Counterparty Risk and Capital Structure

Brent W. Ambrose, Thomas Emmerling, Henry H. Huang
and Yildiray Yildirim [*]

Current version: August 27, 2013

Abstract

The 2007-2009 financial crisis and recession highlighted the role of counterparty risk in financial contracts, many once thought immune to such problems. However, counterparty risk can be significant in a wide variety of contracting situations and can impact capital structure decisions. Using commercial real estate leases as an example, this paper presents a new model that endogenizes the capital structure of both parties to a contract. We follow Grenadier (1996) and Leland and Toft (1996) to examine the interaction between firm capital structures and equilibrium contract pricing. Moreover, in a commercial lease setting, our model demonstrates that consideration of credit risk is instrumental to confirm the complementarity between lease and debt as suggested by Lewis and Schallheim (1992).

Key words: leasing valuation, credit risk, endogenous default
JEL Classification:

1 Introduction

Beginning with the seminal article by Modigliani and Miller (1958) outlining the irrelevance of a firm's capital structure decisions, academic research in finance has expanded in an effort to explain firm capital structures. One important area in this literature is the recognition of the role that relationships among and between a firm's stakeholders can have on shaping a firm's financial decisions. For example, Titman (1984) and Maksimovic and Titman (1991) consider how a firm's capital structure can impact the types of contracts the firm has with its customers. This stream of literature recognizes that when a firm has an interdependent relation with another firm (for example, a unique product that requires investments that might decline in value if the firm liquidates), then the firm may make capital structure decisions in order to maximize the value of these relations. Similiarly, another line in the literature recognizes how firm capital structure decisions can impact management-labor relations. For example, Bronars and Deere (1991), Dasgupta and Sengupta (1993), and Hennessy and Livdan (2009) note that a firm's management can affect their bargaining power over labor unions by altering the firm's debt level to reduce the amount of surplus available to stakeholders. In addition, research using similar logic considers the role that capital structure decisions have on the firm's supply chain relationships (e.g. Kale and Shahrur (2007), Matsa (2010), and Chu (2012).)

More recently, the failure of Lehman Brothers and AIG during the financial crisis of 2007-2009 focused attention on counterparty risk in financial contracts and the impact that a firm's capital structure can have on its counterparties. Furthermore, the depth and length of the recent financial crisis raised awareness of the implications of counterparty risk arising from capital structure decisions to many contracts once thought immune to such problems. For example, on April 16, 2009, General Growth Properties made history as one one of the largest real estate Chapter 11 bankruptcy filings.[1] At the time, General Growth owned or managed over 200 shopping malls with balance sheet assets listed at over $29 billion. While creditors of General Growth were naturally concerned about the prospects of losses arising from the bankruptcy filing, tenants in General Growth malls that have secured leaseholds, which should have made them immune to problems associated with the lessor's bankruptcy, also expressed concern about the impact that

---

[1]Hudson, K. "Mall Titan Enters Chapter 11," *Wall Street Journal* [New York, NY] 16 Apr 2009: B.1.

the bankruptcy filing would have on their leasehold positions.[2] Thus, as the General Growth bankruptcy highlights, counterparty risk can be significant in a wide variety of contracting situations with the result that capital structure decisions can reverberate across seemingly unrelated contracts or relationships. However, to date few papers have explored the complex interactions that arise as a result of counterparty risk in financial contracts and capital structure decisions. We fill this gap by developing a new model that endogenously considers the capital structure of two firms that are linked through a financial contract and thus face counterparty risk on the performance of that contract. We endogenize the counterparty risk within a continuous time structural model that allows us to model firm capital structure decisions without ad hoc assumptions. As a result, our model provides novel insights into how firm capital structures can impact the terms of financial contracts.

As suggested by the General Growth bankruptcy example above, commercial real estate operating leases provide interesting insights into counterparty risks arising from the interlinkages of firm capital structure decisions. Thus, we motivate the discussion and exposition of our model in the context of a commercial real estate lease. In the decade prior to the financial crisis, many real estate owners took advantage of rising property values and declining interest rates to increase their leverage positions. However, during the 2007-2009 financial crisis and recession, the default rate on commercial real estate loans originated prior to the crisis increased substantially. Unfortunately, tenants in properties where the owner faces financial stress are now discovering the risks associated with the default or bankruptcy of their lease counterparty. For example, Sullivan and Kimball (2009) point out that "if the lease was entered into after the landlord's mortgage (or, as is often the case, the lease provides that it is automatically subordinate to any mortgage), the lender's foreclosure action would automatically terminate the lease, wiping out the tenant's right to possession along with its investment in its leasehold improvements." As a result, when a landlord defaults on her mortgage, tenants may find that their leases are terminated. Sullivan and Kimball (2009) also note that in recent years lenders often required standard subordination, nondisturbance and attornment (SNDA) agreements in leases as a condition of obtaining financing. These seemingly benign SNDA agreements

---

[2]Schaefers, A., "General Growth's Bankruptcy Spooks Its Hawaii Tenants," *Hawaii News Archive - Starbulletin.com* Friday, April 17, 2009 (http://archives.starbulletin.com/content/20090417_General_Growths_bankruptcy_spooks_its_Hawaii_tenants).

3

often provide lenders (or purchases at foreclosure) significant rights with respect to the treatment of tenants and leases. For example, a standard lender initiated SNDA may limit the lender's liability in the event of foreclosure to complete lease contracted tenant improvements, or restrict or eliminate any purchase or renewal options specified in the lease.

In addition to risks associated with lessor default and foreclosure on debt, tenants also face the possibility that property owners may file for protection from creditors under the bankruptcy code. If a landlord files for reorganization under Chapter 11 of the bankruptcy code, then the tenant's lease contract is subject to Sections 365 and 363 of the Bankruptcy Code. These sections allow the bankruptcy trustee to either affirm or reject the lease. As a result, tenants with below market rents could find their leases terminated or property services suspended as part of an overall debt restructuring plan.[3]

As the above discussion points out, property owner capital structure decisions clearly impact their future ability to carry out their responsibilities under lease contracts and thus expose tenants to significant counterparty risks. Thus, we present a new model of commercial real estate operating leases that endogenizes both lessee and lessor capital structures into the term structure of lease rates. More specifically, we propose a structural model based on the work developed by Leland and Toft (1996) to effectively link the landlord capital structure to the problem of determining the competitive lease rate.

To preview our results, our model shows the role that potential landlord default plays in determining the competitive lease rate. Specifically, we identify how tenants are compensated (penalized) in the form of lower (higher) lease rates for increasingly (decreasingly) risky financing decisions made by the landlord. We obtain a striking, yet consistent, contrast to previous studies in that *debt and leases can act as complements* when the capital structures of both the landlord and the tenant are considered in the

---

[3]According to Title 11, Chapter 3, Subchapter IV, Section 365 (h)(1), tenants in a lease rejected by the trustee may retain their rights to occupy the space as defined by the lease, but the landlord is released from providing services required under the lease. The tenant will then have to contract separately for those services and may offset the costs of those services from future rent payments. (See Andersen, L., "Impact of Landlord Bankruptcies on Commercial Tenants," *RealEstateLawyers.com* (http://www.realestatelawyers.com/resources/real-estate/commercial-real-estate/landlord-bankruptcy-tenant.htm) and Eisenbach, B., "Commercial Real Estate Leases: How Are They Treated in Bankruptcy?", *In The (Red) The Business Bankruptcy Blog* (Posted October 24, 2006) (http://bankruptcy.cooley.com/2006/10/articles/business-bankruptcy-issues/commercial-real-estate-leases-how-are-they-treated-in-bankruptcy/).) However, Eisenbach (2006) further notes that tenants in a sublease do not have protection under Section 365(h)(1) and thus would have no rights to continue occupying the space if the trustee rejects the original lease. Harvey (1966) also discusses the rights of tenants upon landlord breach under California law.

leasing problem. This finding is consistent with the conclusion obtained by Lewis and Schallheim (1992) in their one-period leasing model. Finally, our numerical implementation also facilitates an examination into the impact of changes in government tax policies upon lease rates. Specifically, we illustrate how differing tax environments can compensate (penalize) counterparties of the lease agreement through the lease rate.

The remainder of the paper is organized as follows. Section 2 summarizes the existing literature on lease valuation. Section 3 presents the setting for determining lease rates and provides for the cases of a risky landlord, risk-less tenant (r_{RN}) and a risky landlord, risky tenant (r_{RR}) as well as a discussion on how they related. Section 4 describes the capital structure setup and Section 5 presents the endogenous decision rules to derive the optimal bankruptcy trigger levels for each firm. Section 6 presents a numerical implementation of the leasing model and discusses the comparative statics to assess the impact of relevant parameters on the term structure of lease rates. Section 7 concludes the discussion of the paper. Finally, an Appendix includes a derivation and formulas pertinent to the proposed model.

2   Literature Review

Traditional models of lease rates, beginning with Lewis and Schallheim (1992) and Grenadier (1996), have long recognized the importance of tenant default and hence tenant credit risk. Recent work by Clapham and Gunnelin (2003), Ambrose and Yildirim (2008), and Agarwal et al. (2011) expanded on these models to explicitly incorporate the interaction of tenant credit risk and capital structure on the endogenous determination of lease rates. These models implicitly and explicitly recognize the risk that tenants may default on their lease obligations. However, as noted above, leases are not one-sided contracts but rather specify rights and responsibilities of both the tenant and the landlord. For example, the typical commercial real estate lease specifies not only the amount of rent owed by the tenant but also the landlord's responsibilities in providing services associated with the contracted space. As a result, the typical lease creates the possibility that either party to the contract might default on the contract exposing both the landlord and the tenant to counterparty risk.

In a recent paper, Agarwal et al. (2011) focus on the tenant's default risk and its

effect on the tenant's capital structure, assuming landlord is default free. Their model is based on the framework originally proposed by Leland and Toft (1996) and examines the interaction of lessee financial decisions and lease rates. Our paper extends this framework to incorporate both lessor and lessee default risks into the term structure of lease rates as well as its endogenous effect on both tenant and landlord capital structures.

