The Term Structure of Lease Rates with Endogenous Default Triggers and Tenant Capital Structure: Theory and Evidence

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Abstract

This paper focuses on the defaultable lease rate term structure with endogenous default. We combine the competitive lease market argument proposed by Grenadier (1996) and the endogenous default structural model proposed by Leland and Toft (1996) to examine the interaction between the lessee's capital structure and the equilibrium lease rate. Under this framework, determining the lease rate is a simultaneous equation problem that captures the tradeoff between debt and lease financing. Using data on 2,482 real estate lease transactions, we empirically confirm the predictions derived from the numerical analysis of the model.

Key words: leasing valuation, credit risk, endogenous default

I Introduction

In December 2005, Lehman Brothers investment bank signed a lease for 160,000 square feet of office space at 1301 Avenue of the Americas in New York, becoming the largest tenant in the building with over 500,000 square feet under its control.[1] At the time, Lehman Brothers was one of the most profitable investment banks and a major user of commercial office space in New York. Less than three-years later, the September 2008 Lehman Brothers bankruptcy filing and the resulting uncertainty regarding its lease at 1301 Avenue of the Americas reinforces the notion that tenant credit risk is an integral component to understanding the term structure of lease rates on commercial property.

The connection between tenant credit risk and lease rates is not new. For example, in a study of the ex post performance of leases, Lease, McConnell, and Schallheim (1990) found significant credit risk in typical lease contracts. The leases in their sample experienced a default rate of approximately 20% and a recovery rate of 38% relative to the original cost of the asset, or 64% relative to the present value of the remaining lease payments plus estimated salvage value. Thus, their analysis suggests that lease valuations must incorporate tenant credit risk just as corporate bond values reflect default risk. Building on this finding, a number a studies have proposed models that attempt to incorporate tenant credit risk into the determination of lease rates.[2] However, these lease valuation models rely on exogenous default triggering mechanisms, and thus, do not account for the substitution of lease financing and debt financing.

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[1]See Geiger, D., "Lehman Brother to lease 160,000 s/f at 1301 Sixth," *Real Estate Weekly,* December 28, 2005.

[2]For example, see Grenadier (1996) for an example of an exogenous structural credit risk model for fairly pricing lease contracts, and Lewis and Schallheim (1992) for a single-period model that incorporates the tenant's capital structure and credit risk.

In his seminal text on corporate leasing, Schallheim (1994) succinctly outlines the economic principles showing that leasing is simply a substitute for debt financing. In the context of an efficient market, the substitutability of debt and lease financing implies that the tenant's capital structure must impact the term structure of lease rates. More recently, Yan (2006) presents evidence confirming the substitutability of debt and leases using information on capitalized leases for large firms.

In this paper, we propose and empirically test an endogenous structural model of the term structure of lease rates that is simultaneously determined by the lessee firm's capital structure and the competitive lease market. Compared with other exogenous defaultable lease contract valuations, this setting is more realistic and can effectively link the lessee's financial decisions with current conditions in the leasing market.

To preview our results, we show that the term structure of lease rates is integral to the lessee's capital structure. The numerical example shows that lessee debt (capital structure) has a direct impact on the equilibrium lease rate. However, the impact arises from multiple sources of interaction resulting from the endogenous default boundary that is a function of both the use of debt and leasing. Finally, our model also provides an opportunity to examine the impact of changes in government tax policies with respect to depreciation. For example, we show the relative sensitivity of lease rates to changes in the marginal tax rates facing both the lessor and lessee.

The remainder of the paper is organized as follows. Section II summarizes the existing literature on lease valuation. Section III then describes the no-arbitrage setting for determining lease rates and provides the defaultable fair lease rate under a competitive lease market. In Section IV we propose a structural model based on the model first developed by Leland and Toft (1996) to express the components

of firm value given that the firm uses debt and lease contracts as its financing instruments. Section V presents the endogenous decision rule to derive the optimal bankruptcy trigger level, and Section VI provides comparative statics and numerical examples to show the impact of relevant parameters on the term structure of lease rates. The numerical analysis continues in Section VII with a discussion of the impact of tax rates and tax policy on the lease term structure. In Section VIII, we present an empirical analysis using real estate lease transactions to validate the predictions derived from the numerical analysis. We conclude in Section IX.

II   Literature Review

The determination of lease rates is widely discussed in the financial literature. In one of the first studies of corporate leasing, Miller and Upton (1976) propose an equilibrium structure for determining the lease versus buy policy of a firm and the determination of lease rates. Following their work, Schallheim and McConnell (1985) and McConnell and Schallheim (1983) established a no-arbitrage framework to price leases and insurance premiums of leased assets in discrete-time cases. Later, Grenadier (1995) was the first to apply a continuous-time model and competitive market rationale to derive the term structure of lease rates. In Grenadier's model, lease rates are endogenously determined, and driven by the trade-off decision between construction cost and developer's profit. Hence, the lease market equilibrium is determined by a firm's decision given a competitive market assumption. However, credit risk is not considered.

More recently, a number of studies have extended the lease valuation models to incorporate more realistic assumptions regarding the nature of the competitive leas-

ing market. For example, Grenadier (2005) uses the same equilibrium approach of Grenadier (1995) but changes the competitive market assumption into a monopolistic assumption by using a game-theoretic based approach. In this paper, Grenadier suggests that the shape of the term structure is determined by two factors. First, the term structure is a function of the expectations of future short term lease rates, similar to the expectation hypothesis in interest rate term structure models. Second, the term structure of lease rates depends upon the number of competitors in the leasing market. Stanton and Wallace (2007) apply the no-arbitrage framework and continuous-time modeling into the determination of lease rates as well and provide an empirical study of the term structure of commercial lease rates.[3] In addition, Clapham and Gunnelin (2003) applied the concept of interest rate swaps to determine lease rates. In their model, the spot interest rate and lease service flow follow continuous-time stochastic processes. Moreover, they regard the lessee paying a fixed lease rent as the fixed interest rate payer in swap contract, and the lessor who promises to provide stochastic service flow as the float interest payer in a swap contract. From this perspective, they use a forward measure derived from pricing option contracts under stochastic interest rates to simplify the expression of the property service flows. They suggest that the expectation hypothesis of lease rates is significantly affected by risk aversion and interest rate uncertainty, which complements the proposition made by Grenadier (1995). Clapham and Gunnelin (2003) also propose a basic reduced form model for determining lease rates under credit risk. However, this model was not fully developed.

Grenadier (1996) incorporates default risk into lease rate determination also by using a competitive market rationale. In this article, the default process is modeled

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[3]Ambrose, Hendershott, and Klosek (2002) use the no-arbitrage framework to derive a closed-form solution to a lease with adjusting rents.

on the first passage time that the lessee firm's asset value hits a given bankruptcy level, which is the so called "structural model" in the broader credit risk literature. In Grenadier's (1996) analysis, the lease rate credit spread, which is defined as the difference between the lease rate and the risk free rate, is influenced by the lease duration, default recovery rate, correlation between leased assets and the lessee firm' assets, etc. However, only the exogenous default level is considered. From this standpoint, the lessee firm's capital structure decision is not relevant in determining lease rates. In addition, the lessee firm's likelihood to default is not endogenously related to how much it pays for its lease service.

In contrast, we assume that the firm's capital structure is taken into consideration when determining the lease rate. Our model is closely related to Lewis and Schallheim (1992), who also consider the lessee firm's capital structure decision and credit risk in determining lease rates. However, their model assumes a simplified single-period framework that overweighs the lease payment in the capital structure decision process. Due to the limitation of their single-period model, the default decision can only be made based on the claim priority in a period, instead of a endogenous default decision, which considers all related cash flows in the future. Similarly, the model developed in Grenadier (1996) is based on an exogenous default triggering level where the lease does not affect the firm's default probability, and thus, does not account for the firm's capital structure. However, the amount of lease financing and the lease rent for leased assets are always a concern in a firm's financial decision making and can not be isolated out of modeling. This is the main issue we address.

One of the major differences between the earlier exogenous default models and our endogenous default model is that the exogenous models do not depend on an

equity maximizing decision. Rather, default is triggered by the zero net worth condition, such as when the collateral asset value (or cash flow) falls to a certain exogenously given barrier. However, endogenous default is similar to an optimal decision that depends on equity maximization. In this model, the endogenous default barrier depends on the model parameters, such as the risk-free rate, lease maturity, interest rate volatility and others. In contrast, an exogenous default barrier is not affected by any of these model parameters.

III   Determination of Lease Rates

We begin by applying the spirit of Brennan and Kraus (1982), McConnell and Schallheim (1983), and Stanton and Wallace (2007) and use the no-arbitrage argument to determine the operating lease rates. For simplicity, we consider the lease contract without any embedded options.[4] The underlying asset value is determined by the present value of its future service flows,[5] and is given by:

$$(1) \qquad \frac{dS_{BD}}{S_{BD}} = \mu_s dt + \sigma_s dW_S$$

where S_{BD} is the service flow before depreciation, μ_s is the drift rate of the service flow process, σ_s is the volatility of this process, and dW_S is the standard Brownian Motion under physical measure $\mathbf{P}$. On the other hand, if we consider the depreciation of the leased asset that is reflected in the drift rate of the service flow process

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[4]McConnell and Schallheim (1985), Stanton and Wallace (2007), Grenadier (1995), Grenadier (1996) incorporate many embedded options in lease contracts to discuss the fair leasing rate. Here we only focus on the interaction between lease rate dynamics and capital structure; therefore, we ignore those options in lease contracts to purify our discussion.

[5]In an economic sense, we can regard the service flow process as the economic value process the leased asset provides over time.

with payout rate q, we can write the service-flow process after depreciation, S_{AD}, as:

$$(2) \qquad \frac{dS_{AD}}{S_{AD}} = (\mu_s - q)dt + \sigma_s dW_S.$$

In equation (2), q represents the property economic depreciation rate, not the accounting depreciation rate. Although the concept of accounting depreciation originally reflected actual economic depreciation, over time, the link between real economic depreciation and accounting depreciation that is set by the government tax policies, has deviated. To see the effect of the difference between accounting and economic depreciation, we assume that accounting depreciation is scaled to a proportion (χ) of economic depreciation. If $\chi = 1$, then accounting and economic depreciation are equivalent. If $\chi < 1$, then government tax policy sets accounting depreciation rules such that the tax deduction associated with depreciation is less than the full economic depreciation. For example, if the building physically depreciates over 25 years and the government sets the depreciable life at 39.5 years, then $\chi = 0.625$.[6] However, in the 1980s, government tax policy provided for accelerated depreciation where the depreciable life was set to 15-years. A 15-year depreciable life implies that $\chi = 1.675$ under the assumption of a 25-year physical life.

In the analysis below, we explore the implications on lease rates of altering the accounting depreciation rate. We also define the lease rate for a risk-free tenant and a risky tenant assuming the lessor writes an operating lease contract of t periods, and the leased asset provides continuous service flows following the stochastic differential equations (1) and (2).

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[6]The typical life expectancy for major structural items, such as a building roof is 20 to 25-years.

A   Case 1: The Risk-Free Lessee

Assume the lessee is risk-free and lessor's net cost of offering a lease contract is the present value of the service flows before depreciation minus the tax-shield benefit associated with the depreciation expense. Thus, the lessor's net cost is defined as:

(3)
$$\widetilde{E}\left\{\int_0^t S_{BD}(u)e^{-ru}du - \chi\text{TAX}_c\left[\left(\int_0^t S_{BD}(u)e^{-ru}du\right) - \left(\int_0^t S_{AD}(u)e^{-ru}du\right)\right]\right\}$$

where, $\int_0^t S_{BD}(u)e^{-ru}du$ represents the present value of the service flows before depreciation from time 0 to time t discounted by the risk-free rate under risk-neutral measure $\widetilde{P}$; similarly, $\int_0^t S_{AD}(u)e^{-ru}du$ represents the present value of the service flow after depreciation under the risk-neutral measure $\widetilde{P}$. $\widetilde{E}(\bullet)$ is the expectation under $\widetilde{P}$. The difference between these two terms is the depreciation cost of the leased asset from time 0 to t. TAX_c is the corporate tax rate applied to the lessor. As introduced above, χ is the depreciation adjustment factor that reconciles the government mandated accounting depreciation to the actual physical depreciation.

