

Do Preferences for Charitable Giving Help Auctioneers?

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Abstract

Preferences for charitable giving in auctions can be modeled by assuming that bidders receive additional utility proportional to the revenue raised by an auctioneer. The theory of bidding in the presence of such preferences results in a very counterintuitive prediction which is that in many cases, bidders having preferences for charitable giving does not lead to a substantial revenue advantage for an auctioneer. We test this theory and this prediction with a series of experiments. In one experiment we induce charitable preferences exactly as specified in the model to see if bidders respond to them as predicted. We find that they do. We then conduct a second experiment in which the revenue from the auctions is donated to actual charities to verify the robustness of the prediction when charitable preferences are generated by a more natural source and find again that the theoretical prediction holds: even strong charitable preferences do not result in substantial revenue increases to the auctioneer.

JEL Codes: D44, D64

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1 Introduction

Standard investigations into auction behavior and design presume that individual bidders gain utility only through their own purchasing of items. There are, however, a variety of reasons why an individual may receive utility from other sources in the allocation process. Such preferences could be derived from indirect and/or non-pecuniary sources such as the bidder possessing charitable preferences towards the auctioneer or through a variety of pecuniary sources such as the bidder being subsidized by the auctioneer (or a third party). In designing auctions for situations like this it is important to understand how such preferences could impact bidding behavior.

Situations involving bidders who may gain utility from an auctioneer receiving higher revenue have been the subject of several recent field investigations of charity auctions leading to mixed results. Elfenbein and McManus (2007) take data from e-Bay's Giving Works charity auctions and compare the prices with similar items auctioned through e-Bay's standard non-charity auctions. Their finding is that there is perhaps a 6% revenue premium for

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the charity auctions. A study of field charity silent auctions in Isaac and Schnier (2005b) finds that bidders in charity auctions rarely bid more than the stated value of an item such as a \$25 gift certificate. While this study did not have a non-charity comparison group, the overall bidding and revenue patterns did not suggest any strong impact on revenue or bidding due to charitable intent. These results might seem counterintuitive as one would expect that auctions for charitable purposes would yield substantially different behavior (and revenue) than non-charity auctions. Popkowski and Rothkopf (2006) presents the results of a field experiment with different findings supporting that intuition. In that study actual goods were auctioned off in an online auction market run by one of the authors with duplicate items being sold in charity and non-charity auctions. They found a substantial premium for the charity auctions and in fact they found such a large premium that they argue that partial charity auctions in which some percentage of the revenue is donated to a charity actually increase the revenue high enough to be profitable to an auctioneer even after the charitable contribution is made. Due to the strikingly different outcomes from these studies there is a need to look deeper into the phenomenon and try to understand how the presence or absence of charitable preferences changes auction behavior and outcomes.

The first step in gaining a better understanding of these results is to examine the relevant theory. These issues were first addressed in Engelbrecht-Wiggans (1994) in which it was proposed that bidders could gain utility in direct proportion to the auction revenue whether they won or lost the auction. This base model was extended in Salmon and Isaac (2006) to allow for an individual bidder to view winning and losing differently as well as for there to exist heterogeneity across bidders.¹ These extensions allow for a more detailed characterization of behavior in a broader class of settings which is useful by itself but can also be exploited in empirical testing to examine the marginal effects of the different motivations. While it was shown in these investigations that the existence of charitable preferences does increase revenue there is an empirical question of “by how much?” While that is not a question that is typically easy to address in a general sense in theoretical work, Salmon and Isaac (2006) presents computational examples which make it clear that if bidders possess what one might consider “standard charity” preferences of just gaining utility from the auctioneer receiving more revenue, then very strong preferences for charitable giving are required to observe any noticeable change in the revenue. The reason for this counterintuitive result is due to the conflicting incentives a bidder sees from the situations in which he wins or loses an auction. While the amount the bidder is willing to pay if he wins increases due to the charitable preferences, the existence of the charitable preferences actually makes losing the auction more attractive and therefore decreases his willingness to pay because the bidder still receives utility from seeing the auctioneer raise money. Thus there are counterbalancing incentives that theoretically net out leading to this surprising result of little change in bidding.

To test this theory and set of predictions we will present two sets of experiments. In the first set we will induce the charitable preferences of the bidders in such a way as to precisely match the model specification to see if they respond as predicted. We present cases in which bidders have symmetric preferences between winning and losing as well as asymmetric preferences in order to determine if they understand the partial effect of

¹In separate work, Engers and McManus (2007) replicate many of the results from Salmon and Isaac (2006) dealing with individuals receiving different utility from winning and losing but this study does not extend to heterogeneous bidders.

each parameter as well as how they balance out overall. The second set of experiments examines bidder behavior in hybrid lab-field charity auctions in which the price preferences are “homegrown” rather than induced to determine if the predictions of the model still hold when the charitable preferences are as they are in the field. One can also look at these experiments as trying to determine the nature of the underlying charitable preferences possessed by the subjects. The intent of both sets of experiments is to uncover a deeper understanding of behavior in auctions in the presence of price proportional preferences and to provide an explanation for some of the perhaps puzzling field results found regarding charity auctions.

There have been prior experimental investigations of some forms of price preference auctions with most focusing on using auctions to raise money for charity. Several recent papers examined the comparative ability of winner-pay auctions versus all-pay auctions and lotteries to raise money in charity auctions (Goeree, Maasland, Onderstal, and Turner (2005) and Davis, Razzolini, Reilly, and Wilson (2003)). While these recent papers focus on all-pay auctions we will not consider them here. The primary reason for that is that our interest is in trying to provide an explanation for the previously described field studies showing relatively little revenue increase in what are essentially second price auctions. As a robustness check we also included first price auctions to help identify the general pattern of how behavior may change between charity and non-charity auctions but we did not see the benefit of also examining all-pay auctions for this purpose. Many charity auctions take the form of multiple unit silent-auctions and there is a series of papers investigating these issues, Isaac and Schnier (2005b) and Isaac and Schnier (2005a). There is also a separate literature looking at the behavior of subsidized bidders, such as Shachat and Swarthout (2002) and Corns and Schotter (1999). While our study relates to these literatures, our motivation is different. These prior papers focus mostly on comparisons across mechanisms while we are more focused on examining the fundamental manner in which bidders respond to price proportional preferences or rather examining behavior across preference regimes. Also, our second set of experiments involving homegrown price proportional preferences provides a useful middle ground between the prior lab and field studies that is intended to serve as a bridge between lab and field results.

Perhaps the closest prior study is Carpenter, Holmes, and Matthews (2008) in which evidence from a field experiment is presented to test theoretical predictions regarding the revenue generation of first price, ascending and all-pay auctions. They find that, contrary to theory, their first price auction raised more money but this result was potentially due to differences in participation rates at their various auction events.² It is also important to note that the aspects of the theory tested in Carpenter, Holmes, and Matthews (2008) are quite different from those that we wish to test. In Carpenter, Holmes, and Matthews (2008) their goal was to examine revenue differences across mechanisms. Our goal is to determine how bidders behave differently between charity and non-charity contexts and in general to determine how different forms of price proportional preferences alter bidding. Both sets of tests are obviously useful and necessary for understanding this phenomenon. Towards this end we have designed a set of experiments which combine the benefits of both field and lab designs to allow us to engage in a more detailed examination of individual behavior.

To address the issue regarding the manner in which bidders respond to given price

²An alternative examination of the effects of endogenous entry on auction revenue can be found in Ivanova-Stenzel and Salmon (2008).

proportional preferences, we conducted sessions in which we induced price proportional preferences by paying subjects bonuses based upon winning and/or losing the auction in the exact manner the theory describes. To then examine the degree to which the model of preferences may match with natural preferences and thus explain behavior in field charity auctions, we conduct several sessions of the experiment in which proceeds from half of the auctions in the sessions are donated to actual charities in the names of the subjects. This latter design is in some sense a hybrid of a lab and a field experiment because we have the full laboratory control over the standard auction-based incentives of the subjects, but they are bidding based upon their homegrown preferences regarding the charities. Our question is the degree to which bidding behavior changes between auctions in which the revenue is donated to a charity and auctions in which the revenue essentially remains with the experimenter (as in the standard lab setting). We have conducted these sessions both with our standard subject pool in which we allow each bidder to choose (from a predetermined list) an individual charity that their auction payments will go to as well as a special self-selected subject pool in which the revenue from all charity auctions was donated to a charity to which these subjects possessed a very strong connection. It is important to point out that this latter subject pool was recruited to the lab with the promise that they would be engaged in a task that would result in their earning money for the relevant charity and thus these subjects self-selected into our experiment for the same reason many participants in field charity auctions self-select into attending those events.