Our paper is also related to the works of correlated default modeling (e.g.: Zhou (2001), Yu (2007), Duffie and Singleton (1999), Das et al. (2007), Duffie et al. (2009)), and counterparty credit risk modeling (e.g.: Jarrow and Yu (2001)). However, none of these works consider lease rate term structure modeling and the implied joint capital structure decisions. In addition, our paper contributes to the research on correlated defaults in that the correlated default probability and lessor and tenant capital structures can be endogenously determined, while previous research is either based on reduced form models or exogenous structural models. The advantage of our model is its flexibility in capturing the credit risk interactions between landlord and tenant. Our model also can explain the credit contagion given the large amount of real estate lease utilized by firms (e.g.: Jorion and Zhang (2009)). For example, Liu and Liu (2012) document that the contracting mechanism associated with retail leases such as percentage rents and co-tenancy clauses provides a mechanism for a credit contagion between landlord and tenant. They find that in a good economy, a tenant bankruptcy has a less negative or more positive effect on landlord's stock return. In contrast, our paper examines the credit contagion between landlord to tenant from the angle of credit risk.

3   Determination of Lease Rates

We begin by defining a simple market environment for the purchase of space that fully captures the basic features of the commercial real estate leasing market. For ease of exposition, we describe the setting in terms of the traditional office leasing market but recognize that our model is easily generalizable to other property types (e.g. retail, industrial, etc.) as well as other assets that are commonly leased (e.g. commercial aircraft, computer equipment, etc.).

The office building owner (the landlord) holds the property in a single-asset entity

financed with debt and equity, with the property's future service flows given by:

$$(1) \qquad \frac{dS_{BD}}{S_{BD}} = \mu_s dt + \sigma_s dW_S.$$

In equation (2), S_{BD} represents the service flow after depreciation, μ_s is the drift rate of the service flow process, σ_s is the volatility of this process, and dW_S is the standard Brownian Motion under physical measure $\mathbb{P}$. On the other hand, if we reflect the economic (not accounting) depreciation of the leased asset in the drift rate of the service flow process (denoted by q), we can write the service-flow process after depreciation, S_{AD}, as:

$$(2) \qquad \frac{dS_{AD}}{S_{AD}} = (\mu_s - q)dt + \sigma_s dW_S.$$

Without loss of generality, we assume that debt is in the form of a traditional mortgage secured by the property, the mortgage is senior to any leasehold. Thus, the landlord is a credit-risk lessor.

The landlord leases the property to a firm (the lessee). In the analysis below, we first define the equilibrium lease rate assuming a risk-free lessee (Case 1) and then extend the model to a credit-risk lessee (Case 2). Thus, our model highlights the complex interactions that result in contracts between credit-risky counterparties. In the following cases, we denote the periodic rent payment on a t-period lease contract between a credit-risk (R) lessor and a risk-free (N) lessee as r_{RN}^t and the rental payment on a lease originated between a credit-risk (R) lessor and a credit-risk (R) lessee as r_{RR}^t.

3.1   Case 1: The Risky Landlord and the Risk-Free Tenant

To derive a full equilibrium lease rate, we first model the case assuming a risk-free lessee and a credit-risk lessor. This scenario resembles the situation where a developer builds and leases an office building to the government. For example, investors treat leases where the lessee is the federal government similar to Treasury bonds. The lessor's credit risk results from his decisions regarding capital structure and thus a risk-free lessor is a special case where the lessor has no debt. Since the landlord's ability to provide the contracted service flow may be impacted by default, his default probability is considered in the formulation of the net cost of the lease contract. We denote the lessor's cumulative survival probability as $p_L(u; V_L, V_{L,B})$, the probability density function of lessor default

as $f_L(u; V_L, V_{L,B})$, and $V_{L,B}$ as the lessor's endogenous default boundary. The lessor's expected net cost of providing the lease is the expected present value of the service flows before depreciation minus the tax-shield benefit associated with the depreciation expense before default plus the expected cost claimed by lessee if the lessor defaults:

$$\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru}[S_{BD}(u)\mathrm{d}u - \chi_L Tax_{L,c}(S_{BD}(u)\mathrm{d}u - S_{AD}(u))]\mathbb{1}_{\{\tau_L > u\}}\mathrm{d}u\right]$$

$$+\widetilde{\mathbb{E}}\left[\rho_L^t\left(\int_{\tau_L}^t e^{-ru}[S_{BD}(u) - \chi_L Tax_{L,c}(S_{BD}(u) - S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau_L \leq t\}}\right]$$

$$= \int_0^t e^{-ru}\left[S_{BD}(u)\mathrm{d}u - \chi_L Tax_{L,c}(S_{BD}(u)\mathrm{d}u - S_{AD}(u))\right]p_L(u; V_L, V_{L,B})\mathrm{d}u.$$

(3) $$+ \int_0^t e^{-ru}\mathrm{Damage}_u f_L(u; V_L, V_{L,B})\mathrm{d}u,$$

where $\int_0^t S_{BD}(u)e^{-ru}du$ represents the present value of the service flows before depreciation from time 0 to time t discounted by the risk-free rate under risk-neutral measure $\widetilde{\mathbb{P}}$, and $\int_0^t S_{AD}(u)e^{-ru}du$ represents the present value of the service flow after depreciation under the risk-neutral measure $\widetilde{\mathbb{P}}$. $\widetilde{\mathbb{E}}(\cdot)$ is the expectation under $\widetilde{\mathbb{P}}$. The difference between these two terms is the depreciation cost of the leased asset from time 0 to t. $Tax_{L,c}$ is the lessor's corporate tax rate and χ_L is the depreciation adjustment factor that reconciles the government mandated accounting depreciation to the actual physical depreciation.[4] The indicator function

$$\mathbb{1}_{\{\tau_L \leq t\}} = \begin{cases} 1, & \text{if } \tau_L \leq t, \\ 0, & \text{otherwise.} \end{cases}$$

highlights when a default occurs. Thus, the first term in equation (3) is the the expected present value of the service flows before depreciation minus the tax-shield benefit associated with the depreciation expense before default, and the second term is the expected cost claimed by tenants upon landlord default.

If the landlord or debtor rejects the lease, we have to consider two scenarios: In scenario 1, the tenant leaves the leased property and files a claim equal to her loss due to landlord's default. That loss might be the due to the inability to use the leased property or the loss associated with having to temporarily stop its business operation. In scenario

---

[4]See Agarwal et al. (2011).

2, the tenant remains in the leased property and the landlord continues to pay the cost of providing the contractual service flow. However, the tenant may be responsible for additional costs, such as power, heat and trash disposal and thus we can assume that damage is a proportion of the present value of future service flows.

Therefore, we define "Damage" in (3) as the landlord's cost upon defaulting on the lease contract, which equals a percentage of the present value of future service flows,

$$(4) \quad \text{Damage}_\tau = \rho_L^t \int_\tau^t e^{-ru} \left[S_{BD}(u)du - \chi_L Tax_{L,c} \left(S_{BD}(u)du - S_{AD}(u) \right) \right] du$$

where τ is landlord's bankruptcy time point. For simplicity, we assume the recovery rate ρ_L^t is constant and independent of t, i.e., $\rho_L^t = \rho_L$. Since the leased property is still in the hands of the landlord, he is responsible for the depreciation expense. Thus, the sum of the two terms in equation (3) is the expected net cost of providing the leased property from the lessor's perspective, recognizing the tax-shield benefit associated with the depreciation expense.

In a competitive market, the expected net cost of the lease exactly equals the present value of the future lease payments if the tenant does not default. Thus the expected cost of the lease is $\int_0^t r_{RN}^t e^{-ru} du = r_{RN}^t (\frac{1-e^{-rt}}{r})$, where r_{RN}^t denotes the operating lease rent with maturity t for a combination of a risky landlord and a risk-free tenant. We can solve for the lease rate r_{RN}^t by setting equation (3) equal to $r_{RN}^t (\frac{1-e^{-rt}}{r})$. Thus, assuming the

asset service flow follows (2), then the lease rate is:[5]

$$
\begin{aligned}
r_{RN}^t \;=\; & \frac{r}{1 - e^{-rt}} \\[4pt]
& \times \Bigg[(1 - \chi_L Tax_{L,c}) \int_0^t S_{BD}(0) e^{(\mu_S - r - \delta\sigma_S)u}(1 - F(u; V_L, V_{L,B})) \mathrm{d}u \\[4pt]
& + \chi_L Tax_{L,c} \int_0^t S_{AD}(0) e^{(\mu_S - r - q - \delta\sigma_S)u}(1 - F(u; V_L, V_{L,B})) \mathrm{d}u \\[4pt]
& + \rho_L^t \Bigg((1 - \chi_L Tax_{L,c}) \\[4pt]
& \times \frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S} \left(e^{(\mu_S - r - \delta\sigma_S)t} F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S - r - \delta\sigma_S)u} f_L(u; V_L, V_{L,B}) \mathrm{d}u \right) \\[4pt]
& + \chi_L Tax_{L,c} \frac{S_{AD}(0)}{\mu_S - r - q - \delta\sigma_S} \\[4pt]
& \times \left(e^{(\mu_S - r - q - \delta\sigma_S)t} F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S - r - q - \delta\sigma_S)u} f_L(u; V_L, V_{L,B}) \mathrm{d}u \right) \Bigg) \Bigg].
\end{aligned}
$$

(5)

where δ denotes the market price of risk for the service value process.