The first term in equation (3) is the expected present value of service flows before depreciation from time 0 to t, and the second term in equation (3) is the tax-shield benefit provided by depreciation expense. The difference between these two terms is the net cost of providing the leased property from the lessor's perspective recognizing the tax-shield benefit associated with the depreciation expense.[7] In a competitive market, the net cost of the lease is exactly equal to the present value of the future lease payments $(\int_0^t r_N^t e^{-ru}du)$ and equals $r_N^t((1 - e^{-rt})/r)$, where r_N^t denotes the

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[7]This expression of operating lease net cost is the same as that provided in Smith and Wakeman (1985), which states that lease rent is equal to the cost of capital and depreciation cost of providing the lease minus the tax deductible benefit of the lease.

non-defaultable operating lease rent with maturity t. Assuming the asset service flow follows the dynamics of equation (2), then the default risk-free lease rate is:[8]

$$(4) \quad r_N^t = S(0) \left(\frac{e^{(\mu_S - \delta\sigma_S - r - q)t} - 1}{\mu_S - \delta\sigma_S - r - q} \right) [(1 - \chi\text{TAX}_c) Q + \chi\text{TAX}_c] \left(\frac{r}{1 - e^{-rt}} \right)$$

where,

$$Q = \left(\frac{e^{(\mu_S - \delta\sigma_S - r)t} - 1}{e^{(\mu_S - \delta\sigma_S - r - q)t} - 1} \right) \left(\frac{\mu_S - \delta\sigma_S - r - q}{\mu_S - \delta\sigma_S - r} \right),$$

$S(0)$ is the initial value of the service flow process, and δ is the market price of risk of the risk factor implicit in the service flow process.[9] In this case, the lease rate does not consider the lessee's credit condition, and only reflects the lessee's corporate tax rate, the government's depreciation tax-shield policy, and the service flow dynamics of leased assets.

B Case 2: The Risky Lessee

We now examine the lease contract for a risky tenant. In this case, the lessor adds a default risk premium to the lease rent to compensate for potential losses. If the lessee defaults and files for bankruptcy at τ $(0 < \tau < t)$, the lessee stops paying its debt claims until the bankruptcy is resolved.[10] Meanwhile, under the petition of the court, the lessee can continue using the leased asset by continuing the lease payment. Therefore, a lease contract under default risk is similar to a defaultable corporate bond with default recovery, and the default recovery is part of the future lease payments.

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[8]This proof can be done by applying the Fubini Theorem in stochastic calculus. For more about this theorem, please refer to Arnold (1974)

[9]The Appendix shows the derivation of equation (4).

[10]See Barclay and Smith (1995), or Graham, Lemmon, and Schallheim (1998). For a practice perspective, see Rosen and Rooney (2002).

Recall that the portfolio value of default-free lease with maturity t is $r_N^t((1 - e^{-rt})/r)$. Similarly, the portfolio value of the default-risky case is the present value the lease rate r_R^t from origination to default time τ, and the recovery of remaining lease rentals from time τ to maturity time t. Under these conditions, we can express the value of the default-risky lease as:

$$(5) \qquad \int_0^t e^{-r\tau} r_R^t \left(1 - F(\tau; V, V_B)\right) d\tau + \int_0^t e^{-r\tau} \rho_R^t R_R^{t-\tau} f(\tau; V, V_B) d\tau$$

where $F(\tau; V, V_B)$ is the cumulative default probability up to time τ under measure $\widetilde{P}$, and $f(\tau; V, V_B)$ is the instantaneous default probability under measure $\widetilde{P}$ at time τ. For simplicity, we assume the recovery rate ρ_R^t is a constant. $R_R^{t-\tau}$ is the present value of the remaining default risky lease payments, and it can be expressed as $r_R^t((1 - e^{-r(t-\tau)})/r)$. The first term in (5) represents the expected discounted lease payment flows from 0 to τ. The second term represents the expected discounted value of the remaining lease payments after default happens.

When the market is in equilibrium, the portfolio value in case 1 is equal to case 2. This argument is similar to the one in Grenadier (1996) where he argues that any two methods of selling the service flow of the asset for t years must have the same value. The first method is to sell the service flow to a credit risk free tenant with a credit risk free lease rate. Alternatively, one can sell the service flow to a credit risky tenant with a credit risky lease rate. The first method provides a series of stationary lease incomes to the lessor for a fixed period. The second method provides a series of credit risk compensated lease incomes to the lessor for a uncertain period. Thus,

following this logic, we combine equations (4) and (5) and express the defaultable lease rate with a maturity of t as:

$$(6) \qquad r_R^t = r_N^t \left[\frac{1 - e^{-rt}}{(1 - e^{-rt}) - (1 - \rho_R^t)(G(t) - F(t)e^{-rt})} \right]$$

where the definitions of $F(t)$, $G(t)$ are expressed in the Appendix.[11] This expression also explains the relation between the risky lease rate and the risk-free lease rate. The denominator represents the discount factor associated with a default-risky lease, and the numerator is the discount factor associated with a risk-free lease. The first part of denominator is the default-free discount factor which is the same as the numerator. The second part is the loss rate, $(1 - \rho_R^t)$, times the difference between lessee's accumulated discounted default probability and discounted accumulated default probability; therefore, it is positive. From this equation, when the lessee's default probability increases, implying $(G(t) - F(t)e^{-rt})$ increases, the value of the denominator decreases.[12] Hence, the risky lease rate increases to compensate for the increase in default probability. In addition, when the expected recovery rate increases, the lessor recovers more when the lessee defaults, and thus, the risky lease rate decreases, all else being equal.

Equation (6) provides an intuitive explanation of the relationship between the risky lease rate and the risk-free lease rate. Note that previous research assumes the lease rate is exogenous to the lessee's capital structure and is only determined by market competition. However, from equation (6), we observe that lease rates are affected by both lease market competition and the lessee's capital structure as reflected in its default probability. Therefore, the lease rate determination problem

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[11]The Appendix also contains the derivation of equation 6.

[12]$(G(t) - F(t)e^{-rt}) = \int_0^t e^{-r\tau} f(\tau; V, V_B) d\tau - \int_0^t e^{-rt} f(\tau; V, V_B) d\tau = \int_0^t (e^{-r\tau} - e^{-rt}) f(\tau; V, V_B) d\tau > 0$

becomes a simultaneous equation problem and is conditional upon the firm's capital structure and the leasing market.

IV   The Capital Structure Setting

As in Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and Leland and Toft (1996), we consider a firm with productive assets whose unleveraged value V follows a continuous diffusion process with constant proportional volatility σ_V:

$$(7) \qquad \frac{dV}{V} = \left(\mu_V \left(t \right) - \delta_V \right) dt + \sigma_V dW_V,$$

where $\mu_V \left(t \right)$ denotes the total expected rate of return on asset value V, δ_V is the constant fraction of value paid out to all security holders, dW_V is the increment of a standard Brownian motion. The value V includes the value of the net cash flows generated by a firm's productive activities.

In the following sections, we derive the firm's endogenous default boundary conditions assuming the firm uses debt and leases to finance its productive assets. In order to provide a tractable framework, we first consider the use of a lease conditional upon the firm's existing capital structure. We then examine the debt contract. Finally, we combine the firm's capital structure with the lease decision to determine the endogenous default boundary.

A   Leasing

Suppose a firm has an existing capital structure (comprising debt and equity) and wishes to acquire the use of an additional asset through lease financing. The firm writes an operating lease contract maturing at t for this asset. Furthermore, assume the firm's value process continues without a time limit unless V falls to a default-triggering value V_B, defined below. If V hits this default-triggering value, the firm defaults on its liabilities. Because of the nature of the lease contract, the lessor can recover part of the remaining lease payments even after default. From the lessor's perspective, the value of a lease contract is given by the expression (5), the fair value of the lease contract. By applying the integration by parts technique to expression (5), we obtain the lease contract value $l_R(V; V_B, t)$:

$$l_R(V; V_B, t) = r_R^t \left(\frac{1 - e^{-rt}}{r} \right) - \frac{r_R^t}{r} \left(G(t) - e^{-rt} F(t) \right)$$

(8)
$$+ \rho_R^t \frac{r_R^t}{r} \left(G(t) - e^{-rt} F(t) \right)$$

The first term on the right hand side of (8) is the present value of future lease payments, the second term is the lease payment loss if the lessee defaults, and the third term is the recovery of future lease payments conditional upon default. Previous studies have assumed that the bankruptcy boundary, V_B, is exogenous. However, Leland and Toft (1996) propose a condition to determine the bankruptcy boundary endogenously. Following their model, the default probability as well as all contingent claims on the firm's assets are affected by this endogenous bankruptcy boundary. Notice that r_R^t in expression (8) is determined by expression (5), which is a function of $G(t)$ and $F(t)$. Therefore, r_R^t is not a constant.

B Debt

We assume for the sake of simplicity that the firm has a single debt issue with maturity t, coupon $c(t)$, and principal $p(t)$. Upon bankruptcy, the bondholder recovers a fraction $\rho_D(t)$ of the firm's net asset value of $\widetilde{V}_B$, where $\widetilde{V}_B$ represents the net asset value after bankruptcy cost and the present value of lessor's recovery lease payments at the time of default. In other words, $\rho_D(t)$ is the recovery rate for a debt with maturity t. Thus, we can write the value of risky debt as:

$$d(V;V_B,t) = \int_0^t e^{-r\tau}c(t)\left(1-F(\tau;V,V_B)\right)d\tau + p(t)e^{-rt}\left(1-F(\tau;V,V_B)\right)$$

$$(9) \qquad\qquad + \int_0^t e^{-r\tau}\rho_D(t)\widetilde{V}_B f(\tau;V,V_B)d\tau$$

If the firm does not declare bankruptcy, then the first term on the right hand side of (9) represents the present value of coupon payments, and the second term represents the present value of the principal payment, respectively. The third term represents the present value of the net asset value accruing to the debt holders if bankruptcy occurs. Thus, we can rewrite equation (9) as:

$$d(V;V_B,t) = \frac{c(t)}{r}\left(1-e^{-rt}\right) - \frac{c(t)}{r}\left(G(t)-F(t)e^{-rt}\right) + e^{-rt}p(t)\left(1-F(t)\right)$$

$$(10) \qquad\qquad + \int_0^t e^{-r\tau}\rho_D(t)\widetilde{V}_B f(\tau;V,V_B)d\tau.$$

C Aggregated Values of Lease and Debt

In order to simplify the analysis, we assume that the firm operates in an environment with stationary debt and lease structure. As a result, the firm continuously sells a constant amount of new debt with maturity T_D from issuance that is redeemed at

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par if the firm is still solvent at T_D. New debt is issued with a principal amount of $p = P/T_D$ per year where P is the total principal amount of outstanding debt. As the same amount of old debt is retired per year, the firm's debt structure is stationary. As long as the firm remains solvent, the total outstanding principal amount at any time s is P, and the firm has a uniform distribution over maturities in the interval (s, T_D). The total coupon payment is C per year where debt with principal pays a constant coupon $c = C/T_D$ and the total annual debt service payment is $C + P/T_D$. Furthermore, we assume equal seniority of all outstanding debt, implying that sum of all fractional claims $\rho_D(t)$ for debt of all maturities outstanding equals 1, and $\rho_D(t) = 1/T_D$ per year for all t.

In addition, the firm continuously signs an operating lease contract with maturity T_L. Suppose that the total lease payments per year is Ω_R. Following the same logic as the stationary debt structure, the operating lease contract with maturity T_L pays a constant rent $r_R^T = \Omega_R/T_L$ per year under the stationary capital structure. The sum of all fractional claims ρ_R^t for the lease contracts of all maturities outstanding equals ρ_R, implying that $\rho_R^t = \rho_R/T_L$. Note that ρ_R is equivalent to the aggregate recovery rate of all outstanding lease contracts. Because the operating lease contract is continuously signed homogeneously over time, the recovery rate for every lease contract with maturity t can be simply represented as ρ_R/T_L.