We find that the behavior in the induced value cases can be explained reasonably well by the model of Salmon and Isaac (2006), although its accuracy does vary among treatments. In particular we find in the basic charity case that revenues respond very sluggishly to increases in the charitable preferences as predicted by the theory. To verify the foundation for that response we conducted sessions in which subjects received the charitable bonus only when they won the auction or only when they lost and we find that subjects responded to these parameters as the theory predicts. Thus the relative lack of response in the basic charity case should not be attributed to the subjects being confused about how to respond to the experiment parameters. In the sessions where the auction revenue is donated to actual charities we also find little difference between the revenue in the rounds in which the revenue is donated to charity and the rounds in which it is not.

Section 2 will provide specialization of the model from Salmon and Isaac (2006) to the current context and identify three main classes of preferences used in treatment design. Section 3 will present the design of the experiments. Section 4 will present the results and section 5 will conclude.

2 Overview of Theory

We will first explain the general version of the problem and then provide specialized bid functions for each of the parametric treatments used in the experiments. Let n be the number of risk neutral bidders in the auction. Each will draw an independent and privately known value for winning the auction, v_i , from a commonly known distribution $F(v)$ with pdf $f(v)$ which will be assumed for convenience to have support over the range $[0, \bar{V}]$ for the general case while we will use the uniform distribution on the range $[0, 100]$ in the experiments. Each bidder will possess two parameters β_i and α_i which describe the utility the bidder gains for each additional unit of revenue the auctioneer raises when the bidder

wins and loses the auction, respectively. We will assume $\alpha_i, \beta_i \in [0, 1]$. This restriction on parameters only implies that the utility effects are non-negative and that the value of a dollar to the auctioneer can not exceed the value of an extra dollar to the bidder.

If we assume for the first price auction that there exists some symmetric bid function $b_f^*(v, \beta_i, \alpha_i)$ that is monotonically increasing and differentiable in v and assume that $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$ for all i and j , then we can show what such a bid function must look like and show that it exists. The fact that we are working with symmetric bid functions allows us to make use of the fact that the probability of winning if i bids as if his value were r is $\Pr(b_f^*(r, \beta_i, \alpha_i) > b_f^*(v_j, \beta_i, \alpha_i), \forall j) = \Pr(r > v_j, \forall j) = F(r)^{n-1}$. This means that the first price auction problem is defined as:

$$\max_r S(v_i, r) = (v_i - (1 - \beta_i)b_f^*(r, \beta_i, \alpha_i))F(r)^{n-1} + \alpha_i \int_r^{\bar{V}} b_f^*(t, \beta_i, \alpha_i)(n-1)f(t)F(t)^{n-2} dt \quad (1)$$

With the equilibrium condition as

$$\frac{\partial(S(v_i, r))}{\partial r} \Big|_{r=v_i} = 0 \quad (2)$$

The first term represents the utility that the bidder receives when he wins the auction multiplied by the probability of that event. The second term is his utility when he loses the auction which requires integrating over all possible prices that the actual winner of the auction might pay multiplied by the probability of each price occurring. The general solution can be shown to be:

$$b_f^*(v_i, \beta_i, \alpha_i) = \frac{\int_0^{v_i} t(n-1)f(t)F(t)^{\frac{(n+\beta_i(2-n)+\alpha_i(n-1)-2)}{1-\beta_i}} dt}{(1-\beta_i)(F(v_i)^{n-1})^{\frac{1-\beta_i+\alpha_i}{1-\beta_i}}} \quad (3)$$

In the case of the uniform distribution on the range $[0, \bar{V}]$, the bid function becomes

$$b_f^*(v_i, \beta_i, \alpha_i) = \frac{n-1}{n(1-\beta_i+\alpha_i)-\alpha_i} v_i \quad (4)$$

Similarly, the second price problem is

$$\max_r S(v_i, r) = \int_0^r (v_i - (1 - \beta_i)b_s^*(t, \beta_i, \alpha_i))(n-1)f(t)F(t)^{n-2} dt + \alpha_i b_s^*(r, \beta_i, \alpha_i)(n-1)F(r)^{n-2}(1-F(r)) + \alpha_i \left(\int_r^{\bar{V}} b_s^*(t, \beta_i, \alpha_i)(n-2)(n-1)F(t)^{n-3}(1-F(t))f(t) dt \right) \quad (5)$$

The first term represents the utility that bidder i gets in the event that he wins the auction and we must integrate over the possible prices he would pay which are the bids an opponent would be expected to make as defined by $b_s^*(t, \beta_i, \alpha_i)$ multiplied by the probability of t being

the second highest value. The second term defines the utility i would receive from placing the second highest bid as he would set the price and would thus get $\alpha_i * b_s^*(r, \beta_i, \alpha_i)$ times the probability his bid is second highest. The final term represents the utility from coming in less than second as he will get α_i times whatever the second highest bidder bids. Solving this results in a general solution of

$$b_s^*(v_i, \beta_i, \alpha_i) = \begin{cases} \frac{1}{\alpha_i} \frac{\int_{v_i}^{\bar{V}} t(1-F(t))^{\frac{(1-\beta_i)}{\alpha_i}} f(t) dt}{(1-F(v_i))^{\frac{1+\alpha_i-\beta_i}{\alpha_i}}} & \text{if } \alpha_i > 0 \\ \frac{v_i}{1-\beta_i} & \text{if } \alpha_i = 0 \end{cases} \quad (6)$$

Again, if we simplify this to the case of the uniform distribution on the range $[0, \bar{V}]$, the bid function becomes

$$b_s^*(v_i, \beta_i, \alpha_i) = \frac{v_i(1 - \beta_i + \alpha_i) + \bar{V}\alpha_i}{(1 - \beta_i + 2\alpha_i)(1 - \beta_i + \alpha_i)} \quad (7)$$

Our experiments will examine three different classes of parameterizations of this model corresponding to three types of preferences. We will refer to the first as the Basic Charity (BC) case which will involve $\alpha_i = \beta_i = \alpha_j = \beta_j$ for all i and j . For this treatment, we can use these two benchmark equilibrium predictions with no modifications. In this treatment bidders derive utility from revenue raised by auctioneer regardless of who obtained the item. Our other two experimental treatments involve cases in which parameter values are heterogeneous across subjects as well as between winning and losing. That complicates the analysis and so we must deal with each of these cases separately.

2.1 See and Be Seen

Our experimental auctions will all have four bidders competing for a single item. This See and Be Seen (SBS) specification involves two of our four bidders possessing values of $\beta_i = .5$ and two having values of $\beta_i = 0$ with all possessing $\alpha = 0$. In general, solving for a closed form solution to this problem when bidders are heterogeneous is difficult, but these difficulties disappear in the case of the uniform distribution. The general approach can be found in Salmon and Isaac (2006) in which it is shown that the solutions shown above for uniformly distributed values are easily adapted into equilibrium bid functions for these cases with heterogeneous bidders.³

The first step towards generating the bid functions in this case involves determining how bidders with a value of $\beta_i > 0$ and $\alpha_i = 0$ would bid in the case of homogeneous preferences. This is found by simplifying equation 4 by setting $\alpha = 0$, which yields $b_f^*(v_i, \beta_i, 0) = \frac{n-1}{n} \frac{v_i}{1-\beta_i}$. Generating the bid functions for the asymmetric case from this is quite straightforward. First, by the general and well-known properties of first price auctions under the assumption of uniformly distributed values, it is the case that for a bidder with $\beta_i = .5$ or $\beta_i = 0$, their best response to the bids of others so long as they are bidding some constant fraction of their value is the same regardless of what that fraction is except for some issues with upper bounds. Consequently solving for the bid functions in the asymmetric case leads to portions

³An important note is that for general value distributions, the techniques for dealing with heterogeneity contained in Salmon and Isaac (2006) lead to approximations of the equilibrium bid functions. As discussed and demonstrated in that paper, under the assumption of uniformly distributed values the techniques yield the actual equilibrium bid functions. So the bid functions used in this paper are not approximations.

of the bid functions for bidders of both types which have simple linear portions that can be obtained by just substituting the relevant values of β_i into the symmetric bid function yielding $b_f^*(v_i, .5, 0) = 1.5v_i$ and $b_f^*(v_i, 0, 0) = 0.75v_i$ for each type. These functions hold over the entire value range except that there is a correction needed to resolve an upper bound issue for the two high β bidders. These bidders know that the two low β bidders will never bid above $b_f^*(100, 0, 0) = 75$. Consequently, for any bids the two high β bidders place above 75, they know that they are only competing against each other. In this region then, their bids will be made assuming only 2 bidders which yields $b_f^*(v_i, .5, 0) = \max(75, \frac{2-1}{2(1-.5)}v_i)$. Since there is no correction necessary for the low β bidders we can see that both full bid functions are as shown in equations 8 and 9.

$$b_f^*(v_i, .5, 0) = \begin{cases} v_i & \text{if } v_i > 75 \\ 75 & \text{if } 50 < v_i \leq 75 \\ 1.5v_i & \text{if } v_i \leq 50 \end{cases} \quad (8)$$

$$b_f^*(v_i, 0, 0) = 0.75v_i \quad (9)$$

Another way of explaining these bid functions is to begin with the realization that the presence of the β can be thought of as just re-scaling a bidder's value. In the presence of the β , when a bidder wins, his utility is $v_i + \beta_i b_i$. One can think of this as just converting his value into $\frac{v_i}{1-\beta_i}$ as this represents his maximum willingness to pay for the item. Notice that using this as the value of the bidder, the solution for the bid function in the presence of the β should look quite familiar to the standard one without nontraditional preferences which is $\frac{(n-1)}{n}v_i$. The solution with the positive value of β just replaces the v_i in that equation with $\frac{v_i}{1-\beta_i}$. The adaptation of this bid function for asymmetric values of β is analogous to the case of the introduction of heterogeneous risk aversion parameters as done in Cox, Smith, and Walker (1988). In both there is the linear portion of the bid function and then for bidders who would bid higher than others, there is a correction necessary at high levels of bidding.