3.2   Case 2: The Risky Landlord and The Risky Tenant

We now examine the lease contract assuming a credit-risk tenant. Recall that the present value of a lease with maturity t to a risk-free tenant is $r_{RN}^t(\frac{1 - e^{-rt}}{r})$. Similarly, the value of the lease when both landlord and tenant have credit risk is the present value the lease rate (r_{RR}^t) from origination to default time τ, and the recovery of remaining lease rentals from time τ to maturity time t. Under these conditions, we can express the value of the default-risky lease as:

$$
(6) \qquad \int_0^t e^{-r\tau} r_{RR}^t \left(1 - F_T(\tau; V_T, V_{T,B}) \right) d\tau + \int_0^t e^{-r\tau} \rho_R^t R_{T,RR}^{t-\tau} f_T(\tau; V_T, V_{T,B}) d\tau
$$

where $F_T(\tau; V_T, V_{T,B})$ is the tenant's cumulative default probability up to time τ under measure $\widetilde{P}$, $f_T(\tau; V_T, V_{T,B})$ is the tenant's instantaneous default probability under measure $\widetilde{P}$ at time τ, and ρ_R^t is the tenant's recovery rate. $R_{T,RR}^{t-\tau}$ is the present value of the remaining lease payments, and it can be expressed as $r_{RR}^t(\frac{1 - e^{-r(t-\tau)}}{r})$. The first term in (6) represents the expected discounted lease payment flows from 0 to τ. The second term represents the expected discounted value of the remaining lease payments after default.

---

[5]See the Appendix for the proof.

Following the arguments in Grenadier (1996) that any two methods of selling an asset's service flow for t-years must have the same value, then in equilibrium the lease values in case 1 must equal case 2. Thus, we can combine equations (5) and (6)

$$r_{RN}^t \left(\frac{1 - e^{-rt}}{r} \right)$$
$$= \int_0^t e^{-r\tau} r_{RR}^t \left(1 - F_T(\tau; V_T, V_{T,B}) \right) d\tau + \int_0^t e^{-r\tau} \rho_R^t R_{T,RR}^{t-\tau} f_T(\tau; V_T, V_{T,B}) d\tau$$

and express the relationship between the lease rates in cases 1 and 2 with a maturity of t as:

$$(7) \qquad r_{RR}^t = r_{RN}^t \left[\frac{1 - e^{-rt}}{(1 - e^{-rt}) - (1 - \rho_R^t)(G_T(t) - F_T(t) e^{-rt})} \right],$$

where[6]

$$(8) \qquad G_T(t) := \int_0^t e^{-r\tau} f_T(\tau; V_T, V_{T,B}) d\tau.$$

Equation (7) shows the relation between the risky lease rate and the risk-free lease rate. The denominator represents the discount factor associated with a default-risky lease, and the numerator is the discount factor associated with a risk-free lease. The first part of denominator is the default-free discount factor which is the same as the numerator. The second part is the loss rate $(1 - \rho_R^t)$ times a difference of discounted default probabilities; a positive quantity [7]. From this equation, when the lessee's default probability increases, implying $(G_T(t) - F_T(t) e^{-rt})$ increases, the value of the denominator decreases. Hence, the risky lease rate increases to compensate for the increase in default probability. In addition, when the expected recovery rate increases, the lessor recovers more when the lessee defaults, and thus, the risky lease rate decreases, all else being equal.

4 Capital Structure

The purpose of our analysis is to incorporate the effects of lease credit risk on lessor and lessee capital structure in order to determine the net effects of capital structure on the

---

[6]The Appendix also contains the derivation of equation 7, similar to Agarwal et al. (2011).

[7]$(G(t) - F(t) e^{-rt}) = \int_0^t e^{-r\tau} f(\tau; V, V_B) d\tau - \int_0^t e^{-rt} f(\tau; V, V_B) d\tau = \int_0^t (e^{-r\tau} - e^{-rt}) f(\tau; V, V_B) d\tau > 0$.

equilibrium term structure of lease rates.[8]

Following Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and Leland and Toft (1996), we assume the lessor owns a property whose unleveraged value V_L follows a continuous diffusion process with constant proportional volatility σ_{V_T}:

$$(9) \qquad \frac{dV_L}{V_L} = \left(\mu_{V_L}(t) - \delta_{V_L}\right) dt + \sigma_{V_L} dW_{V_L},$$

where $\mu_{V_L}(t)$ denotes the landlord's total expected rate of return on asset V_L, δ_{V_L} is the landlord's constant fraction of value paid out to all security holders, and dW_{V_L} is the increment of a standard Brownian motion.

Since the landlord's capital structure is composed of debt and equity, we assume for the sake of simplicity that the landlord has a single debt issue with maturity t, having periodic coupon $(c_L(t))$ and principal $(p_L(t))$ payments. Upon bankruptcy, the bondholder forecloses on the debt and recovers a fraction $\rho_{L,D}(t)$ of the firm's net asset value of $\widetilde{V}_{L,B}$, where $\widetilde{V}_{L,B}$ equals the net asset value after bankruptcy costs plus the present value of lessor's recovery lease payments at the time of default. In other words, $\rho_{L,D}(t)$ is the bondholder's recovery rate for a debt with maturity t. Thus, we can write the value of risky debt as:

$$
\begin{aligned}
d_L(V_L; V_{L,B}, t) &= \int_0^t e^{-r\tau} c_L(t) \left(1 - F_L(\tau; V_L, V_{L,B})\right) d\tau + p_L(t) e^{-rt} \left(1 - F_L(t; V_L, V_{L,B})\right) \\
(10) &\qquad + \int_0^t e^{-r\tau} \rho_{L,D}(t) \widetilde{V}_{L,B} f_L(\tau; V_L, V_{L,B}) d\tau
\end{aligned}
$$

If the firm does not declare bankruptcy, then the first term on the right hand side of (10) represents the present value of coupon payments, and the second term represents the present value of the principal payment, respectively. The third term represents the present value of the net asset value accruing to the debt holders if bankruptcy occurs. Thus, we can rewrite equation (10) as:

$$
\begin{aligned}
d_L(V_L; V_{L,B}, t) &= \frac{c_L(t)}{r} \left(1 - e^{-rt}\right) - \frac{c_L(t)}{r} \left(G_L(t) - F_L(t) e^{-rt}\right) + e^{-rt} p_L(t) \left(1 - F_L(t)\right) \\
(11) &\qquad + \int_0^t e^{-r\tau} \rho_{L,D}(t) \widetilde{V}_{L,B} f_L(\tau; V_L, V_{L,B}) d\tau.
\end{aligned}
$$

---

[8]For the Tenant's capital structure incorporating the lease and debt, refer to section IV of Agarwal et al. (2011).

We assume that when landlord defaults, he receives an automatic liquidation stay from the bankruptcy court. Given this assumption, the landlord's default boundary is:

$$(12) \qquad \widetilde{V}_{L,B} = (1 - \alpha_L) V_{L,B}$$

where α_L is the proportion of firm value loss when landlord firm goes bankrupt, and (12) is consistent with the ordinary trade-off theory of optimal capital structure theory.

5 Determining the Endogenous Default Boundary

We assume that the debt side of the tenant's balance sheet is composed of lease, debt, and equity.[9] Thus, reproducing equation (17) in Agarwal et al. (2011) here, we note that the tenant's endogenous bankruptcy boundary appears as

$$(13) \qquad V_{T,B}^* = \frac{\frac{\Omega_R}{r}(K_1^{T_L} - K_2^{T_L}) - K_3 - K_4 + M - \left(P_T - \frac{C_T}{r}\right) K_1^{T_T,D} - \left(\frac{C_T}{r}\right) K_2^{T_T,D}}{1 + \alpha x_T - (1 - \alpha) K_2^{T_T,D}},$$

with the distinguishing feature that, within our current analysis, the lease rate (Ω_R) is dependent upon the landlord's optimal bankruptcy boundary $V_{L,B}$, which is not present in the corresponding lease rate in Agarwal et al. (2011).[10] Indeed, the lease rate $r_{RR}^{T_L}$ found using equations (5) and (7) is a function of the landlord's bankruptcy boundary $V_{L,B}$. Thus, in this section, we identify the optimal bankruptcy boundary $V_{L,B}^*$ for the landlord that is inserted into equations (5) and (7) in order to determine the lease rate $r_{RR}^{T_L}$.

Similar to the tenant, we assume the landlord trades off the tax benefits and the bankruptcy costs of debt financing. Since, we incorporate lease financing into the capital structure decision, the tax deductibility benefit of the landlord equals the interest expense on the debt and the depreciation expense of the leased asset. Following Leland (1994), the total market value of the landlord $(v_L(V_L; V_{L,B}))$ equals the unleveraged firm value plus the tax benefit of debt and lease financing minus the bankruptcy cost during the

---

[9]In an operating lease, the present value of lease expenses are not listed on the debt side of the balance sheet and the operating lease expenses for the future 5 years are only listed as a footnote of the balance sheet. However, in terms of cash flows, the lessee firm will expend lease payments in exchange for the leased asset's service flows that generate operating cash flows for the firm. Therefore, in terms of cash flows, we treat the present value lease expenses as a part of the lessee firm's debt side on the balance sheet.

[10]The definitions of K_1^T, K_2^T, K_3, K_4, and M in equation (13) appear in the Appendix.

observation period:

$$v_L(V_L; V_{L,B}) = V_L + Tax_{L,c}\left(\frac{C_L}{r} + \frac{\text{Dep}}{r}\right)\left(1 - \left(\frac{V_{L,B}}{V_L}\right)^x\right)$$

$$(14) \qquad\qquad - (\alpha_L V_{L,B} + \text{damage by default})\left(\frac{V_{L,B}}{V_L}\right)^x$$

where V_L is the unleveraged firm value, $Tax_{L,c}$ is the corporate tax rate of the lessor, and $x_L = a_L + z_L$, where a_L and z_L are defined in the Appendix. The second term in (14) represents the tax-benefits associated with interest rate expense and depreciation expense given that the landlord does not default. The third term in (14) is the bankruptcy cost given that the landlord defaults and includes bankruptcy costs documented by Warner (1977) and the damage compensation to the tenant for the landlord's default. To model the periodic depreciation expense, Dep, we assume the leased asset is linearly depreciated and the landlord has a stationary lease structure; thus, the total depreciation expense for the life of the leased asset (T_{Life}) is $\widetilde{\mathbb{E}}\left[\int_0^{T_{\text{Life}}} S_{BD}(u)e^{-ru}du - \int_0^{T_{\text{Life}}} S_{AD}(u)e^{-ru}du\right]$. If we amortize the total expense to a periodic expense, the periodic expense, Dep, is $\widetilde{\mathbb{E}}\left[r \times \left(\int_0^{T_{\text{Life}}} S_{BD}(u)e^{-ru}du - \int_0^{T_{\text{Life}}} S_{AD}(u)e^{-ru}du\right)\right]$ because Dep/r is the total life-long depreciation expense. In this setting, equation (14) is consistent with traditional capital structure trade-off theory that assumes the tax-shield benefit has a positive effect on firm value while bankruptcy costs have a negative effect.