Based on these assumptions, the value of the outstanding lease, $L_R(V; V_B, t)$, can be determined by integrating the lease flows over the period T_L:

$$
(11) \qquad L_R(V; V_B, t) = \int_0^{T_L} l_R(V; V_B, t) dt
$$

$$
= \frac{\Omega_R}{r} \left(1 - \frac{1 - e^{-rT_L}}{rT_L} \right) - \frac{\Omega_R}{r} \left(J(T_L) - I(T_L) \right)
$$

$$
+ \frac{\rho_R}{T_L} \frac{\Omega_R}{r} \left(J(T_L) - I(T_L) \right)
$$

The definitions of $I(T_L)$ and $J(T_L)$ are presented in the Appendix.

Previously, $\widetilde{V}_B$ was defined as the net asset value after bankruptcy and is represented as:

$$
(12) \qquad \widetilde{V}_B = (1 - \alpha) V_B - \rho_R \Omega_R \left(\frac{1 - e^{-r(T_L - \tau)}}{r} \right).
$$

The first term on the right hand side is the asset value after bankruptcy costs, and the second term represents the cash flow recovered by the lessor in the event of bankruptcy. As debt is junior to leases in terms of claim priority, debt holders receive the asset value less the bankruptcy costs and payments to the lessors. We assume that every outstanding lease contract recovers homogeneously in time and amount such that the aggregate recoveries are represented in terms of aggregate lease payments. Thus, the debt maturity is t while the lease contract maturity is T_L. The difference in maturity incorporates the maturity of the lease contract into the debt contract. However, in calculating the loss to the lessor when the lessee defaults, we assume that on average the lessee defaults at the midpoint of the lease

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contract such that the average aggregate loss equals $(1 - \rho_R) \Omega_R \left((1 - e^{-r(T_L/2)})/r \right)$. This assumption is justified by measuring the default time on average.

Given the above assumptions, we rewrite equation (10) as:

$$d(V; V_B, t) = \frac{c(t)}{r} \left(1 - e^{-rt}\right) - \frac{c(t)}{r} \left(G(t) - F(t)e^{-rt}\right) + e^{-rt}p(t)\left(1 - F(t)\right)$$
$$+ \rho_D(t)(1 - \alpha)V_B G(t)$$
$$(13) \qquad - \rho_D(t)\rho_R \left(\frac{\Omega_R}{r}\right) \left(G(t) - e^{-rT_L}F(t)\right),$$

where the fourth and fifth terms in (13) are the additional terms caused by bankruptcy. These terms represent the value that the debtholders recover after bankruptcy cost and reimbursement to the lessor. Let $D(V; V_B, T_D)$ denote the total value of debt, when debt of maturity T_D is issued. Integrating all outstanding debt, we obtain the debt value over period T_D as:

$$D(V; V_B, T_D) = \int_0^{T_D} d(V; V_B, t)dt$$
$$= \frac{C}{r} + \left(P - \frac{C}{r}\right) \left(\frac{1 - e^{-rT_D}}{rT_D} - I(T_D)\right) + \left((1 - \alpha)V_B - \frac{C}{r}\right) J(T_D)$$
$$(14) \qquad - \rho_R \left(\frac{\Omega_R}{r}\right) \left(J(T_D) - \frac{T_L}{T_D}I(T_D)\right).$$

The first three terms on the right hand side of (14) are exactly the aggregate debt value in Leland and Toft (1996), equation (7), assuming debt is the firm's only financing resource. The fourth term represents the debt value loss because of its junior claim priority and is exactly the recovery value of the operating lease when the lessee defaults.

In most interest rate term structure models, such as Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and Leland and Toft (1996), debt is the only financing instrument in the firm's liability structure. In our framework, we incorporate leasing into the debt value determination framework. From equation (14), we see that if the firm uses a lease in addition to debt, the debt value is reduced via the amount of outstanding lease contracts and the fraction of remaining payments the lessors recover. Thus, all relevant debt properties that include the yield-spread, duration, and convexity are also affected by lease financing.

V Determining the Endogenous Default Boundary

In traditional capital structure theory, the firm trades off the tax benefits and the bankruptcy costs of debt financing. Since, we incorporate lease financing into the capital structure decision, the tax deductibility benefit is composed of the interest rate expense of debt, the lease payments, and the depreciation expense. Following Leland (1994), the total market value of the firm equals the unleveraged firm value plus the tax benefit of debt and lease financing minus the bankruptcy cost during the observation period:

$$
\begin{aligned}
v(V; V_B) = V + t_c & \left(\frac{C}{r} + \frac{\Omega_R}{r} \right) \left(1 - \left(\frac{V_B}{V} \right)^x \right) \\
& - \quad \alpha V_B + (1 - \rho_R)\, \Omega_R \quad \left. \frac{1 - e^{-r\frac{T_L}{2}}}{r} \right) \right) \left(\frac{V_B}{V} \right)^x
\end{aligned}
$$

(15)

where $v(V; V_B)$ is the market value of the firm, V is the unleveraged firm value, t_c is the corporate tax rate of the lessee, and $x = a + z$, where a and z are defined in the Appendix. The second term in (15) represents the tax-benefits associated with interest rate expense, lease payments, and depreciation expense given that the firm does not default. The third term in (15) is the bankruptcy cost given that the firm defaults and includes bankruptcy costs documented by Warner (1977) and the lessor's losses. In this setting, equation (15) is consistent with traditional capital structure trade-off theory that assumes the tax-shield benefit has a positive effect on firm value while bankruptcy costs have a negative effect.

To determine the default boundary in this model, we apply the past-smoothing condition in Leland and Toft (1996) and solve the equation to determine the endogenous default boundary, V_B. Let

$$(16) \qquad \left. \frac{\partial E\left(V; V_B, T\right)}{\partial V} \right|_{V=V_B} = 0$$

By solving equation (16), we find the endogenous bankruptcy boundary as:

$$(17) \qquad V_B^* = \frac{\frac{\Omega_R}{r}(K_1^{T_L} - K_2^{T_L}) - K_3 - K_4 + M - \left(P - \frac{C}{r}\right) K_1^{T_D} - \left(\frac{C}{r}\right) K_2^{T_D}}{1 + \alpha x - (1 - \alpha) K_2^{T_D}}$$

where, the definitions of K_1^T, K_2^T, K_3, K_4, and M are listed in the Appendix. Equation (17) reduces to the boundary condition defined in Leland and Toft (1996) when the firm does not use lease financing. Equation (17) shows that when leases are included in the capital structure decision, the amount and the recovery rate of the lease affects the firm's bankruptcy boundary. In contrast to Leland and Toft (1996), our model points out that the default boundary is affected by T_D, T_L, coupon payment amount C, lease payment amount Ω_R, and lease contract recovery

rates ρ_R. In other words, the inclusion of lease financing decreases the bankruptcy boundary through the tax deductibility of lease payments; on the other hand, it also increases the bankruptcy boundary through the perpetual lease payments Ω_R/r and the default recovery rate of the lease contract. Therefore, the lease market condition also affects the lessee firm's capital structure decision.

VI Comparative Statics and a Numerical Example

In this section, we derive the comparative statics and conduct a numerical analysis of the lease rate term structure. Due to the high nonlinearity of equation 6, it is difficult to determine the sign of the comparative statics of r_R^t with respect to the variables exactly. Therefore, we derive the comparative statics of r_R^t in terms of the sign of the comparative statics of V_B. By this method, we can see that most of the comparative statics of r_R^t are determined by the comparative statics of V_B. For the numerical example, we set the firm's initial asset value to \$100 and determine the issuing coupon and principle value assuming debt is issued at par.[13] We examine how the lease rate term structure is affected by the lease contract recovery rate, the lessee's risk aversion attitude toward lease market uncertainty, the service flow volatility of the leased asset, the lessee's corporate tax rate, and the lessee's asset volatility. Since the lease maturity may not equal the debt maturity, we also discuss the relative maturity term changes of the lease and debt contracts. Our discussion provides another dimension to examine the interaction between lease financing and debt financing and how the maturity term of debt financing affects the lease rent.

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[13]To conform with previous studies, we use the parameter values from Leland and Toft (1996).

In addition, the lease market determines the default risk premium on the lease based on the lessee's default risk. Therefore, the determination of the lease rate term structure becomes a simultaneous equation problem concerning the lessee's financing decision and the competitive lease market.

A   The Impact of Expected Lease Default Recovery Rate

We first examine the lease term structure under various assumptions regarding the potential default recovery rate. In order to determine the sign of the comparative static of r_R^t with respect to ρ_R^t (the recovery rate), we show in the Appendix the relation between $\partial r_R^t / \partial \rho_R^t$ and $\partial V_B / \partial \rho_R^t$. Thus, Table 1 shows the impact of changes in ρ_R on V_B. For cases when the lessee issues long-term debt (Panel A), we find that $(\partial V_B / \partial \rho_R^t) > 0$. However, under the assumption that the lessee issues short-term debt (Panel B), we see a non-linear relation depending upon the lease maturity (T_L).

Numerically, Table 1 confirms these comparative statics by showing the lease rates for various expected lease recovery rates (ρ_R) and lease maturities (T_L) assuming the lessee issues debt with maturity of 20-years (Panel A) and 5-years (Panel B). Panels A and B show the impact of changing the expected lease recovery rate from 0% to 60%. Not surprisingly, for short-term leases ($T_L = 2.5$ years) the lease rate is not sensitive to changes in the expected default recovery rate. However, it is interesting to note that the lease rate is only marginally increasing with the expected default recovery rate for long term lease contracts ($T_L = 20$ years). For example, with $\rho_R = 0.6$ the lease rate is only 1.6% higher than when $\rho_R = 0$ (Panel A, $T_L = 20$ years).

The rational for the marginal effect of the expected recovery rate on the lease term structure becomes clear by evaluating the effects of ρ_R on the lessee's optimal bankruptcy boundary condition (equation 17) through the components K_4, M, and Ω_R (as defined in the Appendix). We see that ρ_R affects V_B^* in two directions: First, an increase in ρ_R results in an increase in V_B^* (through K_4 and M). Second, an increase in ρ_R causes a decrease in V_B^* through Ω_R. However, the direction of causation is complicated and non-linear as Ω_R is also affected by V_B^* (because the components $G(t)$ and $F(t)$ are functions of V_B^*). As a result, the net effect of ρ_R^t on r_R^t is minimal.

The increase in the lease rate as the expected recovery rate increases (albeit small) is directly counter to the findings of Grenadier (1996). Grenadier (1996) shows that the term structure of the lease rates decreases strictly with the recovery rate. Thus, the relative lack of sensitivity to changes in the expected recovery rate points out the difference between the endogenous default credit risk model and the exogenous default credit risk model especially under the competitive lease market assumption.

B   The Impact of the Amount of Debt

In our model, the optimal amount of debt is determined endogenously by shareholders under the tradeoff between the tax benefit of debt and the bankruptcy cost, which includes the settlement cost and the possible losses for lessors. In Table 1, for $T_D = 20$ years, the leverage ratio, which is defined as the debt value over the firm value, is optimally determined in the range between 33% to 38% even when the expected lease default recovery rate changes from 0 to 0.6. Similarly, for $T_D = 5$

years, the leverage ratio is stable and located in the range between 48% to 58%. If we examine the case when $T_D = 20$ years and $\rho_R = 0.6$, the leverage ratio decreases with lease maturity while the lease rate increases with lease maturity. This result corresponds to the traditional argument in financial theory that debt and lease are substitutes. As the lease maturity increases, the value of lease increases, and it leads to the decrease of the leverage ratio.

C   The Impact of Lease Maturity

Table 1 also highlights the impact of lease maturity. Not surprisingly, we note that as the lease maturity increases, the lease rate increases. For example, column (1) of Panel A indicates that the market rent for a 20-year lease is 25 percent greater than the market rent for a 2.5-year lease. This finding is consistent with a premium existing for longer term, fixed-rate contracts.

D   The Impact of Debt Maturity

As developed in section IV, the lessee's capital structure will also impact the lease term structure. We highlight the impact of financing decisions on lease rates by examining the lease term structure under the assumption that the lessee issues short-term or long-term debt. Panel A of Table 1 contains the expected lease rates assuming that the lessee issues long-term (20-year) debt while Panel B reports the lease rates assuming the use of short-term (5-year) debt. Holding all else constant, it appears that the lease rate is positively related to lessee debt maturity.   For example, when debt maturity is 20-years the lease rate is 5.8% higher than when

the lessee debt maturity is 5-years (assuming the lease maturity is 20-years and the recovery rate is 0.)