Using this notion that $\frac{v_i}{1-\beta_i}$ is a re-scaled value for the bidder makes explaining the derivation of the bidding strategy in the second price auction even easier to explain. In the case of the second price auction, the proof for the equilibrium bid functions is obvious upon the realization that each bidder will be willing to pay up to $\frac{v_i}{1-\beta_i}$ and this is the same result one gets from simplifying equation 7. This value can just be substituted into the standard proof and the result follows immediately. For example, with $\beta_i = .5$ then the bidder should be willing to bid up to $2v_i$ since the effect of the β is equivalent to the bidder receiving a refund equal to half of the auction payment. So the bidder can profitably win at any price below $2v_i$. Consequently it should be clear that it is a dominant strategy for bidders to bid according to equations 10 and 11 and there need be no corrections to deal with the asymmetry because the bidders wish to follow these strategies regardless of what others do.

$$b_s^*(v_i, .5, 0) = 2v_i \quad (10)$$

$$b_s^*(v_i, 0, 0) = v_i \quad (11)$$

2.2 Raising Rivals' Cost

This case corresponds to a set of parameter values where two of the four bidders possess $\alpha_i = .5$ while the other two possess $\alpha_i = 0$ and all possess $\beta_i = 0$. Again it can be shown that

the equilibrium bid functions in the case of the bidder asymmetry can be derived by taking the general functions above, plugging in the parameters and then dealing with some bound issues for the same reasons as in the SBS case. Taking the general solutions to the first price results above leads to $b_f^*(v_i, 0, .5) = \frac{3}{5.5}v_i$ and $b_f^*(v_i, 0, 0) = \frac{3}{4}v_i$. Here the issue with an upper bound is for the low α bidders who know that the two high α bidders will never bid above $b_f^*(100, 0, .5) = 54.55$. Thus the two low α bidders know that for prices above this level they are only competing against one other potential bidder. This results in a similar function as above in which the bidders will bid according to $b_f^*(v_i, 0, 0) = \max(54.55, \frac{1}{2}v_i)$. Since $v_i \in [0, 100]$, the only operative bound is the first one. Thus the bid functions are

$$b_f^*(v_i, 0, .5) = .55v_i \quad (12)$$

$$b_f^*(v_i, 0, 0) = \begin{cases} 54.55 & \text{if } v_i > 72.73 \\ \frac{3}{4}v_i & \text{if } v_i \leq 72.73 \end{cases} \quad (13)$$

For the second price, we have $b_s^*(v_i, 0, .5) = \frac{1}{2}v_i + \frac{50}{3}$ and $b_s^*(v_i, 0, 0) = v_i$. Technically there can be a lower bound for the bids of the bidders with $\alpha = 0$ of $50/3$. The reason is that those with $\alpha = 0$ know that the lowest bid made by the bidders with $\alpha = .5$ will be $50/3$. Consequently the $\alpha = 0$ bidders are indifferent between $50/3$ and anything less than it. There seems little reason to implement the lower bound though as these bidders gain no utility from increasing the price and thus we have the following bid functions

$$b_s^*(v_i, 0, .5) = \frac{1}{2}v_i + \frac{50}{3} \quad (14)$$

$$b_s^*(v_i, 0, 0) = v_i \quad (15)$$

3 Experiment Design

There were two sets of experiments conducted for this study; one with induced and one with non-induced price proportional preferences. In both sets we have subjects participate in 30 auctions of four bidders using either sealed bid first price or sealed bid second price rules. Bidder groupings were kept fixed throughout a session which leads to each four person group being an independent observation. Subjects were told only that they would be participating in auctions with four bidders and were not told specifically whether they would be facing the same or different opponents each round. Bidder values were denominated in ECUs for convenience and drawn from a uniform distribution over the range $[0, 100]$. Subjects were fully informed about their own preference parameters as well as the parameters of the other bidders against which they were bidding.

3.1 Induced Price Proportional Preferences

For our experiments with induced price proportional preferences, our goal was to determine the marginal as well as combined effects of the bonus parameters α and β . In general the effect of β is to increase bids while α decreases them. So we wanted to investigate cases in which subjects possessed only α s or only β s as well as cases in which subjects possessed positive values of both. Because many situations involving price proportional preferences involve heterogeneity across individuals we wanted also to examine what impact that had

	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
Basic Charity	(0,0)	(.5,.5)	(.15,.15)	(0,0)	(.15,.15)	(.5,.5)
See and Be Seen	(0,0)	(.5,0)(0,0)	(0,0)(.5,0)	(0,0)	(0,0)(.5,0)	(.5,0)(0,0)
Raising Rivals' Cost	(0,0)	(0,.5)(0,0)	(0,0)(0,.5)	(0,0)	(0,0)(0,.5)	(0,.5)(0,0)

Table 1: Summary of parameters used in treatments across 5 period blocks of auctions. Parameters are listed as (β, α) and the two sets indicate the fact that in those blocks half of the subjects in an auction had one set and the other half the other.

on behavior as well so two of our treatments will include parameter heterogeneity across individuals. This led to the design of our three treatments; Basic Charity, See and Be Seen and Raising Rivals Cost.

The Basic Charity (BC) treatment involved all subjects having the same price proportional preferences as α and β were equal for each subject. This treatment is designed to mimic preferences commonly proposed as the baseline for preferences in charity auctions. In this setup bidders receive a utility bonus equal to $\beta * p$ if they win and $\alpha * p$ if they lose. We induced these preferences by paying the subjects these bonuses as monetary values so that their preferences exactly matched with the theoretical model. Over the course of a session we varied the value of these parameters between 0, .15 and .5. The reason is that we wanted to track how behavior changed between having no bonus, a relatively small bonus or a very large bonus. Subjects were given these parameters and they were held constant over five period blocks with the sequence of those blocks being detailed in table 1.

The second treatment called See and Be Seen (SBS) involves only some of the subjects possessing $\beta > 0$ while all others possess $\beta = 0$ and everyone possesses $\alpha = 0$. This case can be used to model a particular form of charitable preferences in which a bidder only receives the utility bonus in the event that they win (e.g., so they only wish to be seen as being generous rather than actually gaining utility from the auctioneer raising money) or it can be used to model the case of subsidized bidders. In terms of the bidding behavior these two motivations are mathematically identical, though they obviously would be implemented differently in practice. In order to magnify the impact of this subsidy we set $\beta = .5$ for two subjects in the bonus rounds and $\beta = 0$ for the other two subjects. The subjects alternated bonus configurations every five periods such that all bidders would receive the bonuses over the course of the experiment with the specific pattern shown in table 1.

The final treatment called Raising Rivals' Cost (RRC) is the complement to the SBS treatment as it involves the same procedures except having $\alpha > 0$ for some subjects and $\beta = 0$ for all. The exact pattern is found in table 1. This treatment is perhaps more difficult to interpret. Our main interest in conducting it is to allow for a complete exploration of the parameter space to understand how each one drives behavior. It is possible to motivate someone having a value of $\alpha > 0$ and $\beta = 0$ if they have a desire to make a rival pay a high price.⁴ One could also imagine this as a (perhaps unwise) way to motivate a shill bidder.

For the BC treatment we do not induce heterogeneity because for that treatment our interest is more in the effect of the overall existence of charitable preferences on bidding and revenue as well as how behavior and outcomes adjust as these preferences vary in strength.

⁴Note that this is not the same motivation as spite explored in Morgan, Steiglitz, and Reis (2002). The difference is subtle and explained in Salmon and Isaac (2006).