Note that "damage by default" is equal to the second term on the left hand side of (3). To simplify the analysis, we assume that on average the lessor defaults at the midpoint of the lease contract such that the damage owed to the tenant equals

$$\text{damage by default} = \rho_L \widetilde{\mathbb{E}}\left[\int_{T_L/2}^{T_L} e^{-r(u-\frac{T_L}{2})}[S_{BD}(u)\mathrm{d}u - \chi_L Tax_{T,c}(S_{BD}(u)\mathrm{d}u - S_{AD}(u))]\mathrm{d}u\right]$$

$$= \rho_L e^{r\frac{T_L}{2}}\left[(1 - \chi_L Tax_{T,c})\frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S}\left(e^{(\mu_S - r - \delta\sigma_S)T_L} - e^{(\mu_S - r - \delta\sigma_S)\frac{T_L}{2}}\right)\right.$$

$$\left. + \frac{S_{AD}(0)}{\mu_S - r - \delta\sigma_S - q}\chi_L Tax_{T,c}\left(e^{(\mu_S - r - \delta\sigma_S - q)T_L} - e^{(\mu_S - r - \delta\sigma_S - q)\frac{T_L}{2}}\right)\right],$$

$$\text{if } \mu_S - \delta\sigma_S - q - r \neq 0$$

where ρ_L is the recovery rate for lost service flows and δ is the market price of risk.

To determine the tenant's default boundary in this model, we apply the smoothing-pasting condition in Leland and Toft (1996) and solve the equation to determine the endogenous default boundary, $V_{L,B}$. Let[11]

$$(15) \qquad \left. \frac{\partial E_L \left(V_L; V_{L,B}, T_{L,D} \right)}{\partial V_L} \right|_{V_L = V_{L,B}} = 0$$

By solving equation (15), we find the endogenous bankruptcy boundary as:

(16)
$$V_{L,B}^* = \frac{(C_L/r)(A/(rT_{L,D}) - B) - AP_L/(rT_{L,D}) - (Tax_{L,c}(C_L + \text{Dep})/r + \text{damage by default})x_L}{1 + \alpha x_L - (1 - \alpha)B},$$

where A and B are defined in the Appendix and coincide with the same identifications established in Leland and Toft (1996). We then simultaneously solve for the landlord and tenant optimal bankruptcy levels by equations (16) and (13).

6 Numerical Implementation

In this section, we discuss a numerical implementation of our model. The construction of our model facilitates a separation of the interdependency between the lessor's and lessee's capital structure in determining the competitive lease rate. We divide the numerical implementation into two parts: First, we find the optimal endogenous bankruptcy boundary for the landlord ($V_{L,B}^*$). Second, we use $V_{L,B}^*$ to calculate, respectively: (a) the risky landlord and risk-less tenant lease rate (r_{RN}) via equation (5); (b) the risky landlord and risky tenant lease rate (r_{RR}) via equation (7); and (c) the tenant's optimal endogenous boundary ($V_{T,B}^*$) via equation (13). We then use the optimal boundaries ($V_{L,B}^*$ and $V_{T,B}^*$) to calculate the tenant and landlord debt and equity values.

Our numerical implementation to determine $V_{L,B}^*$ is similar to the procedure carried out in Leland and Toft (1996). We begin by solving for the lessor's endogenous default boundary values ($V_{L,B}$) for a set of debt contracts characterized by the combination of principal and coupon (P, C) taken over the principal range $[0.5, 100]$ with steps $\triangle P = 0.5$. As in Leland and Toft (1996), we assume the coupon (C) is set so that newly-issued debt sells at par value $(d(V; c, p)|_{V_L = V_L(0)} = p$, where $p = P/T_{L,D}$ and $c = C/T_{L,D}$.) We

---

[11]$v_L = E_L + D_L$, therefore, $E_L = v_L - D_L$

use the bisection method to solve $d = p$ in order to obtain C for a given P. After obtaining the debt contract pair (P, C), we then calculate the corresponding endogenous default boundary $(V_{L,B})$. Once we obtain the set of endogenous default boundaries that correspond to the set of debt principal and coupon contracts, we then select the debt contract (P, C) that maximizes the landlord's value $(v_L(V_L; V_{L,B}))$.

Given the landlord's endogenous boundary $V_{L,B}^*$, we then calculate the equilibrium lease rate and find the tenant's optimal capital structure. The numerical method for doing so, follows the procedure carried out in Agarwal et al. (2011). Recall that Leland and Toft (1996) demonstrate that a firm's optimal default boundary (V_B) can be calculated given the debt contract combination (P, C). However, from (13), we see that the tenant's optimal default boundary $(V_{T,B}^*)$ also depends upon the risky lease rent $r_{RR}^{T_L}$ as well as the debt contract. Thus, even if we fix P, we cannot directly solve for C satisfying $d(V; V_{T,B}, t) = p$ since $r_{RR}^{T_L}$ is also unknown.[12] Note that this equation is equivalent to the requirement that newly issued leases are issued at their "par" value, i.e., equal to the expected present value of service flows, and is the natural extension for leases to the condition in Leland and Toft (1996) that requires new debt to be issued at "par" value. As a result, we input into $d(V; V_{T,B}, t) = p$ the value $r_{RR}^{T_L}$ that satisfies (7). In other words, the numerical task is to find the combination if c and $r_{RR}^{T_L}$ that satisfies both $d(V; V_B, t) = p$ and equation (7) for a given P.

Our extension of Leland and Toft (1996) involves solving a two-dimensional system of nonlinear equations as follows. First, we specify a principal and coupon range: $[0.5, 100] \times [(0.01)P, (0.1)P]$. We then fix the pair (P, C) and numerically solve (via the bisection method) equation (7) for $r_{RR}^{T_L}$.[13] Upon obtaining a solution to (7), we then check whether the value r_{RR} also satisfies $d(V; V_{T,B}, t) = p$. If it does, then the pair $(C, r_{RR}^{T_L})$ represents a solution to the two-dimensional system. If r_{RR} does not satisfy $d(V; V_{T,B}, t) = p$ for the fixed set (P, C), we record this error, increment the coupon by $\triangle C$ and repeat the process.[14] We continue this process until $d(V; V_{T,B}, t) = p$ is satisfied or until $C = (0.1)P$.[15]

---

[12]In order to make the problem tractable, we recognize that we do know the "correct" $r_{RR}^{T_L}$ that satisfies (7).

[13]We note that the functions G_T and F_T are also functions of $r_{RR}^{T_L}$ through $V_{T,B}$. This significantly complicates the equation.

[14]We set $\triangle C = 0.01$.

[15]If $C = (0.1)P$ and we have not found a solution, we consider the pair $(C, r_{RR}^{T_L})$ corresponding to the smallest recorded error to be the approximate solution to the two-dimensional system.

After obtaining P, C, and $r_{RR}^{T_L}$, we then calculate the endogenous tenant default boundary ($V_{T,B}$) and capital structure corresponding to the pair (P, C) that maximizes the firm value $v_T(V_T; V_{T,B})$. The endogenous boundary corresponding to this capital structure is the optimal endogenous boundary for the tenant $V_{T,B}^*$. With this boundary, we then calculate the value of the tenant's debt and equity.

Table 1 presents the base case parameters used in the analysis to follow. Our base case parameters match those in the literature allowing for comparison of our results with previous studies. Table 2 shows the relationship between the probability of default on the lessor's existing debt and the lease term structure. Specifically, we consider three cases of tenant debt: short-term ($T_D = 5$ years), medium-term ($T_D = 10$ years) and long-term ($T_D = 20$ years) across short- and medium-term lease maturities ($T_L = 5, 10$), assuming the landlord's debt maturity remains fixed at 5-years. Later, we relax this assumption and consider the effect of the landlord moving from short-term debt (5-years) to medium-term debt (10-years). In Table 2, the third row within each tenant debt block displays the optimal endogenous default boundaries for the landlord and tenant. The other rows consider alternative exogenous landlord default boundary values and the corresponding implied endogenous tenant default boundary, default probability, and lease rate. As will be noted below, the interactive effects of tenant and landlord default probabilities with lease rates are non-linear and depend upon the lease term (5-years or 10-years).

6.1 Impact of Landlord Default Probability.

The first column in Table 2 shows the landlord's bankruptcy boundary with the third row in each block being the endogenous default boundary. As expected, we note that the landlord's default probability (column 2) increases as the default boundary increases. Columns (3) and (4) show the tenant's implied endogenous default boundary and probability that correspond to the landlord's default boundary while columns (5) and (6) show the equilibrium lease rates that correspond to a risk-free tenant (r_{RN}) and a tenant with credit-risk (r_{RR}), respectively.

As expected, we see that as the landlord's default probability increases, the equilibrium lease rate declines regardless of lease maturity. The effect of a shift in landlord risk is most evident under the case where the tenant is risk-free and the lease is long-term (10-years). In this scenario, the tenant has no default risk and thus the tenant's capital

structure has no impact on the equilibrium lease rate (r_{RN}). As a result, we note that an increase in landlord default probability from 0.1% to 36% results in a 19.35% decrease in the lease rate (from 0.638 to 0.515). However, we note that, as expected, shorter term leases mitigate the impact of landlord credit risk and thus the impact of an increase in counterparty risk is lower. For example, when the lease maturity is only 5-years, the lease rate declines only 8.2% as landlord default probability increases. We also observe a similar, but less dramatic effect when the tenant is not risk-free (r_{RR}). However, the effect is complicated since now the tenant's capital structure also impacts the lease rate. Thus, as intuition suggests we conclude that tenants face lower equilibrium lease rates as their counter-party's risk increases and this risk increases with exposure to the landlord through lease maturity. Furthermore, these results confirm that credit contagion can be amplified through long-term off balance sheet contracts.