E   The Impact of Risk Aversion

To understand the impact of risk aversion on lease rates, we differentiate (6) with respect to δ:

$$
\frac{\partial r_R^t}{\partial \delta} = \underbrace{\left(1 - e^{-rt}\right)}_{\oplus}
$$

$$
\times \left\{ \underbrace{\left(\frac{\partial r_N^t}{\partial \delta}\right) \left[\left(1 - e^{-rt}\right) - \left(1 - \rho_R^t\right)\left(G\left(t\right) - F\left(t\right)e^{-rt}\right)\right]^{-1}}_{\oplus} \right.
$$

$$
+ \underbrace{r_N^t \left[\left(1 - e^{-rt}\right) - \left(1 - \rho_R^t\right)\left(G\left(t\right) - F\left(t\right)e^{-rt}\right)\right]^{-2}}_{\oplus}
$$

$$
\left. \underbrace{\left(1 - \rho_R^t\right)\left(\frac{\partial G\left(t\right)}{\partial V_B} - \frac{\partial F\left(t\right)}{\partial V_B}e^{-rt}\right)}_{\oplus}\left(\frac{\partial V_B}{\partial \delta}\right) \right\}.
$$

Note that the sign of $\partial r_R^t/\partial\delta$ is determined by $(\partial r_N^t/\partial\delta)$ and $(\partial V_B/\partial\delta)$. From equation 4 in Section III.A, we note that when $(\mu_S - \delta\sigma_S - q - r)t$ is small, $e^{(\mu_S - \delta\sigma_S - q - r)t} - 1$ can be approximated by $(\mu_S - \delta\sigma_S - q - r)t$ and $r_N^t = S(0)\left(r/(1 - e^{-rt})\right)t$ such that r_N^t is only slightly affected by δ. However, when $(\mu_S - \delta\sigma_S - q - r)t$ is not small, δ has a noticeable affect on r_N^t.

To illustrate these effects, in Table 2 and Figure 1 we examine the impact of risk aversion in terms of the market price of risk (δ) when the lessee issues long-term debt (Panel A) and short-term debt (Panel B). As in Table 1, the lease rates increase with the lease contract's time to maturity. Furthermore, the lease rate term structure

decreases as the degree of risk aversion increases. For example, the lease rate is 22 percent lower when $\delta = 0.8$ versus when $\delta = 0.2$ (Panel A, $T_L = 20$-years). This result is consistent with Clapham and Gunnelin (2003), who show the same result assuming stochastic interest rates, but without default risk. Intuitively, the results in Table 2 arise from the fact that the drift rate of the risk-neutral service-flow process declines due to the increase in the market price of risk. Thus, the present value of future lease service flows decline regardless of how the lessee firm adjusts its endogenous default level through choice of debt maturity.

F   The Impact of Service Flow Volatility

In Table 3 and Figure 2, we examine impact of changes in the service flow volatility (σ_S) on the lease term structure. Differentiating 6 with respect to σ_S provides an indication of the expected impact of the service flow volatility on the lease rate:

$$
\begin{aligned}
\frac{\partial r_R^t}{\partial \sigma_S} = \underbrace{\left(1 - e^{-rt}\right)}_{\oplus} & \left\{ \left(\frac{\partial r_N^t}{\partial \sigma_S}\right) \underbrace{\left[\left(1 - e^{-rt}\right) - \left(1 - \rho_R^t\right)\left(G\left(t\right) - F\left(t\right)e^{-rt}\right)\right]^{-1}}_{\oplus} \right. \\
& + \underbrace{r_N^t \left[\left(1 - e^{-rt}\right) - \left(1 - \rho_R^t\right)\left(G\left(t\right) - F\left(t\right)e^{-rt}\right)\right]^{-2}}_{\oplus} \\
& \left. \underbrace{\left(1 - \rho_R^t\right)\left(\frac{\partial G\left(t\right)}{\partial V_B} - \frac{\partial F\left(t\right)}{\partial V_B}e^{-rt}\right)}_{\oplus}\left(\frac{\partial V_B}{\partial \sigma_S}\right) \right\}.
\end{aligned}
$$

Again, note that the sign of $(\partial r_R^t / \partial \delta)$ is determined by $(\partial r_N^t / \partial \sigma_S)$ and $(\partial V_B / \partial \delta)$. Similarly, when $(\mu_S - \delta\sigma_S - q - r)t$ is small, $e^{(\mu_S - \delta\sigma_S - q - r)t} - 1$ can be approximated by $(\mu_S - \delta\sigma_S - q - r)t$ and $r_N^t = S(0)\left(r/(1 - e^{-rt})\right)t$ such that r_N^t is only slightly

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affected by σ_S. However, when $(\mu_S - \delta\sigma_S - q - r)t$ is not small, σ_S has a noticeable affect on r_N^t.

Numerically, we find in Table 3 and Figure 2 that the lease rate is decreasing as the volatility of service flow increases. The lease rate increases because the increase in service flow volatility decreases the drift rate of the service flow process under a risk-neutral measure, thus, decreasing the present value of future lease service flows. In this regard, the lessee firm's default risk has little impact on the lease rate term structure when the lease rate term structure is mainly affected by the service flow process (holding all else constant). In addition, comparison of the lease rates in Panels A and B reveal that lease rate changes in response to changes in service flow volatility are not sensitive to the lessee's debt maturity, holding all else constant. For example, the lease rate is 71% lower when $\sigma_S = 0.8$ than when $\sigma_S = 0.2$ (when $T_L = 20$-years) regardless of whether the lessee's debt maturity is 5-years or 20-years.

G   The Impact of the Asset Volatility

In Table 4 and Figure 3, we show the impact of changes to the lessee's asset volatility on the lease rate term structure. To determine the expected effect of changes in asset volatility on the lease rate, we differentiate 6 with respect to σ_V:

$$\frac{\partial r_R^t}{\partial \sigma_V} = \underbrace{-r_N^t \left(1 - e^{-rt}\right) \left[\left(1 - e^{-rt}\right) - \left(1 - \rho_R^t\right) \left(G\left(t\right) - F\left(t\right) e^{-rt}\right)\right]^{-2}}_{\ominus}$$

$$\times \left[\underbrace{-\left(1 - \rho_R^t\right) \left(\frac{\partial G\left(t\right)}{\partial V_B} - \frac{\partial F\left(t\right)}{\partial V_B} e^{-rt}\right)}_{\oplus} \left(\frac{\partial V_B}{\partial \sigma_V}\right)\right].$$

The sign of $(\partial r_R^t / \partial \sigma_V)$ is determined by $(\partial V_B / \partial \sigma_V)$. Consistent with the basic conclusion of structural credit risk models, the numerical results in Table 4 and Figure 3 show that the lease rate increases as the lessee's asset volatility increases. Intuitively, the increase in the asset's risk level should increase the lease rent since holding a lease contract is analogous to selling a put option on the asset. Thus, as the volatility of the underlying asset increases, the premium of the put option increases; therefore, the lease rent will increase. Comparing the percentage increase in Panels A and B reveals that the lease rate is more sensitive to asset volatility when the lessee utilizes long term debt than when it uses short-term debt. For example, when $T_D = $ 20-years (Panel A), the 20-year contract lease rate increases 94% as asset volatility increases from 0.1 to 0.7. However, over the same asset volatility range, the 20-year contract lease rate increases 73% when $T_D = $ 5-years (Panel B).

H   The Impact of Debt Default Probability

Table 5 shows the relation between the probability of default on the lessee's existing debt and the lease term structure. In Panel A, we examine the case where the firm issues short-term debt ($T_D = $ 5 years) and Panel B shows the case where the firm issues long-term debt ($T_D = $ 20 years). The row in italics corresponds to the term structure implied by our endogenous default model. Using this as the base case, we change the default boundary to highlight the impact of the probability of debt default. The results show that the lease rate increases as the default probability increases in every case. In other words, riskier firms (i.e. those firms with higher probabilities of default) face higher equilibrium lease rates. For example, in the case where $T_D = T_L = $ 5 years we see that the lease rate increases by 18% as the default probability increases from 7.65% to 50.25%.

In addition, Table 5 shows the impact of changes in the firm's capital structure that result from the increase in firm risk, holding the lease and debt maturities constant. For example, when the firm uses long-term debt ($T_D = 20$ years), the firm's capital structure (D/V) is not sensitive to changes in the default boundary conditions. However, firm capital structure shows greater sensitivity to changes in the default boundary when the firm uses short-term debt ($T_D = 5$ years).

VII   The Impact of Tax Rates and Tax Policy

In this section, we utilize our model to highlight the complex interactions that arise resulting from differences in lessee and lessor tax rates as well as from changes in overall tax policy. Recall from Section III, we noted that equilibrium lease rate is a function of both the marginal corporate tax rate (TAX_c) as well as the tax treatment of economic depreciation (q) as reflected in the parameter (χ). As noted above, $\chi = 1$ reflects the case that accounting and economic depreciation are equivalent while $\chi < 1$ reflects the condition that the tax deduction associated with depreciation is less that the full economic depreciation. Thus, by varying χ, we can show the impact on lease rates of changes in the depreciation schedules associated with real property.

Table 6 reports the numerical estimates of the lease rates based on changes to the lessor's and lessee's corporate tax rate as well as χ. We first examine the lessee's incentive to use leasing when the lessee's corporate tax rate is higher than lessor's. The results clearly indicate that the equilibrium lease rate is higher when the lessee's tax rate is greater than the lessor's tax rate. For example, in Panel A the lease rate is 32 percent higher when the lessee's tax rate is 70% and the lessor's tax rate is 10%.

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The higher equilibrium lease rate results from the incentive from the high marginal tax rate to utilize more debt because of its higher tax-shield benefit. The increase in the utilization of debt then increases the default probability. The increase in the overall default likelihood results in a higher default premium in the lease rent. Therefore, the lease rent increases with the lessee's corporate tax rate.

Looking now at the impact of the lessor's marginal tax rate, we see that lease rates decline as the lessor's marginal tax rate increases – all else being equal. For example, looking across each row in Panel A we see that the lease rate is approximately 6% lower when the lessor's marginal tax rate is 70% compared to when the lessor's marginal tax rate is 10%.

Finally, turning to the impact of the government's tax policy regarding the depreciation shelter, we see the effect of changing the depreciable life for accounting purposes by comparing the equilibrium lease rates across Panels A, B, and C. As expected, the impact of changing the depreciable life is relatively minor when tax rates are low. For example, if lessor and lessee tax rates are 10%, then allowing accelerated depreciation (moving from Panel A ($\chi < 1$) to Panel C ($\chi > 1$)) results in a 1 to 2% decline in the equilibrium lease rate. However, when marginal tax rates are high, the effect of altering the depreciable life becomes material. For example, if marginal tax rates are 70%, then reducing the depreciable life (allowing the lessor to utilize accelerated depreciation) results in a 14% reduction in the equilibrium lease rate.

VIII   Empirical Analysis

A   Lease Data

In this section, we examine the equilibrium origination lease rates for real estate leases. Using a variety of measures and controls, we are able to test a number of predictions regarding the lease term structure and firm capital structure derived from the theoretical model.

We collect an administrative data set of small business real estate lease contracts financed by a large financial institution. The data set contains detailed information collected from the firms' financial statements, the firms' risk ratings (hard public information) from the national credit bureau (Dunn and Bradstreet), and the financial institution's internal credit score (hard private information) for each business. We also have access to the all-in-price of the lease, lease terms (duration, maturity, and amount), firm industrial code, and location of the firm (zip code). The database captures lease originations between January 2001 to March 2002. In total we have access to 2,482 lease contracts underwritten in the 8 New-England states, Illinois, and Michigan.

Table 7 reports the descriptive statistics for the sample. The leases contained in the data are to small firms. The average firm has assets of $4.78 million and is committing to a lease with an all-in-price (present value) of $423,287. The average lease has a maturity of 4.98 years. In comparison, the average debt maturity for these firms is 2.47 years. The firms have an average of $1.46 million in debt, implying a leverage ratio of 30%. Approximately 40% of the leases are for office space and 31% are for retail space.