	First Price	Second Price
Basic Charity	2 (7)	2 (8)
See and Be Seen	3 (8)	2 (8)
Raising Rivals' Cost	2 (8)	2 (7)

Table 2: Summary of sessions per treatment with the number of independent blocks of bidders in ().

We include the heterogeneity among bidders in the other treatments because our interest is in how bidders respond to these parameters on their own as well as in how those without those parameters respond to others possessing them. The block configuration is intended to allow us to see bidders going through multiple configurations to examine the within subject comparative statics while also breaking up some of the ordering effects. Having each block last five periods seems long enough to allow them some stability with a preference configuration without causing the entire experiment to extend indefinitely. Finally, the specific parameter values we use were chosen because we want to maximize our chances of seeing strong effects. Consequently we chose values for the parameters which we view to be rather large. There are certainly many other configurations of values one can choose but these should be sufficient for mapping out the general manner in which subjects respond to them and determining how well the behavior is captured by the underlying model.

One possible concern about our design is that in the SBS and RRC treatments in moving from the charity to non-charity rounds we might be accused of introducing two elements rather than just one in that we introduce the existence of charitable preferences and we introduce asymmetry among bidders. If behavior differs between the charity and non-charity rounds one might find it difficult to determine if those differences were due to the fact of asymmetry or to the induced charitable preferences. If we did not have the structural model of bidding behavior which gives us explicit predictions about the response to this change that would be a greater concern. Also since we have both treatments which predict different directions of responses to the parameters this also allows us to test if the response we observe is due simply to the existence of heterogeneity versus subjects responding to the parameters in the manner predicted by the model. Further, by having the asymmetry across individuals we can generate an even stronger test of the underlying theory because we can test the differential response from both types of bidders.

In total 184 subjects participated in this set of experiments. A summary of the number of sessions and independent groups of bidders per treatment is in table 2. The exchange rates used to convert earnings into dollars were 1 ECU=\$0.02 for the BC and RRC treatments. For the SBS treatment an adjustment was necessary to generate overall earnings at approximately the same level and so we used the exchange rate of 1 ECU = \$0.06.⁵ Since our interest is mainly on within subject comparisons of behavior, the differential exchange rates used between treatments should cause us no problems. The average earnings of the subjects was approximately \$22. The experiments were programmed using z-Tree (Fischbacher (2007)).

⁵One session of the SBS treatment used a 1 ECU=\$0.09 exchange rate. This session only had two groups in it and no differences were found in behavior attributable to the exchange rate change.

3.2 Non-Induced Price Proportional Preferences

The set of experiments with non-induced price proportional preferences was conducted to determine whether the predictions of the theory still hold with proceeds going to natural charities. We thus investigate the presence of such incentives and their possible effect on bidding behavior in charity auctions with natural non-pecuniary preferences. Towards that end we conducted three sessions in which the proceeds from certain rounds were donated to a charity while in other rounds they were not. For two of these sessions, we used subjects recruited from our standard subject pool which would make them exactly comparable to the subjects used in the induced price proportional preferences sessions. The third session (as described in detail below) was conducted with a specialized subject pool whose members we had reason to believe possessed a very strong connection to the charity to which the proceeds of the auctions were to go. For the two sessions with the standard subject pool, one was first price and the other was second price. The session with the specialized subject pool involved first price auctions only.

For the sessions with the standard subject pool, the subjects were allowed to choose between four options regarding the charitable component of the experiment after they had been given instructions about the experiment. Three of these options were different charities and the fourth was “no charity.” During the charity rounds of the experiment the price paid by the winner would be paid to the charity selected at this point by the winner. The charities were local chapters of the Red Cross, Habitat for Humanity and the Humane Society.⁶ In order to maximize the possibility that subjects knew and cared about the various charities, while going through the experiment instructions we read the mission statement of each charity as reported by the charity review website CharityNavigator.org. We also gave the subjects printouts of a few pages of each organization’s website that provided their standard information regarding what donations are used for.⁷ We allowed subjects to pick “no charity” in event that they disliked all three charities and did not wish any of the three to receive donations on their behalf. The existence of the “no charity” option also allows us to make a revealed preference argument that a subject choosing a charity will receive a non-negative welfare bonus from that charity receiving donations. The subjects were told in the beginning that at the end of the session they would fill out a contribution form and see the check with their contribution filled out to be sent to the charity in their name.

For the session with the specialized subject pool, the subjects were recruited from the FSU Wesley Foundation which is a student group affiliated with a local church. In this session, the revenue generated in a charity round (regardless of who won) was contributed to New Life Children’s Home which is an orphanage in Guatemala. When subjects were recruited, they were told that by attending the experiment they would be able to generate money for this charity. Thus their decision to participate in the experiment was made on very similar considerations to those who would have attended a charity auction held by the church. The New Life Children’s Home is an organization specifically supported by this student group and 8 of the 20 subjects in this session had visited the orphanage on a mission trip. 8 more of the subjects had either been on similar mission trips or had plans

⁶To be specific, the Capital Area Red Cross, Big Bend Habitat for Humanity and The Jacksonville Humane Society. We chose the Jacksonville Humane Society over the local Tallahassee one because the Jacksonville facility had experienced a recent fire ruining their facilities. This was a well publicized event which we thought might intensify charitable preferences towards the organization.

⁷Full copies of materials with instructions are available from the authors upon request.

to do so. Consequently there is reason to believe that these subjects have strong charitable preferences towards the orphanage of the sort that could generate strong price proportional preferences as modeled in the theory.

In all three of these sessions we had 20 subjects leading to 5 independent groups of four bidders each per session. There were 30 rounds total with rounds 1-5 and 11-20 being non-charity rounds and 6-10 and 21-30 being charity rounds with proceeds going to charity. The auction parameters were exactly the same across all 30 rounds with the only difference being that in the charity rounds the price they paid for winning went to a charity instead of effectively back to the experimenters as in the non-charity rounds. To increase the ability to make a valid inference on the treatment effect of charity vs. non-charity rounds, each subject used one set of 15 randomly drawn values across both treatments. This was done by drawing values for the 15 non-charity rounds and then randomly resorting those values to reappear for the subjects in a different order during the charity rounds. The new order was the same for all subjects meaning that the subjects competed against the same group of people all having the same values once in the charity rounds and once again in the non-charity rounds.

We had 60 subjects participate in this set of sessions. Earnings were converted into dollars at the rate of 1 ECU= \$0.05. Average total earnings to the subjects was around \$19 with subjects generating on average an additional \$11.50 contribution to the relevant charity.⁸ Of the 40 subjects from the standard pool across both sessions, 9 chose to contribute to Habitat for Humanity generating \$126, 10 chose the Red Cross generating \$103 and 14 chose the Humane Society generating \$175. The remaining 7 subjects chose the “No Charity” option. The 20 subjects in the specialized pool generated a total of \$240 in contributions for the New Life Children’s Home. These sessions were also conducted using z-Tree (Fischbacher (2007)).

4 Results

4.1 Induced Price Proportional Preferences Sessions

We will first present the results from the experiments with induced price proportional preferences and then those from the non-induced sessions. For the induced sessions the general questions of importance involve what effect the parameters have on aggregate measures of revenue and efficiency as well as how they impact individual bidding behavior. We will address both in succession.

4.1.1 Aggregate Results

Summary statistics of the impacts of the bonus regimes on the two institutions can be found in tables 3 and 4. These tables contain the average revenue over the last 3 blocks of periods. They contain both the actual average revenue as well as the revenue predicted according to the theoretical models described above. While a visual inspection of these numbers allows one to get a general understanding of the treatment effects, tables 5 and 6 provide the statistical analysis of the revenue differences. We note that while we included only the data

⁸In the standard subject pool sessions, individual average earnings were \$19.60 with \$10 contribution (average was \$12 per subject if we drop out the 0’s from those who chose “No Charity”) while in the specialized subject pool the average earnings were \$16.90 with a \$14.30 contribution.

	Actual			Predicted		
	No Bonus	Low	High	No Bonus	Low	High
BC	69.60	69.77	69.31	59.84	62.37	65.30

	No Bonus	B 5	B 6	Avg	No Bonus	B 5	B 6	Avg
	SBS	73.95	88.92	77.10	83.01	61.80	76.94	72.63
RRC	69.30	53.80	57.50	55.65	60.67	47.58	49.60	48.59

Table 3: Average actual and theoretically predicted revenue in first price auctions across all treatments.

	Actual				Predicted			
	No Bonus	Low	High		No Bonus	Low	High	
BC	70.02	74.99	73.38		61.20	64.69	70.35	

	No Bonus	B 5	B 6	Avg	No Bonus	B5	B6	Avg
	SBS	71.26	85.28	80.27	82.92	63.92	87.75	83.87
RRC	66.67	57.89	52.42	55.15	64.30	52.17	49.85	51.01

Table 4: Average actual and theoretically predicted revenue in second price auctions across all treatments.

from the last three blocks in tables 3 and 4, this was mostly to conserve space and allow the reader to focus on the periods after any initial learning. All of the regression analysis which form the basis of the results of the paper incorporates all periods of the data.