Finally, columns (7) through (10) show the endogenous tenant capital structure that results from contracting with a risky landlord. Overall, we observe that the use of leverage increases as the lease counter-party risk increases. For example, when both lease and tenant debt are long-term, the tenant's leverage ratio increases 74.58% (from 0.1503 to 0.2624) in response to an increase in the landlord's default probability. This phenomenon conforms with intuition that tenant firms have an increasing preference for debt as landlord riskiness increases.

6.2   Impact of Tenant Debt Maturity.

Table 2 also allows us to consider the impact on lease rates to changes to tenant debt maturity. To do so, we focus on the optimal endogenous landlord default boundary case (row three in each tenant debt maturity block highlighted in italics). First and intuitively obvious, we see that tenant default probabilities increase with longer debt maturities (rising from 14.9% to 66.2% as maturity increases from 5-years to 20-years). Next, we consider the increase in tenant debt maturity from 5-years to 20-years and note that the tenant's capital structure also impacts the competitive lease rate. Comparing lease rates for short-term debt and long-term debt evaluated at the landlord's optimal endogenous default boundary (and holding all else constant), we see that lease rates are positively related to tenant debt maturity irrespective of the lease maturity date and the landlord default boundary. In other words, the results show that landlords are compensated in

18

the form of higher lease rates for riskier tenant firms; this intuitive phenomenon was also observed in the risk-less landlord case examined by Agarwal et al. (2011).

6.3   Impact of Landlord Debt Maturity.

Just as the tenant's capital structure impacts the competitive lease rate, we also expect financing decisions made by the landlord to influence lease rates. To illustrate these effects, Table 3 compares the landlord and tenant default probabilities and lease rates assuming the landlord issues short or medium term debt (5-years and 10-years) while the tenant issues short, medium, and long-term debt (5, 10, and 20-years). The results in Table 3 clearly demonstrate that, as predicted, long-term debt issuance by the landlord, which effectively increases the landlord's credit risk, reduces the lease rate, i.e., the tenant lease payment is reduced for riskier landlord firms. For example, when the lease is short term (5-years) and the landlord and tenant use short-term debt, the endogenous landlord default boundary is 43.84 with an implied default probability of 11.6%. However, as the landlord's debt maturity increases to 10-years the endogenous landlord default boundary increases to 47.17 with an implied default probability of 38.7%. This increase of landlord default risk translates into a lower lease rate (0.787 versus 0.781). However, we observe an interesting non-linear phenomenon on lease rates as tenant debt maturity changes. For example, holding landlord debt maturity constant at 10-years we see that the 5-year maturity lease rate first declines (from 0.781 to 0.779) as tenant debt maturity increases from 5 to 10-years and then rises (from 0.779 to 0.789) as debt maturity increases to 20-years. As a result, Table 3 reveals interesting new insights regarding the term structure of lease rates that have been ignored in previous studies that did not consider the endogenous counter-party risks.

6.4   The Term Structure of Leases

Figure 1 highlights the effects of changes in landlord and tenant debt maturities and riskiness on the equilibrium term structure of lease rates. The figure highlights the lease term structure that prevails under assumptions that both the landlord and tenant have short-term (5-year) debt and long-term (10-year) debt. In addition, we highlight the shift in the lease term structure that occurs when the landlord becomes risky. While we see that the lease term structure is downward sloping, Figure 1 reveals two interesting

results. First, we note that when we move from a risk-free to a risky landlord the lease term structure becomes steeper, indicating that long-term leases are discounted in the presence of landlord risk. The intuition for the discount is that long-term leases increase tenant exposure to potential landlord default and, in equilibrium, the reduction in rent compensates the tenant for this increase in risk. Second, in the case of risky landlord and tenant, we observe that the impact of debt maturity dissipates as the lease term increases (the equilibrium rental rates converge). Notice that for risky landlord and tenant, convergence of rates begin approximately after the 10 year lease maturity; for the riskless landlord, convergence begins after the 15 year lease maturity. This phenomenon reflects how lengthening of lease maturities beyond both parties' debt maturity renders both short and medium term debt to be viewed as similar risks.

6.5   Impact of Tenant Default

Table 4 shows the relation between the probability of default on the tenant's existing debt and the lease term structure. As before, we examine the lease term structure when the tenant firm issues short-term debt (5-year), intermediate-term debt (10-year), and long-term debt (20-year). Italicized entries in each block indicate optimal endogenous default boundaries for tenant and landlord. These italicized entries along with the corresponding lease rate are the base case within each block. Within each block, we change the default boundary to highlight the impact of the probability of debt default. For a fixed optimal landlord boundary $(43.84, 43.75)$ and suboptimal tenant boundary $(30, 40, 60)$ we calculate the lease rate r_{RR} satisfying equation (7) via the bisection method. Notice this task is much easier using suboptimal boundaries since the right-hand side of (7) is now independent of r_{RR}.

In Table 4, we first notice that the lease rate is increasing in the tenant default boundary. For example, when $T_L = 5$ and $T_D = 5$ the lease rate increases from 0.770 to 0.849 as the tenant default boundary increases. This is expected since the landlord should be compensated for increased tenant default likelihood. Additionally, we observe that the lease rates in the rows corresponding to the default boundaries of 30, 40, and 60 of each block do not change (holding lease maturity constant) as tenant debt maturity increases $(T_D = 5, 10, 20)$. This is due to the fact that once the tenant default boundary is determined, the tenant debt maturity does not enter into equation (7); the time variable

in (7) refers to lease length T_L.

6.6 Debt and Lease as Complements

When there is no landlord default risk, Agarwal et al. (2011) observe the traditional argument in financial theory that debt and lease are substitutes; namely, the default probability of the tenant decreases with lease maturity while the lease rate increases with lease maturity. However, when landlord default risk is a possibility, we no longer see this phenomenon. Indeed, in Table 4 an increase in the tenant's debt maturity (from 5 years to 20 years) increases the tenant debt probability ($\lambda_{D,T}$) from 0.149 to 0.662 while the lease rate increases from 0.787 to 0.795, holding lease maturity constant at 5-years. Furthermore, if we keep the tenant debt maturity fixed (for example, $T_{D,T} = 20$), then we find that an increase in the lease maturity from 5 years to 10 years decreases the lease rate from 0.795 to 0.668, and has little impact on tenant debt default probability. This complementary behavior between debt and lease is also observed in the one-period analysis conducted by Lewis and Schallheim (1992). However, this effect is absent from the traditional literature that examines the term structure of lease rates. Thus, our analysis confirms that incorporating the capital structure of the landlord is instrumental to observing this complementary behavior.

6.7 Impact of Taxes and Depreciation

Table 5 highlights how differences in landlord and tenant tax rates and changes in overall tax policy can affect the equilibrium lease rate. Recall from our model that the lease rate is a function of the landlord and tenant marginal corporate tax rates as well as the tax treatment of economic depreciation (q) as reflected in χ. As noted above, $\chi = 1$ reflects the case that accounting and economic depreciation are equivalent, while $\chi < 1$ reflects the condition that the tax deduction accepted with depreciation is less than that of the full economic depreciation. Thus, by varying χ, we can observe how changes in the depreciation schedules associated with the leased asset impact lease rates.

First, we consider how changes in the tenant's tax rate affect the lease rate. We see that for a fixed landlord tax rate, the lease rate increases for higher tenant tax rates. For instance, when $\chi = 0.5$, and the landlord tax rate is 0.25, the lease rate increases from 0.782 to 0.819 as the tenant's tax rate increases from 0.25 to 0.40. This behavior, also

21

observed in Agarwal et al. (2011), results from the incentives that higher tax rates create for the tenant to utilize more debt, which in turn, makes the firm riskier.

Similarly, we can observe how changes in the landlord's tax rate affect the lease rate. The results indicate that for a fixed tenant tax rate, the lease rate decreases for higher landlord tax rates; notice that the lease rates decreases from 0.782 to 0.748 as one moves along the first row of the table (holding tenant tax rate constant at 0.25). Once again, the incentive to utilize debt to take advantage of tax shields makes the landlord riskier for which the tenant is compensated in the form a lower lease rate.

However, changes in tax policies normally impact both firms simultaneously. By examining the diagonal elements in each block in Table 5, we can see the impact on the equilibrium lease rate of increasing corporate taxes. Since the increase in corporate taxes alters both the landlord's and tenant's incentives to use debt in the same direction, Table 5 shows that the equilibrium lease rate remains virtually unchanged as tax rates increase. Thus, our analysis confirms that when both parties to a contract face the same tax environment, changes in tax policies should have no impact on the contract pricing. It is only in cases where changes in tax policy differentially impact one party over another that we should observe changes in the equilibrium contract pricing.

Finally, the effect of allowing the landlord to accelerate depreciation of the leased asset ($\chi = 0.5$ to $\chi = 1.5$) results in lower equilibrium lease rates; compare for example 0.782 (first row, first column) to 0.763 (seventh row, first column). This phenomenon is expected as tax benefits to the landlord are passed to the tenant in the form of lower equilibrium lease rates.

7   Conclusion

Using commercial real estate as the motivating example, we develop a continuous time structural model to consider how the endogenous capital structure decisions of landlords and tenants interact to determine the equilibrium lease rates. Thus, we provide a mechanism to illustrate the credit contagion that results between tenant and landlord through the lease contract. Our analysis also highlights a little known aspect of how the riskiness of counter-parties to a firm's off-balance sheet financing tools (such as leases) can impact the firm's capital structure decisions. As a result, our model illustrates the complexity

and associated endogenous relationships that accompany corporate financing decisions.