B   Estimation Results

Our empirical test of the predictions from the model developed in Section IV follows the spirit of the reduced-form regression framework of Collin-Dufresne, Goldstein, and Martin (2001). The lease equilibrium term structure is reflected in the lease's effective yield to the financial institution. We then calculate the lease pricing spread as the difference between the lease effective yield and the 3-year constant maturity treasury yield at the date of origination. We estimate the following regression equation:

$$(18) \qquad\qquad r^* - r_{Treas} = X'\beta + \varepsilon$$

where $r^* - r_{Treas}$ is the lease pricing spread, and X represents the set of lease and firm characteristics including: the present value of the lease contract discounted at the bank's cost of capital (L_R), the lease maturity (t_L), the firm's debt maturity at lease origination (t_D), the firm's asset size at lease origination (V), the firm's asset volatility measured as the standard deviation in asset values over the previous three years (σ_V), the firm's debt-asset ratio at lease origination (D/V), the volatility in the debt-asset ratio measured as the standard deviation in the debt-asset ratio over the previous three years ($\sigma_{D/V}$), and a set of control variables reflecting the property type being leased (office, agricultural, retail, or other) and the firm's risk rating. We also include zip code fixed effects to control of location specific factors, month fixed effects to control of temporal difference in the economic environment over the sample period, and SIC fixed effects to control for differences in the firm's industry segment.

Table 8 reports the results for the ordinary least squares (OLS) regression of equation (18). The model has a high degree of fit as reflected in the R^2 indicating that the model is able to explain approximately 63% of the cross-sectional variation in lease rates.

Turning to the individual coefficients of interest based on our theoretical lease term structure model, we first note that the estimated coefficient for lease contract maturity is positive and significant (at the 1% level); confirming the prediction of a positive relation between lease maturity and the lease spread (Section C). We also see that the estimated coefficient for debt maturity is also positive and significant (at the 1% level) indicating that firms with longer-term debt face higher leasing costs. Furthermore, the dummy variable indicating that the firm utilizes long-term debt (greater than or equal to 5-years) is also positive and significant at the 1 percent level. Thus, the estimated coefficients for debt maturity confirm the prediction in section D of a positive relation between lease rates and debt maturity.

In section G, we noted that our lease term structural predicts a positive relation between the firm's asset volatility and the lease rate, which is consistent with the traditional credit risk models. The estimated coefficient for asset volatility reported in Table 8 is positive and significant at the 1% level, confirming the model's prediction.

Consistent with the debt/lease tradeoff arguments in financial theory (section B), the regression results indicate a positive and significant (at the 1% level) relation between the use of debt (leverage ratio) and lease rate. In other words, as the firm's leverage (debt/asset) level increases, the cost of leasing increases. Thus, this result empirically verifies the tradeoff between debt and leases.

Turning to the firm risk ratings, we see that the coefficients for the internal and external risk credit scores are significant (at the 1% level) and have the appropriate sign. First, the coefficient for the bank's internal risk rating measure is positive indicating that firms with higher risk factors (as measured by the bank) face higher lease rates. Second, the negative coefficient for the external credit score (higher scores correspond to lower risk) again implies that lower risk firms face lower lease rates.

Finally, we note that the property type control variables are highly significant and consistent with expectation regarding the asset risk. For example, we note that the estimated coefficient for Office properties is negative and significant (at the 1% level) consistent with prior expectations that office space should have higher recovery rates (i.e. be easier to re-lease in case of default) than specialized property types contained in the "other" category. In addition, we also see that retail and agricultural properties command a lease rate premium that is consistent with the lower expected recovery rates on these property types. For example, tenants in retail spaces are most likely to default during economic downturns affecting all users of retail space.

As a robustness check, we also estimate equation (18) separately for each property type (office, agricultural, retail, and other) in order to verify that the results identified above are not simply a function of differences in contracts across the various property types. Table 9 reports the estimated coefficients from these regressions. We note that all the coefficients are statistically significant and have the same signs as the coefficients reported in Table 8. Thus, the separate regressions again confirm the theoretical predictions of our model. We also note that the magnitude of the estimated coefficients differ across property types reflecting the idiosyncratic prop-

erty types. For example, the coefficient for debt maturity in the 'other' property type is roughly half the magnitude of the 'office' property type coefficient. Furthermore, we note that the adjusted R^2's indicate that the 'agricultural' and 'other' property type models have approximately half the explanatory power of the 'office' and 'retail' models. However, the lower explanatory power is to be expected given the greater heterogeneity of uses, and thus risk, associated with agricultural and generic property types.

IX Conclusion

The lease rate term structure model has been widely discussed in literature due to its similarity with interest rate term structure. This paper focuses on defaultable lease rate term structure with the application of endogenous default model. This credit risk issue is not new in fixed-income pricing. However, only a few papers, e.g., Grenadier (1996), Clapham and Gunnelin (2003), Ambrose and Yildirim (2008), formally model this issue in terms of a lease contract. This paper applies the competitive lease market argument and endogenous default structural model proposed by Leland and Toft (1996) to examine the interaction between a lessee firm's financial decision and lease market, thus, the lease rate determination problem becomes a simultaneous equation problem in contrast with previous studies, which assume the lessee firm's default risk is independent from the lease market.

We examine the lease rate term structure under different scenarios. The results are consistent with previous models except the recovery rate may not play a significant role in lease rate determination because the impact of the exogenous recovery rate can be offset by the lessee firm's endogenous financial decisions. Further, we

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examine the interaction between debt's maturities and lease rate term structure. Shorter debt maturity leads to less cash outflows and lessens the financial burden of debt, thus decreasing the lessee firm's default risk and resulting in a smaller lease rate term structure.

These results provide testable hypotheses about lease rate determination. They include a negative relationship between market price of risk and lease rate, a negative relationship between service flow volatility and lease rate, a negative relationship between lessee's tax rate and lease rate, a positive relationship between lessee's asset volatility and lease rate, and a positive relationship between maturity of existing debt and lease rate. Our empirical analysis of 2,482 small business leases for commercial property confirm these predictions. Overall, our empirical model is able to explain approximately 63 percent of the cross-sectional variation in lease rates.

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A Appendix

A Derivation of Equation (4).

The closed form solution for the S.D.E in equation 1 can be represented as:

(A-1)
$$S_{BD}(t) = S_0 e^{(\mu_S - \frac{1}{2}\sigma_S^2)t + \sigma_S W_S(t)}$$

where S_0 is the initial service flow of the leased asset, and $W(t)$ is the standard Brownian motion at time t under the true measure. Because the leased asset may not be very liquid or easily traded, in order to obtain the expected present value of the discounted service flow, we employ the market price risk adjustment to derive the service flow process under the risk-neutral measure:

$$S_{BD}(t) = S_0 e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)t + \sigma_S \widetilde{W}_S(t)}$$

where δ is the market price of risk for service flow, and $\widetilde{W}(t)$ is the standard Brownian motion under the risk-neutral measure. Using the equation (A-1), the present value of the future service flows is:

$$
\begin{aligned}
&\widetilde{E}\left(\int_0^t S_{BD}(u)e^{-ru}du\right) \\
&= \widetilde{E}\left(\int_0^t S_0 e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)u + \sigma_S \widetilde{W}_S(u)}e^{-ru}du\right) \\
&= S_0\widetilde{E}\left(\int_0^t e^{(\mu_S - \delta\sigma_S - \frac{1}{2}\sigma_S^2)u + \sigma_S \widetilde{W}_S(u)}e^{-ru}du\right).
\end{aligned}
$$

Furthermore, by applying Fubini's Theorem (refer to Arnold (1974)),

$$
\widetilde{E}\left(\int_0^t e^{(\mu_S-\delta\sigma_S-\frac{1}{2}\sigma_S^2)u+\sigma_S\widetilde{W}_S(u)}e^{-ru}du\right)
$$

$$
=\int_\Omega\int_0^t e^{(\mu_S-\delta\sigma_S-\frac{1}{2}\sigma_S^2)u+\sigma_S\widetilde{W}_S(u)}e^{-ru}du\,d\widetilde{P}
$$

$$
=\int_0^t e^{(\mu_S-\delta\sigma_S-\frac{1}{2}\sigma_S^2-r)u}\left(\int_\Omega e^{\sigma_S\widetilde{W}_S(u)}d\widetilde{P}\right)du
$$

$$
=\int_0^t e^{(\mu_S-\delta\sigma_S-\frac{1}{2}\sigma_S^2-r)u}\widetilde{E}\left(e^{\sigma_S\widetilde{W}_S(u)}\right)du
$$

where Ω is the sample space of the random variable $\widetilde{W}(u)$, and $\widetilde{E}(e^{\sigma_S\widetilde{W}_S(u)})$ is the moment generating function of the Brownian motion $\widetilde{W}_S(u)$, which is equal to $e^{\frac{u}{2}\sigma_S^2}$. Therefore, the expected present value of the service flow from 0 to t can be written as:

(A-2) $$\widetilde{E}\left(\int_0^t S_{BD}(u)e^{-ru}du\right)=\frac{S(0)}{(\mu_S-\delta\sigma_S-r)}\left(e^{(\mu_S-\delta\sigma_S-r)t}-1\right)$$

$$\text{if}\quad(\mu_S-\delta\sigma_S-r)\neq 0.$$

By applying the same technique to the after-depreciation case, we can obtain a similar result:

(A-3) $$\widetilde{E}\left(\int_0^t S_{AD}(u)e^{-ru}du\right)=\frac{S(0)}{(\mu_S-\delta\sigma_S-q-r)}\left(e^{(\mu_S-\delta\sigma_S-q-r)t}-1\right)$$

$$\text{if}\quad(\mu_S-\delta\sigma_S-q-r)\neq 0.$$

Inserting equations (A-2) and (A-3) into equation (3) in the Section III.A and equating to $r_N^t\left((1-e^{-rt})/r\right)$, we obtain the expression for r_N^t in equation (4).

B Definitions of $G(t)$, $F(t)$ and Relevant Equations.

$$G(t) = \int_0^t e^{-r\tau} f(\tau; V, V_B) d\tau$$

$$F(t) = \int_0^t f(\tau; V, V_B) d\tau$$

and by Leland and Toft (1996), we can represent $G(t)$, $F(t)$ as:

$$F(t, V, V_B) = N(h_1(t)) + \left(\frac{V}{V_B}\right)^{-2a} N(h_2(t))$$

$$G(t, V, V_B) = \left(\frac{V}{V_B}\right)^{-a+z} N(q_1(t)) + \left(\frac{V}{V_B}\right)^{-a-z} N(q_2(t))$$

$N(\cdot)$ is the cumulative standard normal distribution, and

$$q_1(t) = \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}; \qquad q_2(t) = \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}}$$

$$h_1(t) = \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}; \qquad h_2(t) = \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}$$

$$a = \frac{r - \delta - \left(\frac{\sigma^2}{2}\right)}{\sigma^2}; \qquad b = \ln\left(\frac{V}{V_B}\right); \qquad z = \frac{\left((a\sigma^2)^2 + 2r\sigma^2\right)^{0.5}}{\sigma^2}$$

C Derivation of Equation (6).

The default-risky lease contract value can be represented as equation (5) in Section III.B. Since $R_R^{t-\tau}$ is equal to $r_R^t((1 - e^{-r(t-\tau)})/r)$, we can further reduce equation (6) in Section III.B to be equation (8) in Section IV.A. In equilibrium, the default-risky lease contract value is equal to the default risk-free lease contract value, which is

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$r_N^t((1 - e^{-rt})/r)$. By equalizing equation (8) and $r_N^t((1 - e^{-rt})/r)$, we obtain the relation between the default risk-free lease rate and the default risky lease rate in equation (6).

D   Definitions of $I(t)$, $J(t)$ and Relevant Equations.

$$I(T) = \frac{1}{T} \int_0^T e^{-rt} F(t) dt; \qquad\qquad J(T) = \frac{1}{T} \int_0^T G(t) dt$$

In Leland and Toft (1996), they show that $I(t)$, $J(t)$ also can be represented as:

$$I(T) = \frac{1}{rT} \left(G(T) - e^{-rT} F(T) \right)$$

$$J(T) = \frac{1}{z\sigma_V \sqrt{T}} \left(-\left(\frac{V}{V_B}\right)^{-a+z} N\left(q_1(T)\right) q_1(T) + \left(\frac{V}{V_B}\right)^{-a-z} N\left(q_2(T)\right) q_2(T) \right)$$

It is important to note that the value of $J(T)$ is greater than the value of $I(T)$; we can observe that from the definition of these two functions.