Table 5 contains the results of random effects panel regressions of auction revenue on dummy variables for the bonus regime, auction format and auction format interacted with bonus regime as well as the highest and second highest values held by bidders in that round. These results are intended to give us some indication of how bonus regimes and the auction institution impact revenue generation. Table 6 contains the results of fixed effects panel regressions of auction revenue on the theoretical prediction of revenue interacted with bonus regime to help understand how accurately the theoretical model predicts the revenue. The random and fixed effects are specified at the group level since that is the unit of observation for revenue.

Examining table 5 by treatment reveals a number of interesting results. In the BC treatment we see that the Low Bonus regime leads to no statistically significant effect on revenue while the High Bonus regime does lead to a small increase that is borderline significant. Revenue in the second price auction is not significantly different from the first price auction. For the second price case, both (High and Low) bonus regimes have a positive and significant impact on revenue. These effects must be determined by adding the base coefficients to the interacted coefficients to get the overall effect and testing whether or not the total is significantly different from 0. The Low Bonus case yields a p -value <0.001 and the High Bonus case also yields a p -value <0.001 . This is quite a counterintuitive result because the preferences being modeled here even in the Low treatment represent what should be considered extreme preferences for generating money for the charity and

	BC		SBS		RRC	
	Coeff	p-Value	Coeff	p-Value	Coeff	p-Value
Constant	8.93	0.08	-1.37	0.88	12.53	<0.01
LowBonus	2.79	0.29	-	-	-	-
HighBonus	5.11	0.06	-	-	-	-
Bonus	-	-	14.98	<0.01	-10.72	<0.01
MaxValue	0.39	<0.01	0.56	<0.01	0.37	<0.01
SecondPrice	1.71	0.71	-.63	0.94	-3.55	0.33
SP_Low	3.80	0.30	-	-	-	-
SP_High	2.62	0.47	-	-	-	-
SP_Bonus	-	-	-1.07	0.84	3.62	0.29
SecondValue	0.45	<0.01	0.45	<0.01	0.42	<0.01
R² Overall	0.52		0.33		0.56	

Table 5: Random effects panel regressions of auction price to determine differences between bonus regimes and formats.

yet the revenue increases are either nonexistent or modest.

The SBS and RRC cases yield exactly opposite effects from each other due to the introduction of the different forms of bonuses. In the SBS case the effect of introducing the bonus yields a strongly positive and significant effect while the RRC case leads to a strongly negative and significant effect. This effect holds for both auction formats. In neither treatment do we see a significant difference in revenue between formats.

We can examine the results in table 6 to determine how accurate is the theoretical model in predicting revenue. The regressions contained in the table are fixed effect panel regressions of actual revenue on theoretically predicted revenue with dummy variable interactions of the revenue prediction with the bonus regime. If the theory is completely accurate we should see a coefficient of 1 on TH Rev and 0 elsewhere. If there exist significant coefficients on the bonus regime terms then there is an indication that the theory performs differently during that regime than during the no bonus periods. Careful inspection reveals that the theory is quite accurate in some cases but less accurate in others. Both of the BC cases yield coefficients on theoretical revenue that can not be rejected as being equal to 1 with the only significant interaction term being for the high bonus phase in the first price auction treatment. There are still deviations from the theory coming from the positive and significant constants, but overall the deviations from predictions are not more severe during the bonus than the non-bonus phases. In the SBS case we see a coefficient on predicted revenue greater than 1 in the first price case and less than 1 in the second price case. In the first price SBS, there is a drop in the coefficient on theoretical predicted revenue back towards 1 but there is no statistically significant shift in the second price auction treatment. In the RRC treatment, we see a coefficient on predicted revenue above 1 for the first price treatment but no significant interaction during the bonus phase and then for the second price auction there are so significant deviations from the theoretical predictions.

It is important to note that the inaccuracies in the model are generally over the entire set of data and not just in the bonus periods since in all but a few cases the coefficients on the bonus periods are not different from 0. It is well established in prior literature

		BC		SBS		RRC	
		Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value
First Price	Constant	8.57	0.03	-0.15	0.98	-9.06	0.09
	TH Rev*	1.02	0.79	1.19	0.03	1.28	<0.01
	TH_High	-0.08	0.01	-	-	-	-
	TH_Low	-0.04	0.21	-	-	-	-
	TH_Bonus	-	-	-0.08	0.02	0.05	0.20
<i>R</i>² Overall		0.66		0.69		0.65	
Second Price	Constant	14.68	<0.01	37.56	<0.01	-0.56	0.90
	TH Rev*	0.91	0.22	0.55	<0.01	1.04	0.56
	TH_High	-0.06	0.17	-	-	-	-
	TH_Low	0.01	0.81	-	-	-	-
	TH_Bonus	-	-	-0.03	0.68	0.05	0.21
<i>R</i>² Overall		0.50		0.15		0.58	

* *p*-values indicate test of significance from a coefficient of 1.

Table 6: Random effects panel regressions of auction price to determine deviations from theory.

First Price				Second Price				
	No Bonus	Low	High	No Bonus	Low	High		
BC	0.99	0.96	0.99	0.97	0.90	0.91		
	No Bonus	B 5	B 6	Avg	No Bonus	B5	B6	Avg
SBS	0.99	0.92	0.94	0.93	0.91	0.91	0.88	0.90
RRC	0.98	0.92	0.92	0.92	0.96	0.89	0.92	0.91

Table 7: Average efficiency from auctions with induced price proportional preferences.

dating back to Cox, Roberson, and Smith (1982) that standard risk neutral models do not predict behavior perfectly accurately in first price and second price auctions so the overall inaccuracies here are no surprise. The fact that our theory performs no worse in the bonus periods suggests that on average the model is accurately predicting the comparative static response in how revenue changes as the bonus changes even though overall predictions of precise levels may be biased up or down. This implies that overall the model is doing a reasonable job of capturing how revenue may differ due to the introduction of one of these bonus regimes.

Table 7 shows the average efficiencies for each treatment and phase. We provide these summary statistics to help the reader understand the underlying data but will only comment on the nature of these results without providing full statistical analysis to conserve space.⁹ One point of general interest is that in almost all cases there are no statistically significant differences in the efficiency of first price versus second price auctions even in the bonus periods. We do see that in the bonus periods in the SBS and RRC treatments are less

⁹Full statistical analysis is available from authors upon request.

	First Price		Second Price	
	Coeff	p-Value	Coeff	p-Value
Constant	-0.79	0.65	4.21	0.19
Value	0.85	<0.01	1.036	<0.01
Low Bonus	1.45	0.54	16.83	<0.01
High Bonus	0.17	0.94	14.02	<0.01
Low Value	-0.03	0.49	-0.19	0.01
High Value	0.02	0.564	0.11	0.15
R^2 (overall)	0.85		0.53	

Table 8: Regressions of bid functions in the Basic Charity treatment. Regressions are done with individual level fixed effects.

efficient than the non-bonus periods but this is expected. Theoretically these rounds should not generate 100% efficiency while all of the rounds without bonuses or with symmetric bonuses, as in the BC treatment, should generate 100% efficiency.

4.1.2 Individual Bidding Behavior

To get a finer understanding of how subjects respond to the bonuses we next examine the nature of the bid functions exhibited by the subjects. For each treatment we will present scatterplots of the bid functions to present a visual representation of the behavior as well as panel regressions with individual level fixed effects to generate a statistical characterization of the results.

Basic Charity Looking back at equation 4, we can see the theoretical prediction for the first price auction in the BC case is $b^*(v_i, 0, 0) = .75v_i$, $b^*(v_i, .15, .15) = 0.78v_i$, and $b^*(v_i, .5, .5) = 0.86v$. This suggests there should be little difference between the no bonus and low bonus regime but there should be a detectable difference in the high bonus regime. Figure 1 shows scatterplots of how the bid functions change as the bonus parameters change and visually it is difficult to detect any difference across regimes. Table 8 contains the results of panel regressions with individual fixed effects of these bid functions. The regression regresses bid on a constant, the bidder’s value and then dummy variables for each bonus regime interacted with the bidder’s value. If the theory is accurate then the constant and the bonus regime dummies should all be 0. The coefficient on value should be .75 and the interacted values should have values of .03 for the Low Bonus and 0.11 for the High Bonus. All of the constants are not significantly different from 0 but neither are the value interactions. The coefficient on value is higher than predicted, as is normally found in first price auctions, but there is no detectable increase in aggregate bidding behavior as the bonus increases.