Our numerical analysis provides a number of empirical predictions. First, our model predicts that tenants face lower equilibrium lease rates as their counterparty's risk increases and this risk increases with exposure to the landlord through lease maturity. In addition, the numerical results show that credit contagion can be amplified through long-term off balance sheet contracts. In other words, when the landlord's credit condition deteriorates, tenant debt default probability increases through the interaction of the lease contract and the firm's capital structure. Second, our model indicates that the use of leverage should increase as the firm's counterparty's risk increases. In the context of real estate leases, this suggests that tenants have an increasing preference for debt as their landlord riskiness increases. Third, our model confirms the intuitive phenomenon that landlords should be compensated with higher lease rates when renting to riskier firms. Fourth, our model provides the novel prediction that the downward sloping term structure of lease rates should become steeper as the landlord risk increases, indicating that long-term leases are discounted more heavily in the presence of landlord risk. Finally, we also show that debt and lease are complimentary as suggested by Lewis and Schallheim (1992) when landlord default is possible. Thus, our analysis confirms that incorporating the capital structure of the landlord is instrumental to observing this complementary behavior.

References

Agarwal, S., Ambrose, B. W., Huang, H., and Yildirim, Y. The term structure of lease rates with endogenous default triggers and tenant capital structure: Theory and evidence. *Journal of Financial and Quantitative Analysis*, **46**(2):553–584 (2011).

Ambrose, B. W. and Yildirim, Y. Credit risk and the term structure of lease rates: A reduced from approach. *Journal of Real Estate Finance and Economics*, **37**:281–297 (2008).

Bronars, S. G. and Deere, D. R. The treat of unionization, the use of debt, and the preservation of shareholder wealth. *Quarterly Journal of Economics*, pages 231–254 (1991).

Chu, Y. Optimal capital structure, bargaining, and the supplier market structure. *Journal of Financial Economics*, **106**:411–426 (2012).

Clapham, E. and Gunnelin, A. Rental expectations and the term structure of lease rates. *Real Estate Economics*, **31**:647–670 (2003).

Das, S. R., Duffie, D., Kapadia, N., and Saita, L. Common failings: How corporate defaults are correlated. *Journal of Finance*, **62**:93–117 (2007).

Dasgupta, S. and Sengupta, K. Sunk investment, bargaining and choice of capital structure. *International Economic Review*, **34**:203–220 (1993).

Duffie, D. and Singleton, K. Simulating correlated defaults. *Working Paper* (1999).

Duffie, D., Eckner, A., Horel, G., and Saita, L. Frailty correlated default. *Journal of Finance*, **64**:2089–2123 (2009).

Grenadier, S. R. Leasing and credit risk. *Journal of Financial Economics*, **42**:333–364 (1996).

Hennessy, C. A. and Livdan, D. Debt, bargaining, and credibility in firm-supplier relationships. *Journal of Financial Economics*, **93**:382–399 (2009).

Jarrow, R. and Yu, F. Counterparty risk and the pricing of defaultable securities. *Journal of Finance*, **56**:1765–1799 (2001).

Jorion, P. and Zhang, G. Credit contagion from counterparty risk. *Journal of Finance*, **64**:2053–2087 (2009).

Kale, J. R. and Shahrur, H. Corporate capital structure and the characteristics of suppliers and customers. *Journal of Financial Economics*, **83**:321–365 (2007).

Leland, H. E. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance*, **49**:1213–1252 (1994).

Leland, H. and Toft, K. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, **51**:987–1019 (1996).

Lewis, C. and Schallheim, J. Are debt and leases substitutes? *Journal of Financial and Quantitative Analysis*, **27**(4):497–511 (1992).

Liu, C. H. and Liu, P. P. Is what's bad for the goose (tenant), bad for the gander (landlord): A retail real estate perspective. *Journal of Real Estate Research (to appear)* (2012).

Maksimovic, V. and Titman, S. Financial policy and reputation for product quality. *Review of Financial Studies*, pages 175–200 (1991).

Matsa, D. Capital structure as a strategic variable: evidence from collective bargaining. *Journal of Finance*, **65**:1197–1232 (2010).

Modigliani, F. and Miller, M. H. The cost of capital, corporation finance and the theory of investment. *American Economic Review*, **48**:261–297 (1958).

Titman, S. The effect of capital structure on a firms liquidation decision. *Journal of Financial Economics*, **13**:1–19 (1984).

Warner, J. B. Bankruptcy, absolute priority, and the pricing of risky debt claims. *Journal of Financial Economics*, **4**(3):239–276 (1977).

Yu, F. Correlated defaults in intensity-based models. *Mathematical Finance*, **17**:155–173 (2007).

Zhou, C. An analysis of default correlation and multiple default. *Review of Financial Studies*, **14**:555–576 (2001).

A   Derivations

A.1   Derivation of Lease Rate

Recall, δ is the market price of risk for the service value process. In this section, suppose τ is the time of default for the lessor and F is the cumulative distribution function for the lessor default time. We begin with four calculations which will assist in determining the lease rate. Using Fubini's theorem, we have

$$
\begin{aligned}
\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{BD}(u)\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] &= \widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{BD}(0)e^{\left(\mu_S-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u+\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] \\
&= \int_0^t S_{BD}(0)e^{\left(\mu_S-r-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u}\times\widetilde{\mathbb{E}}\left[e^{\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\right]\mathrm{d}u \\
&= \int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u}\times(1-F(u;V_L,V_{L,B}))\mathrm{d}u,
\end{aligned}
$$

where the last line follows by assuming the independence of τ and $\widetilde{W}_S(\cdot)$ since

$$
\widetilde{\mathbb{E}}\left[e^{\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\right] = e^{\frac{u}{2}\sigma_S^2}\times(1-F(u;V_L,V_{L,B})).
$$

Similarly,

$$
\begin{aligned}
\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{AD}(u)\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] &= \widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{AD}(0)e^{\left(\mu_S-q-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u+\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] \\
&= \int_0^t S_{AD}(0)e^{\left(\mu_S-r-q-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u}\times\widetilde{\mathbb{E}}\left[e^{\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\right]\mathrm{d}u \\
&= \int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u}\times(1-F(u;V_L,V_{L,B}))\mathrm{d}u
\end{aligned}
$$

Let $n(\cdot)$ denote the density of the standard normal distribution. Using Fubini's theorem

and the independence of τ and $\widetilde{W}_S(\cdot)$, we have

$$\widetilde{\mathbb{E}}\left[\int_\tau^t e^{-rs}S_{BD}(s)\mathbb{1}_{\{\tau\le t\}}\mathrm{d}s\right]$$

$$= S_{BD}(0)\int_{-\infty}^\infty\int_0^t\int_u^t e^{(\mu_S-r-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\mathrm{d}s\times f_L(u;V_L,V_{L,B})n(x)\ \mathrm{d}u\mathrm{d}x$$

$$= S_{BD}(0)\int_0^t\int_u^t\int_{-\infty}^\infty e^{(\mu_S-r-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\times f_L(u;V_L,V_{L,B})\times\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\mathrm{d}x\mathrm{d}s\mathrm{d}u$$

$$= S_{BD}(0)\int_0^t\int_u^t e^{(\mu_S-r-\delta\sigma_S)s}\mathrm{d}s\ f_L(u;V_L,V_{L,B})\mathrm{d}u$$

$$= S_{BD}(0)\int_0^t\frac{1}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}-e^{(\mu_S-r-\delta\sigma_S)u}\right)\ f_L(u;V_L,V_{L,B})\mathrm{d}u$$

$$= \frac{S_{BD}(0)}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u;V_L,V_{L,B})\mathrm{d}u\right),$$

if $\mu_S-r-\delta\sigma_S\ne 0$. Similarly, if $\mu_S-r-q-\delta\sigma_S\ne 0$,

$$\widetilde{\mathbb{E}}\left[\int_\tau^t e^{-rs}S_{AD}(s)\mathbb{1}_{\{\tau\le t\}}\mathrm{d}s\right]$$

$$= S_{AD}(0)\int_{-\infty}^\infty\int_0^t\int_u^t e^{(\mu_S-r-q-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\mathrm{d}s\times f_L(u;V_L,V_{L,B})n(x)\ \mathrm{d}u\mathrm{d}x$$

$$= S_{AD}(0)\int_0^t\int_u^t\int_{-\infty}^\infty e^{(\mu_S-r-q-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\times f_L(u;V_L,V_{L,B})\times\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\mathrm{d}x\mathrm{d}s\mathrm{d}u$$

$$= S_{AD}(0)\int_0^t\int_u^t e^{(\mu_S-r-q-\delta\sigma_S)s}\mathrm{d}s\ f_L(u;V_L,V_{L,B})\mathrm{d}u$$

$$= S_{AD}(0)\int_0^t\frac{1}{\mu_S-r-q-\delta\sigma_S}(e^{(\mu_S-r-q-\delta\sigma_S)t}-e^{(\mu_S-r-q-\delta\sigma_S)u})\times f_L(u;V_L,V_{L,B})\mathrm{d}u$$

$$= \frac{S_{AD}(0)}{\mu_S-r-q-\delta\sigma_S}\left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u}\times f_L(u;V_L,V_{L,B})\mathrm{d}u\right)$$

Recall from (3), the lessor's expected net cost of providing lease services is

(17)
$$\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru}[S_{BD}(u)\mathrm{d}u-\chi_L Tax_{L,c}(S_{BD}(u)\mathrm{d}u-S_{AD}(u))]\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right]+$$
$$+\widetilde{\mathbb{E}}\left[\rho_L^t\left(\int_\tau^t e^{-ru}[S_{BD}(u)-\chi_L Tax_{L,c}(S_{BD}(u)-S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau\le t\}}\right],$$

where the last term is the damage caused to the tenant, i.e., a proportion (perhaps greater than 1) of the future service flows. The four above calculations allow us to resolve this

net cost of lease services. Thus, the first term in (17) is equal to

$$\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru}[S_{BD}(u)\mathrm{d}u - \chi_L Tax_{L,c}(S_{BD}(u)\mathrm{d}u - S_{AD}(u))]\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right]$$

$$= (1 - \chi_L Tax_{L,c})\int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u} \times (1 - F(u; V_L, V_{L,B}))\mathrm{d}u+$$

$$+ \chi_L Tax_{L,c}\int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u} \times (1 - F(u; V_L, V_{L,B}))\mathrm{d}u.$$