E   Definitions of K_1^T, K_2^T, K_3, K_4, and M.

$$K_1^T = \frac{1}{rT}\left[(-a+z)N(-z\sigma_V\sqrt{T}) + \frac{n(-z\sigma_V\sqrt{T})}{\sigma_V\sqrt{T}} + (-a-z)N(z\sigma_V\sqrt{T}) + \frac{n(z\sigma_V\sqrt{T})}{\sigma\sqrt{T}}\right]$$

$$- \frac{e^{-rt}}{rT}\left[\frac{n(-a\sigma_V\sqrt{T})}{\sigma_V\sqrt{T}} - 2aN(a\sigma_V\sqrt{T}) - \frac{n(z\sigma_V\sqrt{T})}{\sigma_V\sqrt{T}}\right]$$

$$K_2^T = (-a+z)N(-z\sigma_V\sqrt{T}) - \frac{n(-z\sigma_V\sqrt{T})}{\sigma_V\sqrt{T}} + N(-z\sigma_V\sqrt{T})\frac{1}{z\sigma_V^2 T} + (-a-z)N(z\sigma_V\sqrt{T})$$

$$- n(z\sigma_V\sqrt{T})\frac{1}{\sigma_V\sqrt{T}} - N(z\sigma_V\sqrt{T})\frac{1}{z\sigma_V^2 T}$$

$$K_3 = (C + \Omega_R)\left(\frac{t_c}{r}\right)x$$

$$K_4 = (1 - \rho_R)\,\Omega_R\left[\frac{1 - e^{-r\frac{T_L}{2}}}{r}\right]$$

$$M = \left(\frac{\Omega_R}{r}\rho_R\right)\left(K_1^{T_L}\frac{T_L}{T_D} - K_2^{T_D}\right) - \left(\frac{\Omega_R}{r}\frac{\rho_R}{T_L}\right)\left(K_1^{T_L} - K_2^{T_L}\right)$$

where $N(\cdot)$ is the cumulative standard normal distribution, and $n(\cdot)$ is the probability density function of standard normal distribution. Other variables are defined in the main context of this paper.

F Derivation of Comparative Static in Section VI.

In equation (6), we note that

$$r_R^t$$

$$= r_N^t \left[\frac{1 - e^{-rt}}{(1 - e^{-rt}) - (1 - \rho_R^t)(G(t) - F(t)e^{-rt})} \right]$$

$$= r_N^t (1 - e^{-rt}) \underbrace{\left[(1 - e^{-rt}) - (1 - \rho_R^t)(G(t) - F(t)e^{-rt}) \right]^{-1}}_{\oplus}$$

Therefore,

$$\left[(1 - e^{-rt}) - (1 - \rho_R^t)(G(t) - F(t)e^{-rt}) \right]^{-1} > 0,$$

and

$$\left[(1 - e^{-rt}) - (1 - \rho_R^t)(G(t) - F(t)e^{-rt}) \right]^{-2} > 0.$$

Thus, the comparative static of r_R^t with respect to ρ_R^t (the recovery rate) is

$$\frac{\partial r_R^t}{\partial \rho_R^t}$$

$$= \underbrace{-r_N^t (1 - e^{-rt}) \left[(1 - e^{-rt}) - (1 - \rho_R^t)(G(t) - F(t)e^{-rt}) \right]^{-2}}_{\ominus}$$

(A-4) $$\times \left[\underbrace{(G(t) - F(t)e^{-rt})}_{\oplus} - \underbrace{(1 - \rho_R^t)\left(\frac{\partial G(t)}{\partial V_B} - \frac{\partial F(t)}{\partial V_B} e^{-rt} \right)}_{\oplus} \left(\frac{\partial V_B}{\partial \rho_R^t} \right) \right].$$

Since

$$(G(t) - F(t)e^{-rt}) = \left(\int_0^t e^{-r\tau} f(\tau; V, V_B)d\tau - e^{-rt} \int_0^t f(\tau; V, V_B)d\tau \right)$$

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and

$$\frac{\partial f(\tau; V, V_B)}{\partial V_B} > 0,$$

the sign of $\partial r_R^t / \partial \rho_R^t$ is determined by

$$\left[\left(G(t) - F(t) e^{-rt} \right) - \left(1 - \rho_R^t \right) \left(\frac{\partial G(t)}{\partial V_B} - \frac{\partial F(t)}{\partial V_B} e^{-rt} \right) \left(\frac{\partial V_B}{\partial \rho_R^t} \right) \right],$$

which is determined by $\left(\partial V_B / \partial \rho_R^t \right)$. When $sgn \left(\partial V_B / \partial \rho_R^t \right) < 0$,

$$sgn \left[\underbrace{\left(G(t) - F(t) e^{-rt} \right)}_{\oplus} - \underbrace{\left(1 - \rho_R^t \right) \left(\frac{\partial G(t)}{\partial V_B} - \frac{\partial F(t)}{\partial V_B} e^{-rt} \right) \left(\frac{\partial V_B}{\partial \rho_R^t} \right)}_{\oplus} \right] > 0.$$

When $sgn \left(\partial V_B / \partial \rho_R^t \right) > 0$,

$$sgn \left[\underbrace{\left(G(t) - F(t) e^{-rt} \right)}_{\oplus} - \underbrace{\left(1 - \rho_R^t \right) \left(\frac{\partial G(t)}{\partial V_B} - \frac{\partial F(t)}{\partial V_B} e^{-rt} \right) \left(\frac{\partial V_B}{\partial \rho_R^t} \right)}_{\oplus} \right]$$

is determined by the amount of every item in the brackets.

Table 1: **Impact of Lease Recovery Rate on the Lease Rate Term Structure**

Panel A: $T_D = 20$ years

	$\rho_R = 0$			$\rho_R = 0.2$			$\rho_R = 0.4$			$\rho_R = 0.6$		
T_L	Leverage Ratio	Default Boundary	Lease Rate	Leverage Ratio	Default Boundary	Lease Rate	Leverage Ratio	Default Boundary	Lease Rate	Leverage Ratio	Default Boundary	Lease Rate
2.5	57.03%	41.31	1.02	55.45%	41.58	1.02	53.84%	41.84	1.02	52.60%	42.48	1.02
5	56.35%	40.90	1.04	54.73%	41.23	1.04	53.48%	41.95	1.04	51.39%	41.94	1.04
10	56.05%	40.47	1.13	54.03%	40.71	1.13	52.38%	41.32	1.13	50.70%	41.95	1.13
15	55.65%	39.71	1.21	53.70%	40.27	1.21	51.29%	40.46	1.21	49.26%	41.03	1.21
20	55.23%	38.84	1.27	53.36%	39.71	1.28	51.03%	40.22	1.29	48.65%	40.75	1.29

Panel B: $T_D = 5$ years

	$\rho_R = 0$			$\rho_R = 0.2$			$\rho_R = 0.4$			$\rho_R = 0.6$		
T_L	Leverage Ratio	Default Boundary	Lease Rate	Leverage Ratio	Default Boundary	Lease Rate	Leverage Ratio	Default Boundary	Lease Rate	Leverage Ratio	Default Boundary	Lease Rate
2.5	36.53%	34.51	1.01	35.73%	34.66	1.01	34.92%	34.82	1.01	34.11%	34.97	1.01
5	35.24%	33.45	1.03	35.30%	34.14	1.03	35.36%	34.82	1.03	34.51%	34.65	1.03
10	34.84%	33.00	1.09	34.94%	33.41	1.09	35.51%	34.26	1.09	35.63%	34.67	1.09
15	33.93%	31.88	1.14	35.09%	33.09	1.14	35.36%	33.44	1.15	36.63%	34.65	1.16
20	33.46%	31.13	1.20	34.33%	31.92	1.20	35.75%	33.14	1.20	37.78%	34.81	1.23

Note: This table displays the lease rate term structure, default boundary values, and leverage ratios for lease maturities (T_L) ranging from 2.5 years to 20 years assuming the risk free interest rate $r = 7.5\%$, the bankruptcy costs $\alpha = 50\%$, the lessor's corporate tax rate $TAX_c = 35\%$, the lessee's corporate tax rate $t_c = 35\%$, the firm's asset volatility $\sigma_v = 20\%$, the firm's payout rate $\delta_V = 7\%$, the drift term of the service flow process $\mu_S = 6\%$, the depreciation rate for the service flow $q = 5\%$, the initial service flow $S_0 = 1$, the risk aversion $\delta = 0.83$, and the level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$.

Table 2: **The Impact of Risk Aversion on the Lease Rate Term Structure**

Panel A: $T_D = 20$ **years**

| | $\delta = 0.2$ | | $\delta = 0.4$ | | $\delta = 0.6$ | | $\delta = 0.8$ | |
| | Default | Lease | Default | Lease | Default | Lease | Default | Lease |
T_L	Boundary	Rate	Boundary	Rate	Boundary	Rate	Boundary	Rate
2.5	42.08	1.05	42.40	1.04	42.34	1.03	42.29	1.02
5	41.78	1.12	42.02	1.10	42.27	1.07	42.15	1.05
10	41.57	1.30	41.26	1.24	41.72	1.19	41.82	1.14
15	40.90	1.48	40.75	1.39	41.02	1.30	41.34	1.23
20	40.14	1.66	40.13	1.52	40.58	1.41	40.73	1.30

Panel B: $T_D = 5$ **years**

| | $\delta = 0.2$ | | $\delta = 0.4$ | | $\delta = 0.6$ | | $\delta = 0.8$ | |
| | Default | Lease | Default | Lease | Default | Lease | Default | Lease |
T_L	Boundary	Rate	Boundary	Rate	Boundary	Rate	Boundary	Rate
2.5	35.17	1.05	35.14	1.04	35.10	1.03	35.07	1.02
5	34.43	1.11	34.82	1.09	34.77	1.06	35.16	1.04
10	34.44	1.25	34.38	1.20	34.77	1.15	34.72	1.10
15	34.38	1.41	33.90	1.32	34.73	1.25	35.13	1.17
20	33.63	1.57	34.03	1.45	34.44	1.34	34.41	1.24

Note: This table displays the lease rate term structure and default boundary values for lease maturities (T_L) ranging from 2.5 years to 20 years assuming the risk free interest rate $r = 7.5\%$, the bankruptcy costs $\alpha = 50\%$, the lessor's corporate tax rate $\text{TAX}_c = 35\%$, the lessee's corporate tax rate $t_c = 35\%$, the firm's asset volatility $\sigma_v = 20\%$, the firm's payout rate $\delta_V = 7\%$, the drift term of the service flow process $\mu_S = 6\%$, the depreciation rate for the service flow $q = 5\%$, the initial service flow $S_0 = 1$, the recovery rate $\rho_R = 0.62$ and the level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$.

Table 3: **The Impact of Service Flow Volatility on the Lease Rate Term Structure**

Panel A: $T_D = 20$ years

| | $\sigma_S = 0.2$ | | $\sigma_S = 0.4$ | | $\sigma_S = 0.6$ | | $\sigma_S = 0.8$ | |
| | Default | Lease | Default | Lease | Default | Lease | Default | Lease |
T_L	Boundary	Rate	Boundary	Rate	Boundary	Rate	Boundary	Rate
2.5	42.37	0.87	42.43	0.73	42.65	0.61	42.99	0.52
5	42.31	0.79	42.70	0.57	42.75	0.43	43.05	0.34
10	42.36	0.69	42.80	0.42	42.87	0.29	43.26	0.22
15	42.50	0.63	42.87	0.35	43.01	0.24	43.06	0.18
20	42.33	0.59	42.71	0.32	43.28	0.22	42.97	0.17

Panel B: $T_D = 5$ years

| | $\sigma_S = 0.2$ | | $\sigma_S = 0.4$ | | $\sigma_S = 0.6$ | | $\sigma_S = 0.8$ | |
| | Default | Lease | Default | Lease | Default | Lease | Default | Lease |
T_L	Boundary	Rate	Boundary	Rate	Boundary	Rate	Boundary	Rate
2.5	36.00	0.87	36.48	0.73	36.61	0.61	37.24	0.52
5	35.99	0.78	36.90	0.56	37.52	0.43	37.79	0.34
10	36.48	0.66	37.98	0.41	38.30	0.28	38.24	0.22
15	37.02	0.60	38.20	0.34	38.59	0.23	39.00	0.18
20	37.29	0.56	38.11	0.31	38.52	0.21	38.95	0.16

Note: This table displays the lease rate term structure and default boundary value for lease maturities (T_L) ranging from 2.5 years to 20 years assuming the risk free interest rate $r = 7.5\%$, the bankruptcy costs $\alpha = 50\%$, the lessor's corporate tax rate $\text{TAX}_c = 35\%$, the lessee's corporate tax rate $t_c = 35\%$, the firm's payout rate $\delta_V = 7\%$, the drift term of the service flow process $\mu_S = 6\%$, the depreciation rate for the service flow $q = 5\%$, the initial service flow $S_0 = 1$, the risk aversion $\delta = 0.83$, the recovery rate $\rho_R = 0.62$, and the level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$.