For the second price auction, equation 7 presents the theoretical prediction. We should observe $b_s^*(v_i, 0, 0) = v_i$, $b_s^*(v_i, .15, .15) = 0.87v_i + 13.04$, and $b_s^*(v_i, .15, .15) = 0.67v_i + 33.33$. So in this case, we should see still see a constant of 0 but the bonus dummies should be positive, 13 and 33 respectively, and then the coefficient on value should be 1 with values of -0.13 and $-.33$ for the low and high bonus regimes. Figure 2 shows the graphical characterization of the bidding behavior and indeed as the bonus rises above 0 we do observe

First Price BC

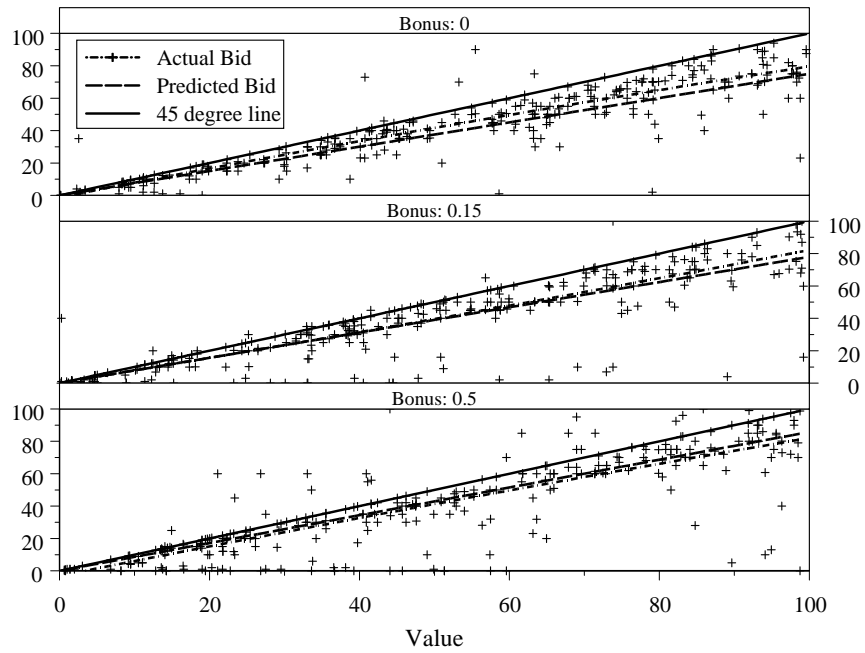


Figure 1: Scatterplot of bids versus values in the first price auction, Basic Charity treatment.

many more low value bidders bidding above their value as predicted by the theory, but there does not appear to be a high enough shift up in the High Bonus regime. This is confirmed in the regression results again shown in table 8. We do indeed get a constant not significantly different from 0 and a coefficient on value not significantly different from 1, p -value=0.62. The Low Bonus Dummy is not significantly different from 13, p -value=0.38, and the interaction with value is not significantly different from -0.13, p -value=0.43. In the High Bonus regime both the dummy and the value interaction terms are significantly different than the predicted values, both p -values<0.01.

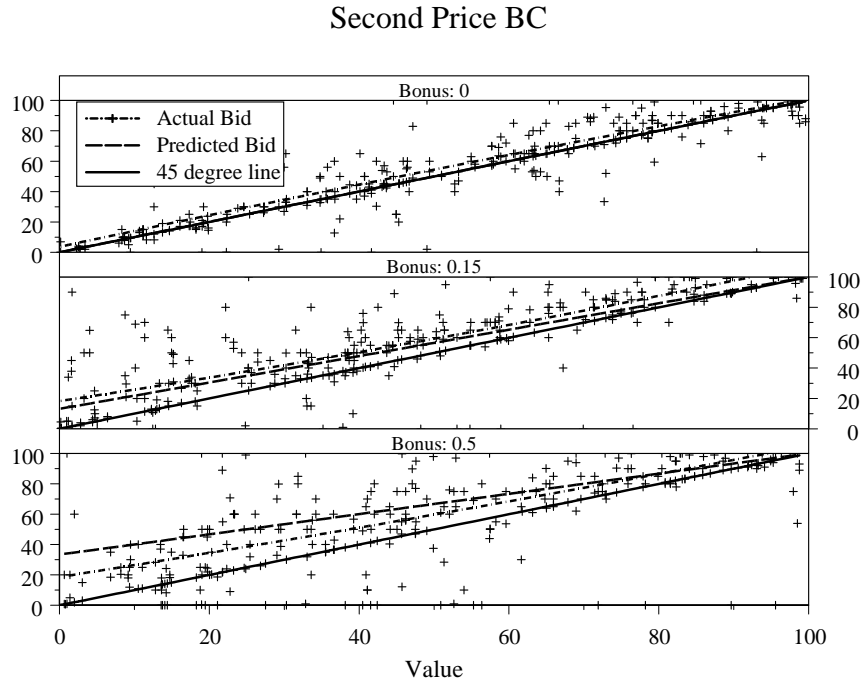


Figure 2: Scatterplot of bids versus values in the second price auction, Basic Charity treatment.

See and Be Seen Of course one possible argument for why the effects of the bonuses were so modest in the BC treatment is that perhaps subjects were confused by them and did not understand the incentives that each of the different bonuses created. Since the theoretical prediction is also of a modest change it is not possible from just the BC data to determine if the modest change is due to subjects responding to the incentives as theory predicted or to the subjects just ignoring the bonuses. Part of the reason for examining these other two cases is to determine if subjects understand the individual bonuses themselves and respond as predicted when there are not contradictory incentives and so there is a strong predicted effect.

To examine how subjects respond to a utility bonus in the event that they win the auction, we examine the results from the SBS treatment. The bid function for the first price auction under the SBS parameterization is more complex in the bonus regime as seen

in equation 8. In the non-bonus regime the bid function is simply $b_f^*(v_i, 0, 0) = 0.75v_i$. Due to the complications in the bid function we must use multiple variables for value when the bonus is in effect corresponding to the ranges of below 50, between 50 and 75 and then above 75 with accompanying intercept shift terms. We have also have added a dummy variable with a value interaction, $\text{Against}\beta$, to indicate when a bidder has a $\beta = 0$ but is facing bidders with $\beta = .5$. This is to determine how unsubsidized bidders may react to facing subsidized bidders. The theoretical prediction is that the constant will be 0, coefficient on value 0.75, Bonus_Value should be .75 (or $\text{Value}+\text{Bonus_Value}=1.5$), MidVal_Value should be equal to $-.75$ (or $-\text{Value}$) and HighVal_Value should be equal to 0.25 or $(\text{HighVal_Value}+\text{Value}=1)$. The two variables regarding how bidders without bonuses respond to those that do should be 0.

The base intercept is insignificant and the coefficient on value is greater than .75 and is significantly different than .75, $p\text{-value}<0.01$, as we would normally expect. The combined coefficient $\text{Bonus_value}+\text{Value}=1.2$ which is significantly different than 1.5, $p\text{-value}<0.01$, while the combined $\text{MidVal_Value}+\text{Value}=.32$ but not significantly from 0, $p\text{-value}=0.19$, and $\text{Value}+\text{HighVal_Value}=.61$ but not significantly different from 1, $p\text{-value}=0.13$. The bidders without bonuses exhibit no response to those with bonuses which matches the theoretical prediction. On the whole bidders do respond as theory suggests (except with high values) though again the theory fails to precisely predict the levels of bidding. The theory underpredicts bidding in the standard rounds as it always does but bidders fail to respond by bidding as aggressively as they “should” when they receive the bonus.

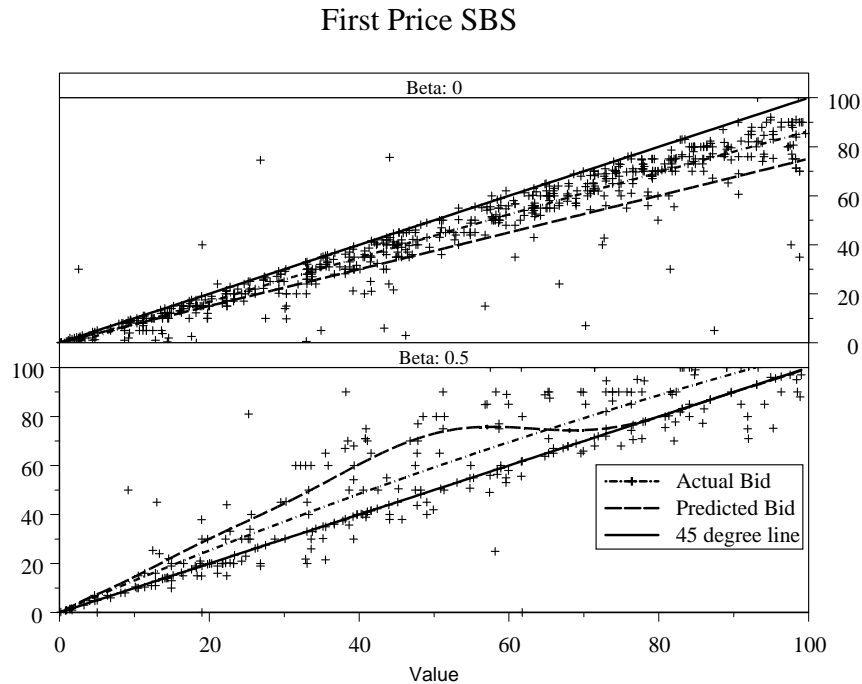


Figure 3: Scatterplot of bids versus values in the first price auction, SBS treatment.