With regard to the second term in (17), we have

$$\widetilde{\mathbb{E}}\left[\rho_L^t\left(\int_\tau^t e^{-ru}[S_{BD}(u) - \chi_L Tax_{L,c}(S_{BD}(u) - S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau\le t\}}\right]$$

$$= \rho_L^t\Bigg((1 - \chi_L Tax_{L,c})\times$$

$$\times \frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u; V_L, V_{L,B})\mathrm{d}u\right) +$$

$$+ \chi_L Tax_{L,c}\frac{S_{AD}(0)}{\mu_S - r - q - \delta\sigma_S}\times$$

$$\times \left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u} \times f_L(u; V_L, V_{L,B})\mathrm{d}u\right)\Bigg).$$

Now, equating (17) with $r_{RN}^t\left(\frac{1-e^{-rt}}{r}\right)$ yields,

(18)
$$r_{RN}^t\left(\frac{1 - e^{-rt}}{r}\right) =$$

$$= (1 - \chi_L Tax_{L,c})\int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u} \times (1 - F(u; V_L, V_{L,B}))\mathrm{d}u+$$

$$+ \chi_L Tax_{L,c}\int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u} \times (1 - F(u; V_L, V_{L,B}))\mathrm{d}u +$$

$$+ \rho_L^t\Bigg((1 - \chi_L Tax_{L,c})\times$$

$$\times \frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u; V_L, V_{L,B})\mathrm{d}u\right) +$$

$$+ \chi_L Tax_{L,c}\frac{S_{AD}(0)}{\mu_S - r - q - \delta\sigma_S}\times$$

$$\times \left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u} \times f_L(u; V_L, V_{L,B})\mathrm{d}u\right)\Bigg).$$

Solving the above equation for r_{RN}^t yields the risky lessor, risk-free tenant lease rate:

(19)
$$r_{RN}^t = \frac{r}{1 - e^{-rt}} \times$$

$$\times \left[(1 - \chi_L Tax_{L,c}) \int_0^t S_{BD}(0) e^{(\mu_S - r - \delta\sigma_S)u} \times (1 - F(u; V_L, V_{L,B})) \mathrm{d}u + \right.$$

$$+ \chi_L Tax_{L,c} \int_0^t S_{AD}(0) e^{(\mu_S - r - q - \delta\sigma_S)u} \times (1 - F(u; V_L, V_{L,B})) \mathrm{d}u +$$

$$+ \rho_L^t \left((1 - \chi_L Tax_{L,c}) \times \right.$$

$$\times \frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S} \left(e^{(\mu_S - r - \delta\sigma_S)t} F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S - r - \delta\sigma_S)u} f_L(u; V_L, V_{L,B}) \mathrm{d}u \right) +$$

$$+ \chi_L Tax_{L,c} \frac{S_{AD}(0)}{\mu_S - r - q - \delta\sigma_S} \times$$

$$\left. \left. \times \left(e^{(\mu_S - r - q - \delta\sigma_S)t} F(t; V_L, V_{L,B}) - \int_0^t e^{(\mu_S - r - q - \delta\sigma_S)u} \times f_L(u; V_L, V_{L,B}) \mathrm{d}u \right) \right) \right].$$

A.2   Definitions

$$a_T := \frac{r - \delta_{V_T} - (\sigma_{V_T}^2/2)}{\sigma_{V_T}^2}; \ a_L := \frac{r - \delta_{V_L} - (\sigma_{V_L}^2/2)}{\sigma_{V_L}^2};$$

$$b_T := \ln\left(\frac{V}{V_{T,B}}\right); \ b_L := \ln\left(\frac{V}{V_{L,B}}\right);$$

$$z_T := \frac{((a_T \sigma_{V_T}^2)^2 + 2r\sigma_{V_T}^2)^{1/2}}{\sigma_{V_T}^2}; \ z_L := \frac{((a_T \sigma_{V_L}^2)^2 + 2r\sigma_{V_L}^2)^{1/2}}{\sigma_{V_L}^2};$$

$$x_T := a_T + z_T; x_L := a_L + z_L;$$

$$A := 2a_L e^{-rT_{L,D}} N(a_L \sigma_{V_L} \sqrt{T_{L,D}}) - 2z N(z\sigma_{V_L} \sqrt{T_{L,D}})$$

$$- \frac{2}{\sigma_{V_L} \sqrt{T_{L,D}}} n(z\sigma_{V_L} \sqrt{T_{L,D}}) + \frac{2e^{-rT_{L,D}}}{\sigma_{V_L} \sqrt{T_{L,D}}} n(a_L \sigma_{V_L} \sqrt{T_{L,D}}) + (z_L - a_L);$$

$$B := - \left(2z_L + \frac{2}{z_L \sigma_{V_L}^2 T_{L,D}} \right) N(z\sigma_{V_L} \sqrt{T_{L,D}}) - \frac{2}{\sigma_{V_L} \sqrt{T_{L,D}}} n(z\sigma_S \sqrt{T_{L,D}}) + (z_L - a_L) + \frac{1}{z_L \sigma_{V_L}^2 T_{L,D}};$$

$$K_1^T := A/(rT)$$

$$K_2^T := B$$

$$K_3 := (C_T + \Omega_R)\left(\frac{Tax_{T,c}}{r}\right)x_T;$$

$$K_4 := (1 - \rho_R)\Omega_R\left(\frac{1 - e^{-r\frac{T_L}{2}}}{r}\right);$$

$$M := \left(\frac{\Omega_R}{r}\rho_R\right)\left(K_1^{T_L}\frac{T_L}{T_{T,D}} - K_2^{T_{T,D}}\right) - \left(\frac{\Omega_R}{r}\frac{\rho_R}{T_L}\right)(K_1^{T_L} - K_2^{T_L}),$$

where $N(\cdot)$ is the cumulative standard normal distribution, and $n(\cdot)$ is the probability density function of the standard normal distribution.

Table 1: Initial Parameter Values

Base Case Parameters				
Market Parameters			Landlord Parameters	
r	0.075		μ_{V_L}	0.05
$Tax_{LT,c}$	0.35		σ_{V_L}	0.06
μ_S	0.06		$V_L(0)$	100
σ_S	0.2	Cost of bankruptcy		0.5
q	0.05	Default Recovery		0.62
S_0	1			
market price of risk (δ)	0.83	Tenant Parameters		
			μ_{V_T}	0.05
			σ_{V_T}	0.2
			δ_{V_T}	0.06
			$V_T(0)$	100
		Cost of bankruptcy		0.5
		Default Recovery		0.62

Table 2: The Impact of Landlord Default Probability on Lease Rate Term Structure

Landlord Default		Tenant Default		Lease Rates		Tenant Capital Structure			
Boundary	Probability	Boundary	Probability	r_{RN}	r_{RR}	P	C	Value	Debt/Value
Lease Maturity = 5-years, Tenant Debt Maturity = 5-years									
20	0.0010	46.63	0.1499	0.7754	0.7935	46.50	4.19	126.4834	0.3749
30	0.0167	46.63	0.1499	0.7754	0.7935	46.50	4.19	126.4837	0.3749
43.84	*0.1163*	*46.57*	*0.1492*	*0.7693*	*0.7871*	*46.50*	*4.19*	*126.3739*	*0.3753*
60	0.3600	46.47	0.1480	0.7121	0.7284	47.00	4.23	125.2764	0.3827
Lease Maturity = 5-years, Tenant Debt Maturity = 10-years									
20	0.0010	45.67	0.3625	0.7754	0.7913	44.00	3.96	126.1787	0.3467
30	0.0167	45.67	0.3625	0.7754	0.7913	44.00	3.96	126.1789	0.3467
43.84	*0.1163*	*45.56*	*0.3607*	*0.7693*	*0.7849*	*44.00*	*3.96*	*126.0976*	*0.3472*
60	0.3600	45.22	0.3553	0.7121	0.7258	45.00	4.05	125.2830	0.3587
Lease Maturity = 5-years, Tenant Debt Maturity = 20-years									
20	0.0010	49.63	0.6639	0.7754	0.8019	51.00	4.59	126.2204	0.3676
30	0.0167	49.63	0.6639	0.7754	0.8019	51.00	4.59	126.2205	0.3676
43.84	*0.1163*	*49.48*	*0.6622*	*0.7693*	*0.7951*	*51.00*	*4.59*	*126.1682*	*0.3686*
60	0.3600	48.40	0.6495	0.7121	0.7330	51.50	4.64	125.6277	0.3803
Lease Maturity = 10-years, Tenant Debt Maturity = 5-years									
20	0.0010	48.28	0.1720	0.6387	0.7031	49.00	4.90	140.2134	0.3735
30	0.0167	48.23	0.1713	0.6339	0.6976	49.00	4.90	140.0145	0.3739
43.75	*0.1151*	*48.35*	*0.1728*	*0.6011*	*0.6621*	*49.50*	*4.95*	*138.5962*	*0.3810*
60	0.3600	49.25	0.1855	0.5151	0.5715	51.50	5.15	134.7787	0.4060
Lease Maturity = 10-years, Tenant Debt Maturity = 10-years									
20	0.0010	47.86	0.3975	0.6387	0.7008	31.50	3.15	134.3939	0.2452
30	0.0167	47.63	0.3939	0.6339	0.6944	31.50	3.15	134.2619	0.2457
43.75	*0.1151*	*47.87*	*0.3977*	*0.6011*	*0.6597*	*33.50*	*3.35*	*133.3225*	*0.2628*
60	0.3600	48.26	0.4040	0.5151	0.5670	38.50	3.85	130.8456	0.3069
Lease Maturity = 10-years, Tenant Debt Maturity = 20-years									
20	0.0010	50.70	0.6762	0.6387	0.7174	24.50	2.21	130.1120	0.1503
30	0.0167	50.77	0.6769	0.6339	0.7125	25.00	2.25	130.0106	0.1541
43.75	*0.1151*	*49.53*	*0.6627*	*0.6011*	*0.6684*	*26.50*	*2.39*	*129.3810*	*0.1718*
60	0.3600	54.39	0.7160	0.5151	0.5997	38.00	3.80	127.7404	0.2624

Notes: Table 2 examines the relationship between the lease term structure and the probability of default on the lessor's debt.