Table 4: **The Impact of Asset Volatility on the Lease Rate Term Structure**

Panel A: $T_D = 20$ years

| T_L | $\sigma_V = 0.1$ | | $\sigma_V = 0.3$ | | $\sigma_V = 0.5$ | | $\sigma_V = 0.7$ | |
	Default Boundary	Lease Rate	Default Boundary	Lease Rate	Default Boundary	Lease Rate	Default Boundary	Lease Rate
2.5	58.50	1.01	33.50	1.02	25.92	1.05	19.76	1.09
5	58.22	1.03	33.70	1.07	25.56	1.18	20.35	1.33
10	57.33	1.08	33.29	1.21	24.05	1.45	19.58	1.78
15	57.30	1.13	32.33	1.34	22.64	1.66	17.48	2.08
20	57.60	1.18	31.56	1.44	21.32	1.82	15.72	2.29

Panel B: $T_D = 5$ years

| T_L | $\sigma_V = 0.1$ | | $\sigma_V = 0.3$ | | $\sigma_V = 0.5$ | | $\sigma_V = 0.7$ | |
	Default Boundary	Lease Rate	Default Boundary	Lease Rate	Default Boundary	Lease Rate	Default Boundary	Lease Rate
2.5	53.10	1.01	28.49	1.02	26.23	1.05	22.24	1.12
5	52.66	1.03	28.31	1.05	25.69	1.19	22.55	1.38
10	52.75	1.07	28.18	1.16	23.63	1.44	19.32	1.77
15	52.84	1.11	27.53	1.27	20.65	1.60	15.25	1.98
20	52.84	1.15	26.21	1.35	17.40	1.68	10.42	1.99

Note: This table displays the lease rate term structure and default boundary values for lease maturities (T_L) ranging from 2.5 years to 20 years assuming the risk free interest rate $r = 7.5\%$, the bankruptcy costs $\alpha = 50\%$, the lessor's corporate tax rate $\text{TAX}_c = 35\%$, the lessee's corporate tax rate $t_c = 35\%$, the firm's asset volatility $\sigma_v = 20\%$, the firm's payout rate $\delta_V = 7\%$, the drift term of the service flow process $\mu_S = 6\%$, the depreciation rate for the service flow $q = 5\%$, the initial service flow $S_0 = 1$, the risk aversion $\delta = 0.83$, the recovery rate $\rho_R = 0.62$, and the level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$.

Table 5: **The Impact of Probability of Debt Default on the Lease Rate Term Structure**

| | $T_L = 5$ **years** | | | | $T_L = 20$ **year** | | |
Default Boundary	Default Probability	D/V	Lease Rate	Default Boundary	Default Probability	D/V	Lease Rate
Panel A: $T_D = 5$ years							
20.00	7.65%	33.63%	1.03	20.00	7.65%	35.05%	1.13
35.14	*18.82%*	*34.97%*	*1.03*	30.00	14.61%	36.70%	1.19
40.00	23.14%	35.55%	1.04	*34.84*	*18.56%*	*37.83%*	*1.23*
45.00	27.93%	36.24%	1.05	40.00	23.14%	39.33%	1.28
50.00	33.05%	37.04%	1.07	45.00	27.93%	41.09%	1.35
55.00	38.48%	38.00%	1.10	50.00	33.05%	43.26%	1.43
60.00	44.22%	39.22%	1.15	55.00	38.48%	45.96%	1.54
65.00	50.25%	40.82%	1.22	60.00	44.22%	49.43%	1.68
Panel B: $T_D = 20$ years							
20.00	7.65%	51.82%	1.03	20.00	7.65%	48.14%	1.13
40.00	23.14%	51.44%	1.04	30.00	14.61%	48.21%	1.19
42.08	*25.09%*	*51.38%*	*1.04*	40.00	23.14%	48.24%	1.28
45.00	27.93%	51.31%	1.05	*40.64*	*23.73%*	*48.24%*	*1.29*
50.00	33.05%	51.24%	1.07	45.00	27.93%	48.32%	1.35
55.00	38.48%	51.27%	1.10	50.00	33.05%	48.51%	1.43
60.00	44.22%	51.47%	1.15	55.00	38.48%	48.86%	1.54
65.00	50.25%	51.86%	1.22	60.00	44.22%	49.43%	1.68

Note: This table examines the relation between the lease rate term structure and the probability of default on the lessee's debt. We assume the risk free interest rate $r = 7.5\%$, the bankruptcy costs $\alpha = 50\%$, the lessor's corporate tax rate $\text{TAX}_c = 35\%$, the lessee's corporate tax rate $t_c = 35\%$, the firm's asset volatility $\sigma_v = 20\%$, the firm's payout rate $\delta_V = 7\%$, the drift term of the service flow process $\mu_S = 6\%$, the depreciation rate for the service flow $q = 5\%$, the initial service flow $S_0 = 1$, the risk aversion $\delta = 0.83$, the recovery rate $\rho_R = 0.62$, and the level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$.

Table 6: **The Impact of Tax Policies on the Lease Rate Term Structure**

Panel A: $\chi = 0.5$

	Lessor's Tax Rate			
Lessee's Tax Rate	10%	30%	50%	70%
10%	1.08	1.06	1.04	1.02
30%	1.11	1.08	1.06	1.04
50%	1.49	1.44	1.43	1.40
70%	1.43	1.40	1.21	1.35

Panel B: $\chi = 1.0$

	Lessor's Tax Rate			
Lessee's Tax Rate	10%	30%	50%	70%
10%	1.07	1.03	0.99	0.95
30%	1.09	1.05	1.01	0.96
50%	1.48	1.41	1.36	1.30
70%	1.42	1.36	1.31	1.25

Panel C: $\chi = 1.5$

	Lessor's Tax Rate			
Lessee's Tax Rate	10%	30%	50%	70%
10%	1.06	1.00	0.93	0.87
30%	1.08	1.02	0.95	0.89
50%	1.46	1.37	1.29	1.21
70%	1.40	1.32	1.24	1.15

Note: This table displays the lease rate term structure for lease maturities (T_L) ranging from 2.5 years to 20 years assuming the risk free interest rate $r = 7.5\%$, the bankruptcy costs $\alpha = 50\%$, the lessor's corporate tax rate $\mathrm{TAX}_c = 35\%$, the lessee's corporate tax rate $t_c = 35\%$, the firm's asset volatility $\sigma_v = 20\%$, the firm's payout rate $\delta_V = 7\%$, the drift term of the service flow process $\mu_S = 6\%$, the depreciation rate for the service flow $q = 5\%$, the initial service flow $S_0 = 1$, the risk aversion $\delta = 0.83$, and the recovery rate $\rho_R = 0.62$.

Table 7: **Summary Statistics of the Leasing Data**

Summary Statistics	Means	Std
Lease Amount	$423,297.30	$447,782.90
Lease Contract Maturity (Years)	4.98	4.19
Debt Maturity (Years)	2.47	2.18
Firm Size (Assets)	$4,784,322	$3,915,630
Debt	$1,458,751	$1,201,745
Internal Risk Rating	3.9385	1.5082
External Credit Score	717.18	81.91
Property Type Office Space	39.71%	
Property Type Agriculture Land	13.36%	
Property Type Retail Space	31.35%	
Property Type Others	15.58%	

Note: This table reports the descriptive statistics for the administrative data set of real estate lease contracts originated from a large financial institution between January 2001 to March 2002. The dataset covers 2,482 lease contracts underwritten in the 8 New-England states, Illinois, and Michigan. Lease amount is the present value of the lease contract. External Credit Score refers to the firm's risk rating (hard public information) obtained from the national credit bureau (Dunn and Bradstreet). Internal Risk Rating is the bank's proprietary credit score based on hard private information about the firm.

Table 8: **Empirical Analysis of Lease Term Structure**

Variable	Coeff Value	Std Err	t-stat
Intercept	8.1822	1.9272	4.25***
Log (Lease Amount)	-1.4934	0.2273	-6.57***
Lease Contract Maturity (Years)	0.3704	0.0413	8.97***
Debt Maturity (Years)	0.3586	0.1571	2.29**
Log(Firm Size (Assets))	-1.6450	0.4011	-4.10***
Asset Volatility (Std Dev over three years)	0.5997	0.1830	3.28***
Log(Debt/Assets)	2.3428	0.5117	4.58***
Debt/Asset Volatility (Std Dev over three years)	-0.2038	0.1299	-1.57*
Long Term Debt=1	0.2721	0.0391	6.96***
Property Type Office Space	-0.1172	0.0232	-5.04***
Property Type Agriculture Land	0.3737	0.0377	9.88***
Property Type Retail Space	0.3548	0.0389	9.12***
Internal Risk Rating	0.9782	0.1883	5.19***
Log (External Credit Score (D&B))	-1.0821	0.3798	-2.85**
Zip Code Fixed Effects	Yes		
Month Fixed Effects	Yes		
SIC Fixed Effects	Yes		
Number of Obs	2482		
R-Sq	0.6291		

Note: This table reports the estimated coefficients from the ordinary lease squares regression of the lease term structure for real estate lease contracts originated from a large financial institution between January 2001 to March 2002. The dependent variable is the all-in lease spread, defined as the lease effective yield less the 3-year constant maturity treasury yield. Lease amount is the present value of the lease contract. External Credit Score refers to the firm's risk rating (hard public information) obtained from the national credit bureau (Dunn and Bradstreet). Internal Risk Rating is the bank's proprietary credit score based on hard private information about the firm. *** - significant at the 1% level. ** - significant at the 5% level. * - significant at the 10% level.

Table 9: **Empirical Analysis of Lease Term Structure by Property Type**

	Office	Agriculture	Retail	Other
Intercept	6.9214***	4.7522***	5.3689***	7.1273***
	(2.59)	(1.46)	(1.55)	(3.34)
Log (Lease Debt Amount)	-1.6713***	-1.1308**	-1.6276***	-1.1828**
	(0.44)	(0.60)	(0.54)	(0.62)
Lease Contract Maturity (Years)	0.8783***	0.4370**	0.6028***	0.2095***
	(0.27)	(0.22)	(0.25)	(0.09)
Debt Maturity (Years)	0.8499***	0.5970**	0.7664***	0.4310***
	(0.26)	(0.31)	(0.24)	(0.17)
Log(Firm Size (Assets))	-2.2271***	-1.4154***	-2.5936***	-3.1919***
	(0.19)	(0.64)	(0.53)	(0.50)
Asset Volatility (Std Dev over three years)	0.4654***	0.3974***	0.2020***	0.3745***
	(0.11)	(0.13)	(0.05)	(0.07)
Log(Debt/Assets)	1.3825***	1.2144***	2.0924***	3.1505***
	(0.49)	(0.47)	(0.59)	(0.49)
Debt/Asset Volatility (Std Dev over three years)	-0.3800**	-0.3361*	-0.3938*	-0.1289***
	(0.19)	(0.23)	(0.24)	(0.05)
Long Term Debt=1	0.1627***	0.4862***	0.3710***	0.4823***
	(0.04)	(0.07)	(0.10)	(0.07)
Internal Risk Rating	0.7195***	0.5216***	0.9341***	1.2714***
	(0.16)	(0.18)	(0.27)	(0.23)
Log (External Credit Score (D&B))	-1.2570***	-0.9670***	-0.8269**	-1.3652**
	(0.58)	(0.15)	(0.46)	(0.71)
Zip Code Fixed Effects	Yes	Yes	Yes	Yes
Month Fixed Effects	Yes	Yes	Yes	Yes
SIC Fixed Effects	Yes	Yes	Yes	Yes
Number of Obs	986	331	778	387
R-Sq	0.5918	0.2012	0.4991	0.1827

Note: This table reports the estimated coefficients from the ordinary lease squares regression of the lease term structure for real estate lease contracts originated from a large financial institution between January 2001 to March 2002. The dependent variable is the all-in lease spread, defined as the lease effective yield less the 3-year constant maturity treasury yield. Lease amount is the present value of the lease contract. External Credit Score refers to the firm's risk rating (hard public information) obtained from the national credit bureau (Dunn and Bradstreet). Internal Risk Rating is the bank's proprietary credit score based on hard private information about the firm. Standard errors are reported in parentheses. *** - significant at the 1% level. ** - significant at the 5% level. * - significant at the 10% level.