	First Price		Second Price	
	Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value
Constant	-.03	0.99	10.47	0.03
Value	0.88	<0.01	0.96	<0.01
Bonus	-1.70	0.57	6.16	0.07
Bonus_value	0.40	<0.01	0.12	0.04
MidValue	32.93	0.03	-	-
MidVal_Value	-0.56	0.03	-	-
HighValue	12.18	0.57	-	-
HighVal_Value	-0.27	0.29	-	-
Againstβ	-3.09	0.21	-6.54	0.31
Againstβ_Value	0.04	1.14	0.14	0.20
R^2 (overall)	0.88		0.48	

Table 9: Regressions of bid functions in the SBS treatment. Regressions are done with individual level fixed effects.

The second price bid functions as shown in equations 10 and 11 are quite simple. When not receiving a bonus bidders should bid their value and when receiving a bonus they should bid twice their value. Figure 4 shows that bidders do bid approximately their value when receiving no bonuses (though with some overbidding) and when receiving bonuses many bid higher but most appear reluctant to go all the way to $2v_i$. The regression results in table 9 again verify the visual result is valid. The coefficient on value is not significantly different than 1, p -value=0.62, but Value+Bonus_Value=1.08 is far below 2 and significantly different, p -value<0.01. The bidders do become slightly more aggressive due to the positive and significant intercept shift, but overall they are not nearly as aggressive as theory predicts. They do, however, fail to respond to seeing other bidders with subsidies which is as predicted by the theory.

Raising Rivals' Cost To determine how subjects respond when they receive a utility bonus for losing but not winning, we examine the RRC treatment. For the first price auctions the bid function for those receiving the bonus is straightforward, $b_f^*(v_i, 0, .5) = 0.55v_i$, but the bid function for those not receiving a bonus when facing those with a bonus has a slope change for high values as seen in equation 13. Overall the prediction is that bidders should become less aggressive when they receive the bonus when losing the auction as this simply makes losing more attractive. Figure 5 shows that indeed the bidders learned to bid substantially less aggressively when receiving the bonus though there was a large amount of variation in the degree to which bidders learned to become less aggressive. As shown in table 10, once again bidders in the standard rounds bid higher than predicted on average and the coefficient is statistically significantly different from .75, p -value=0.01. For bidders with the bonus, while the predicted value of the combined coefficient Value+Bonus_Val=.55, we found it to be .045 which is significantly different from 0.55, p -value<0.01. Bidders with no bonus were not supposed to respond to the values of others for low value and the lack of significance of the Against α terms confirms that they did not. There was supposed to be a response for high values but neither effect

Second Price SBS

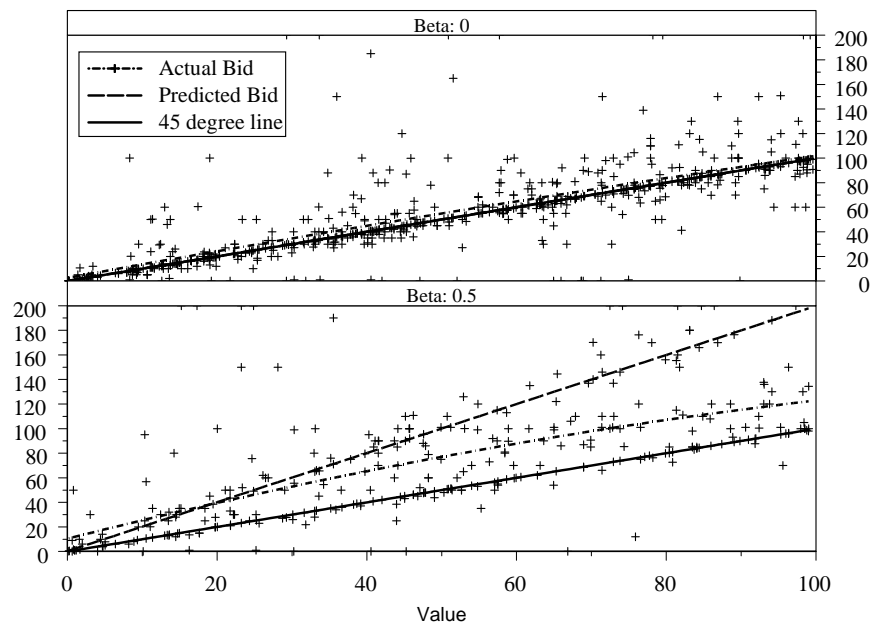


Figure 4: Scatterplot of bids versus values in the second price auction, SBS treatment.

was significant.

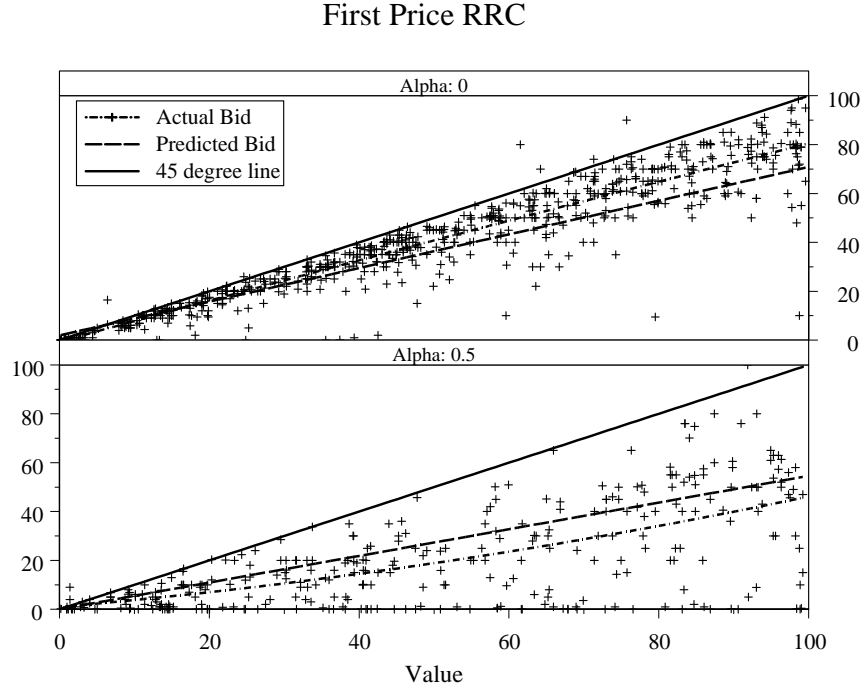


Figure 5: Scatterplot of bids versus values in the first price auction, RRC treatment.

In the case of the second price auction, bidders with no bonus had a dominant strategy to bid their value, as usual, but bidders with a bonus had the bid function $b_s^*(v_i, 0, .5) = 0.5v_i + 16.667$ indicating that they should be more aggressive for low values but less aggressive at high values. Figure 6 shows that some bidders displayed such a shift while others still appeared to bid as if they had no bonus. The base intercept is not significant and the coefficient on value is not significantly different from 1, p -value=0.54. The bidders with a bonus should possess an intercept of 16.67 but $\text{Constant} + \text{Bonus} = 6.61 - 0.22 = 6.39$ which is significantly different, p -value<0.01. The combined coefficient $\text{Value} + \text{Bonus_Val}$ should be 0.5 and it is 0.66 which is significantly different from .5, p -value<0.01.

4.2 Non-Induced Price Proportional Preferences Sessions

These sessions were conducted to study bidding for actual charities when (unlike in the field) we observe bidders' monetary valuations for the items. The design allows us to detect possible deviations in bidding when the bonus values were not induced by the experimenter, but rather "homegrown" or induced by the subjects' natural inclinations towards the charities. In this treatment, the revenue from the auctions is donated to an actual charity in half of the periods. In one sub-treatment, subjects were drawn from our standard subject pool of undergraduate students and chose from a list of three different charities (along with a "no charity" option) where the revenue from any auction they won would be paid as described in Section 3. In another sub-treatment, the subjects were drawn from a specialized subject

	First Price		Second Price	
	Coeff	p -Value	Coeff	p -Value
Constant	1.69	0.33	-0.22	0.94
Value	0.82	<0.01	1.03	<0.01
Bonus	-3.67	0.11	6.61	0.09
Bonus_Val	-0.37	<0.01	-0.37	<0.01
HighVal	18.85	0.38	-	-
HighVal_Value	-0.25	0.33	-	-
Againstα	-2.55	0.32	0.72	0.86
Againstα_Value	-0.02	0.72	-0.01	0.92
R^2 (overall)	0.81		0.67	

Table 10: Regressions of bid functions in the RRC treatment. Regressions are done with individual level fixed effects.

