and three short-term debt securities. Net short-term debt held by the government is $B_t^G = TR_t^F + TR_t^T + RS_t^F$. Net short-term debt held by bankers is $B_t^I = D_t^I + RS_t^I$, netting out both in their budget constraint and, due to zero risk weights, in the capital requirements, restoring the original leverage constraint. And net short-term debt held by savers is $B_t^S = D_t^S + TR_t^S$, netting out in the savers' budget constraint.

Compare how the Fed conducts large-scale asset purchases in the alternative model with how the government conducts LSAPs in the main model. First, consider LSAPs of Treasuries. The Fed increases its holdings of treasuries by ΔTR_{t+1}^F financing this increase with additional excess reserves $\Delta RS_{t+1}^F = \Delta TR_{t+1}^F$. In equilibrium, this leads to fewer government bonds held by the saver $\Delta TR_{t+1}^S = -\Delta TR_{t+1}^F$ and more reserves held by the banker $\Delta RS_{t+1}^I = -\Delta RS_{t+1}^F$. Given zero risk weight on reserves, the banker can retain his already optimal consumption and other choices by financing the additional reserve asset with additional deposit liability i.e. $\Delta RS_{t+1}^I = -\Delta D_{t+1}^I$. In equilibrium, the additional deposits come from savers $\Delta D_{t+1}^S = -\Delta D_{t+1}^I$. But after consolidating these changes by balance sheet and security into the main model, these changes all net to 0. In a consolidated government balance sheet, reduction in Treasury bond liability is offset by Fed reserve liability. The increase in bankers' reserve asset is offset by an increase in deposit liability. And the decrease in savers' Treasury bond holdings is offset by an increase in bank deposits. When all government debt is short-term, LSAPs of Treasuries are trivially neutral.

Now, consider LSAP of mortgages yielding $\Delta A_{t+1}^G = \Delta A_{t+1}^F$. For both models, in equilibrium the mortgage holdings of the bankers must fall $\Delta A_{t+1}^I = -\Delta A_{t+1}^G$. This purchase in the main model is financed by additional government debt ΔB_t^G, changing equilibrium short-term debt positions of bankers and savers. These changes are model outcomes, but I can denote them as ΔB_t^I and ΔB_t^S, respectively, as long as they satisfy market clearing $\Delta B_t^G + \Delta B_t^S + \Delta B_t^I = 0$.

In the disaggregated model, these purchases ΔA_{t+1}^F are financed by additional reserves ΔRS_{t+1}^F such that $q_t^{\$} RS_{t+1}^F = q_t^{\$,B} A_{t+1}^F$. In equilibrium, the additional reserves are held by banks $\Delta RS_{t+1}^I = -\Delta RS_{t+1}^F$. Given the new asset portfolios, bankers change their deposits by some amount ΔD_{t+1}^I. In equilibrium, these changes are absorbed by savers $\Delta D_{t+1}^I = \Delta D_{t+1}^S$. After consolidation, the net change in the government's short-term position is ΔRS_{t+1}^F; the net change in bankers' short-term position is $\Delta RS_{t+1}^I - \Delta D_{t+1}^I$; and the net change in savers' short-term position is ΔD_{t+1}^S.

If $\Delta RS_{t+1}^F = \Delta B_{t+1}^G$ and $\Delta RS_{t+1}^I - \Delta D_{t+1}^I = \Delta B_t^I$, then $\Delta D_{t+1}^S = \Delta B_t^S$, and both models produce identical consumption allocations and prices. These restrictions guide my calibration, as described in the later section.