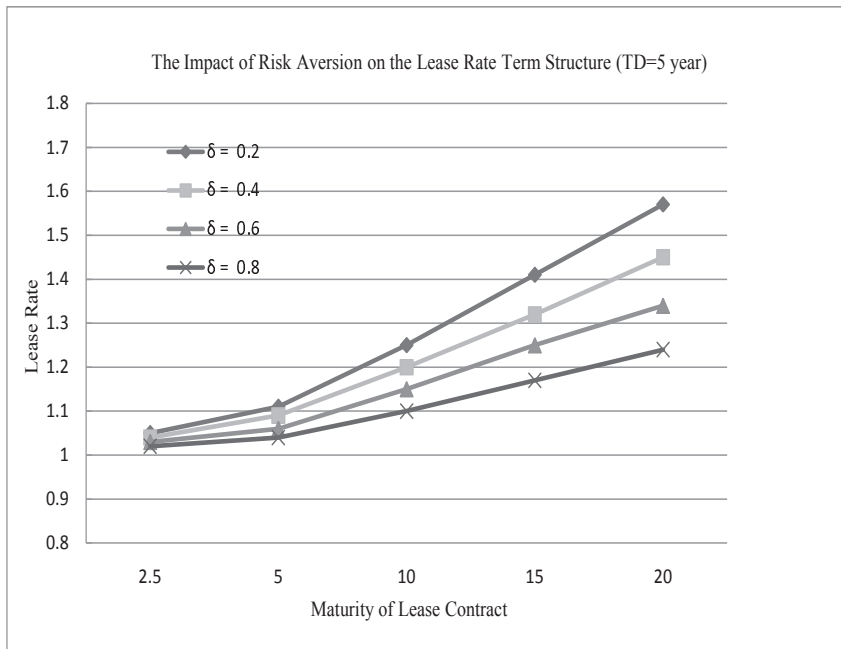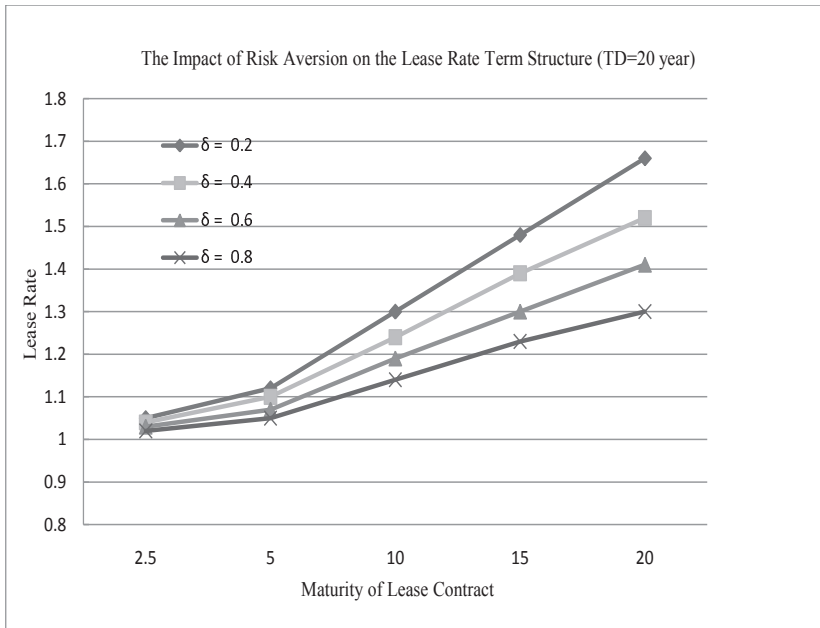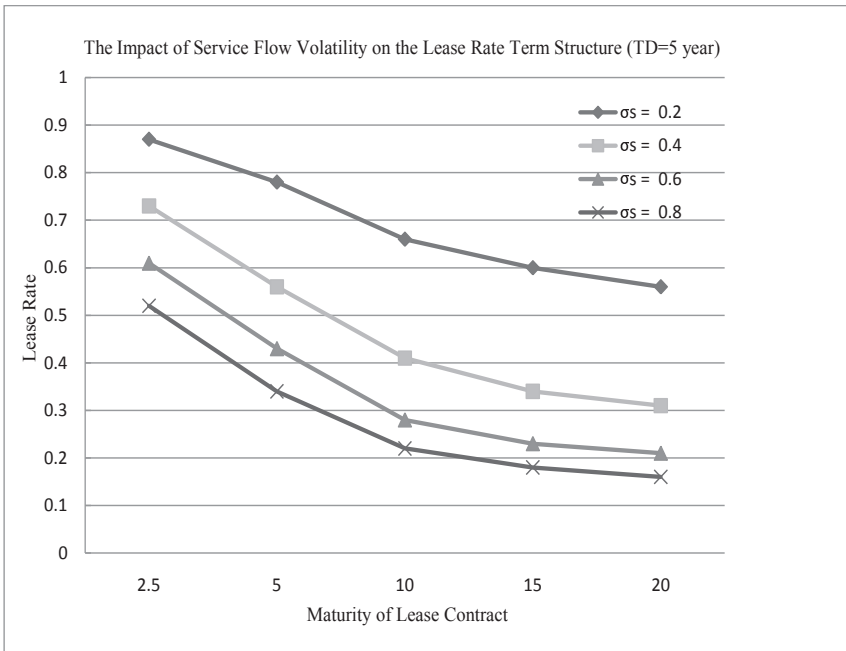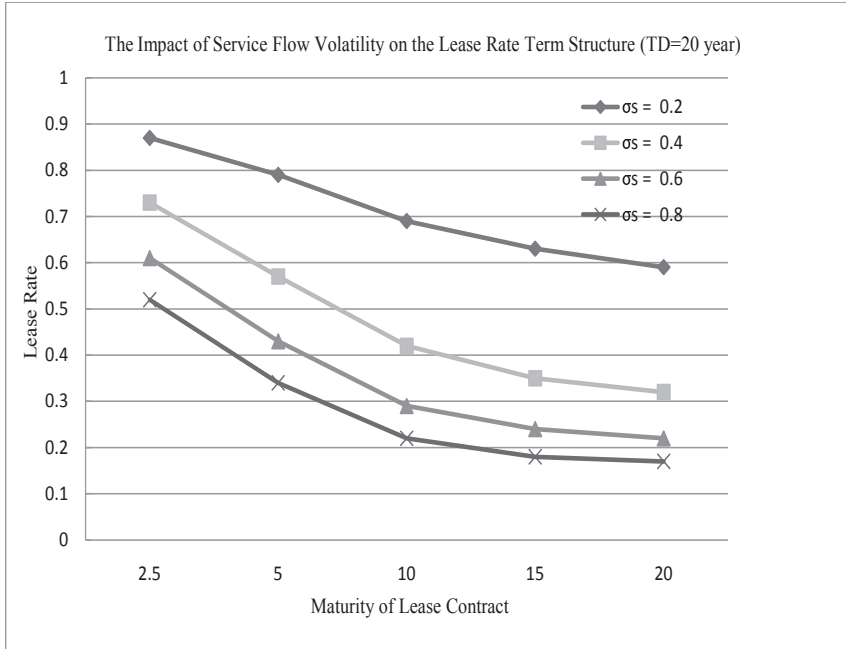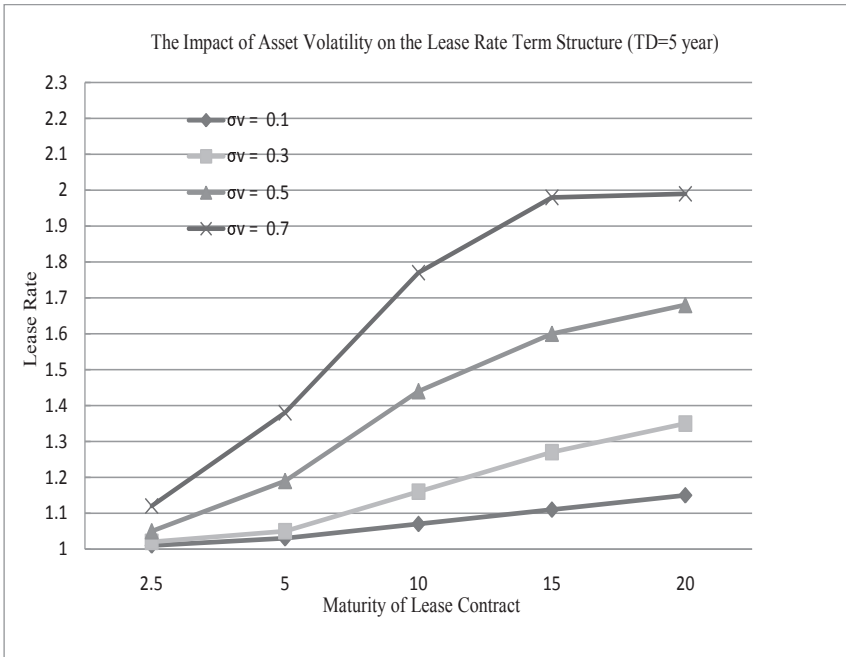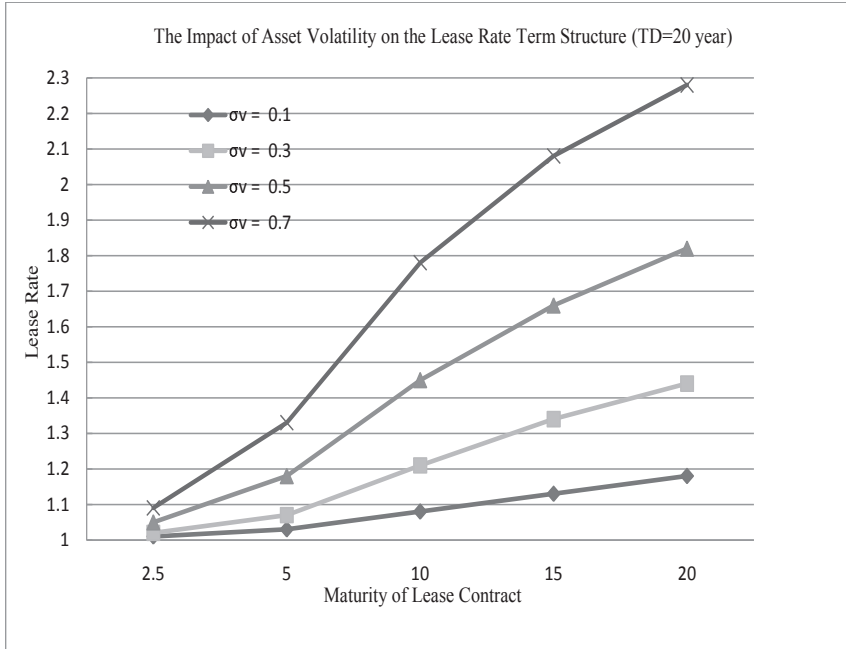Figure 1: **The impact of risk aversion.**

Figure 2: **The impact of service flow volatility.**

Figure 3: **The impact of asset volatility.**