Parameters: $r = 0.075$, corporate tax rate $Tax_{L/T,c} = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_D = 5, 10, 20$; debt maturity for landlord $L_D = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$. Principal and Coupon Window used in the implementation: $P = [0.5, 100]$ using 0.5 as the principal step size, $C = [(0.01)P, (0.1)P]$ with 0.01 as the coupon step size.

Description: The first and second columns are the endogenous landlord bankruptcy boundary and the landlord's default on debt probability (not scaled by 100), i.e., $\mathbb{P}_{V_L(0)=100}[V_L(\tau_L) \leq T_D^L]$. The third and fourth columns are the endogenous tenant bankruptcy boundary and the tenant's default on debt probability (not scaled by 100), i.e., $\mathbb{P}_{V_T(0)=100}[V_T(\tau_T) \leq T_D]$. The fifth and sixth columns are the risky landlord, risk-free tenant lease rates. The seventh through tenth columns make up the tenant's optimal capital structure. Specifically, the seventh and eight columns are the optimal Principal and Coupon for the tenant firm. The ninth column is the optimal tenant firm value and the tenth column is the optimal leverage ratio for the tenant firm.

Note that, in each block, the italicized third row indicates the optimal capital structure for the landlord and tenant firms when $T_D^L = 5$ and $T_D^L = 10$, respectively.

Run-time Information: The calculations for this table were implemented on the Syracuse University Matlab cluster matlabts.syr.edu using parallel processing with a total of 36 core processors. Run-time to calculate all entries in the table was approximately 40 minutes.

Table 3: The Impact of Short-term and Long-term Leases

	Landlord Default		Tenant Default		Leases	
	Boundary	Probability	Boundary	Probability	r_{RN}	r_{RR}
Lease Maturity = 5-years, Tenant Debt Maturity = 5-years						
Landlord Debt Maturity = 5-years	43.84	0.1163	46.57	0.1492	0.7693	0.7871
Landlord Debt Maturity = 10-years	47.17	0.3866	46.52	0.1485	0.7638	0.7814
Lease Maturity = 5-years, Tenant Debt Maturity = 10-years						
Landlord Debt Maturity = 5-years	43.84	0.1163	45.56	0.3607	0.7693	0.7849
Landlord Debt Maturity = 10-years	47.17	0.3866	45.45	0.3590	0.7638	0.7790
Lease Maturity = 5-years, Tenant Debt Maturity = 20-years						
Landlord Debt Maturity = 5-years	43.84	0.1163	49.48	0.6622	0.7693	0.7951
Landlord Debt Maturity = 10-years	47.17	0.3866	49.34	0.6606	0.7638	0.7890
Lease Maturity = 10-years, Tenant Debt Maturity = 5-years						
Landlord Debt Maturity = 5-years	43.75	0.1151	48.35	0.1728	0.6011	0.6621
Landlord Debt Maturity = 10-years	47.05	0.3846	48.65	0.1771	0.5872	0.6484
Lease Maturity = 10-years, Tenant Debt Maturity = 10-years						
Landlord Debt Maturity = 5-years	43.75	0.1151	47.87	0.3977	0.6011	0.6597
Landlord Debt Maturity = 10-years	47.05	0.3846	48.11	0.4015	0.5872	0.6456
Lease Maturity = 10-years, Tenant Debt Maturity = 20-years						
Landlord Debt Maturity = 5-years	43.75	0.1151	49.53	0.6627	0.6011	0.6684
Landlord Debt Maturity = 10-years	47.05	0.3846	49.33	0.6605	0.5872	0.6519

Table 4: The Impact of Tenant Default Boundary on Lease Rate Term Structure

	$T_L = 5$					$T_L = 10$			
DB_L	DB_T	$\lambda_{D,T}$	$\lambda_{D,L}$	r_{RR}	DB_L	DB_T	$\lambda_{D,T}$	$\lambda_{D,L}$	r_{RR}
$T_{D,T} = 5$					**$T_{D,T} = 5$**				
	30			0.7702		30			0.6086
	40			0.7760		40			0.6286
43.84	*46.57*	0.1492	0.1163	0.7871	*43.75*	*48.35*	0.1728	0.1151	0.6621
	60			0.8487		60			0.7491
$T_{D,T} = 10$					**$T_{D,T} = 10$**				
	30			0.7702		30			0.6086
	40			0.7760		40			0.6286
43.84	*45.56*	0.3607	0.1163	0.7849	*43.75*	*47.87*	0.3977	0.1151	0.6597
	60			0.8487		60			0.7491
$T_{D,T} = 20$					**$T_{D,T} = 20$**				
	30			0.7702		30			0.6086
	40			0.7760		40			0.6286
43.84	*49.48*	0.6622	0.1163	0.7951	*43.75*	*49.53*	0.6627	0.1151	0.6684
	60			0.8487		60			0.7491

Parameters: $r = 0.075$, corporate tax rate $Tax_{L/T,c} = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_D = 5, 10, 20$; debt maturity for landlord $L_D = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

Description: The first column is the endogenous landlord bankruptcy boundary (DB_L). The second column is the tenant bankruptcy boundary (DB_T). The third row in each block (where block refers to entries corresponding to a $(T_L, T_{D,T})$ pair, $T_{D,T}$ is the maturity of tenant's debt) is the optimal endogenous boundary found using the landlord's endogenous boundary in the first column. The third and fourth columns are the default probabilities of debt for both tenant ($\lambda_{D,T}$) and landlord ($\lambda_{D,L}$) respectively. The fifth column is the lease rate implied using the first two columns. Columns six through ten are repeats of columns one-three using $T_L = 10$. Italicized table entries indicate optimal tenant and landlord boundary values.
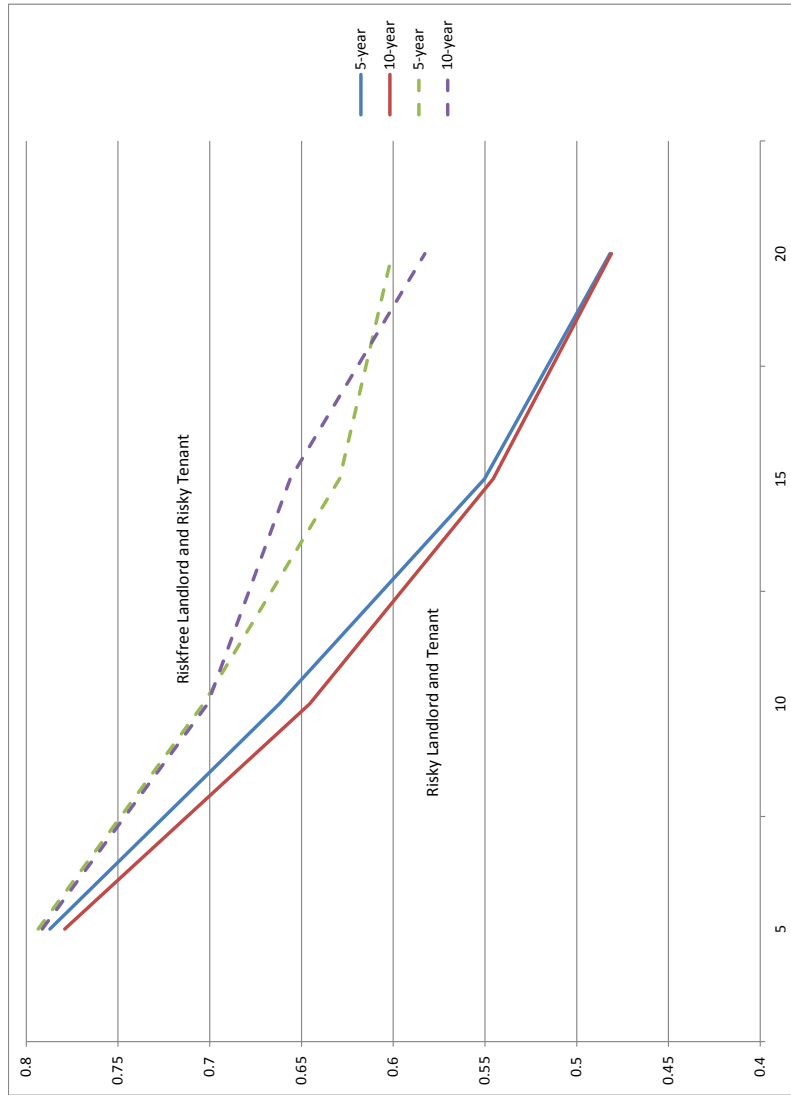
Figure 1: Term Structure of Lease Rates

Notes: Figure 1 highlights the term structure of lease rates using the model parameters from our base case.

Parameters: $r = 0.075$, corporate tax rate $Tax_{L/T,c} = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_D = 5, 10, 20$; debt maturity for landlord $L_D = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

Table 5: The Impact of Taxes and Depreciation on Lease Rate Term Structure

Tenant Tax Rate	Landlord Tax Rate		
$\chi = 0.5$	0.25	0.35	0.40
0.25	0.7821	0.7721	0.7478
0.35	0.7974	0.7871	0.7618
0.40	0.8187	0.8080	0.7817
$\chi = 1$			
0.25	0.7723	0.7597	0.7397
0.35	0.7873	0.7740	0.7534
0.40	0.8082	0.7944	0.7748
$\chi = 1.5$			
0.25	0.7628	0.7476	0.7317
0.35	0.7772	0.7615	0.7461
0.40	0.7977	0.7814	0.7662

Notes: Table 5 highlights how differences in landlord and tenant tax rates as well changes in overall tax policy affect the lease rate.

Parameters: $r = 0.075$, corporate tax rate $Tax_{L/T,c} = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5$; debt maturity for tenant $T_D = 5$; debt maturity for landlord $L_D = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

Description: The assumed tax rates $0.25, 0.35, 0.4$ for the tenant appear as rows and the assumed tax rates for the landlord $0.25, 0.35, 0.4$ appear as columns. Entries of the table are the lease rate for a risky landlord and tenant.

Run-time Information: The calculations for this table were implemented on the Syracuse University Matlab cluster matlabts.syr.edu using parallel processing with a total of 4 core processors. Run-time to calculate all entries in the table was approximately 4 hours.