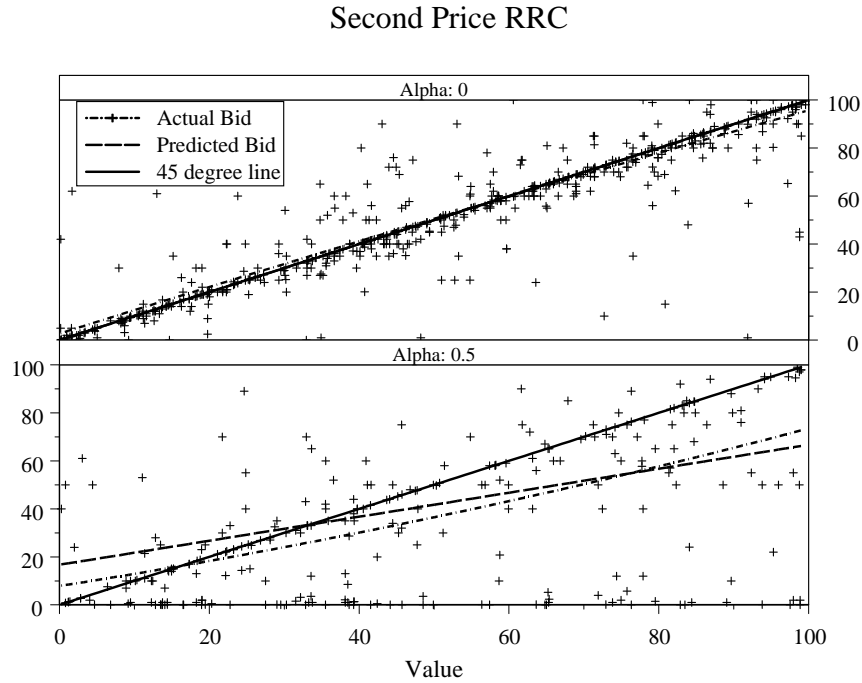


Figure 6: Scatterplot of bids versus values in the second price auction, RRC treatment.

	General Pool				Specialized Pool	
	Second Price		First Price		First Price	
	Non-Ch	Ch	Non-Ch	Ch	Non-Ch	Ch
Group 1	63.13	62.59	60.64	62.29	63.30	68.20
Group 2	61.45	61.92	57.67	58.20	75.45	80.81
Group 3	59.96	59.13	62.64	71.44	75.62	75.10
Group 4	61.42	64.54	77.20	77.02	70.37	73.79
Group 5	51.68	51.94	68.72	69.52	67.46	83.10
Overall	59.53	60.02	65.37	67.70	70.44	76.20

Table 11: Average revenue for each independent group of charity and non-charity auction rounds.

pool and there was a single charity to which all subjects had a very strong connection. One might expect that in this treatment (in all three sessions), during the charity rounds charitable preferences of some sort were operative although since these preferences were not induced we do not know exactly what they were. It is reasonable to form a hypothesis though that the charitable preferences of those in the specialized subject pool would be strong on an absolute basis and certainly stronger in relative terms than those held by the subjects from the standard pool.

4.2.1 Aggregate results.

These sessions were constructed to allow direct comparison between charity and standard rounds by having bidders interact with the same value draws in both sets of rounds against bidders also possessing the same value draws. Thus there were 15 values randomly drawn for each bidder and the bidder used each of those values twice; once in a standard auction and once in a charity auction with the cross bidder value pairings being maintained as well. This removes the possibility that any differences in revenue could be due to differences in value draws. Table 11 shows the average revenue of the charity and non-charity auctions for each independent four bidder group. In most cases, there is only a very small difference between the revenue raised in the two regimes. In group 5 of the specialized pool there is a fairly substantial difference which appears to have been from one subject demonstrating very clear and strong charitable preferences as well as sophisticated bidding in order to maximize the contribution to the charity. Looking across all 10 independent pairs of charity and non-charity revenue averages in the two first price sessions, the average revenue in the charity phase is larger than the non-charity phase. Most of these differences are small though which, as we will show, leads to an overall lack of a statistically significant effect when taking the magnitudes of those differences into accounts. Overall, there is a weakly positive effect, but not a strong one.

A statistical characterization of the differences in revenue can be found in table 12. This table contains the results from a fixed effects panel regression of revenue on either the highest or second highest value depending on the format and then on whether or not the round is a charity round with the two variables interacted. In all three cases both the Charity Round dummy variable and the interaction with the relevant value are not statistically significant. This is an indication that there is no significant difference in revenue whether

	General Subject Pool				Specialized Pool	
	Second Price		First Price		First Price	
	Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value
Constant	-0.66	0.819	6.10	0.033	6.67	0.170
Relevant Value	1.03	<0.001	0.76	<0.001	0.81	<0.001
Charity Round	-0.70	0.863	3.59	0.223	0.36	0.958
Charity_Value	0.02	0.754	-0.02	0.641	0.07	0.412
R^2 (overall)	0.83		0.73		0.73	

Table 12: Panel regressions with individual level fixed effects of revenue.

	General Pool				Specialized Pool	
	Second Price		First Price		First Price	
	Non-Ch	Ch	Non-Ch	Ch	Non-Ch	Ch
Group 1	0.93	0.91	0.96	0.98	1.00	0.99
Group 2	0.99	0.97	0.99	0.98	0.99	0.99
Group 3	0.94	0.92	0.99	0.94	0.94	0.98
Group 4	0.96	0.96	0.99	0.99	0.99	0.99
Group 5	0.97	1.00	1.00	1.00	1.00	0.93
Overall	0.96	0.95	0.99	0.98	0.98	0.98

Table 13: Average efficiency for each independent group of charity and non-charity auction rounds.

or not the revenue accrues to a charity or the experimenter. This is true even in the case of the specialized subject pool whose members have externally demonstrated very strong charitable preferences in regards to the charity in question. This result, while perhaps unexpected, comports with both our theoretical prediction and the results from the lab experiment with very strong induced charitable preferences.

To further examine these results we can first examine the efficiency achieved in the auctions. Table 13 shows the average efficiency achieved by each group in both regimes. By and large the auctions are quite efficient and again there seems to be little difference between the charity and non charity rounds. The most substantial difference is again in group 5 of the special subject pool which is from the one noted bidder bidding well above their value and outbidding higher valued bidders, one presumes, in order to insure large donations to the charity.

4.2.2 Individual Results

Table 14 shows the overall effect of the charity rounds on bidding behavior by providing fixed effects panel regression regarding the bid functions exhibited by subjects. In the specialized pool there is a small but statistically significant intercept shift in the bidding behavior while there are no overall significant impacts on the slopes of the bid function. The coefficient on the intercept shift for the charity rounds is small but positive in all 3 treatments and is significant at the 10 and 5% levels for the First Price General Pool and the First Price Specialized Pool sessions respectively. To gain perspective on the intercept shift we note

	General Subject Pool				Specialized Pool	
	Second Price		First Price		First Price	
	Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value	Coeff	<i>p</i> -Value
Constant	1.122	0.471	0.01	0.994	1.92	0.03
Relevant Value	0.997	<0.001	0.83	<0.001	0.86	<0.001
Charity Round	2.72	0.300	1.85	0.086	2.57	0.039
Charity_Value	-0.012	0.788	-0.002	0.914	0.03	0.127
R^2 (overall)	0.72		0.89		0.89	

Table 14: Panel regressions with individual level fixed effects of bid functions.

that values are on the range $[0, 100]$ so an upward shift of 2.57 as with the Specialized pool is about 3.2% of expected winning valuation and thus perhaps not economically significant which may help explain the noted lack of a statistically significant effect on revenue already demonstrated.

If we conduct regressions of the form shown in table 14 separately for each individual bidder then we can determine if there are individual bidders who respond to the charity rounds even if overall the effect is insignificant. In the general subject pool we find that in the second price auction only 1 out of 20 subjects exhibited such a reaction. In the first price auction case we find that 3 out of our 20 subjects exhibited positive and significant (at the 5% level or better) coefficients on the interaction term between the charity dummy and value. While there may be relatively few subjects exhibiting positive shift or rotation in bidding behavior, we also note that no subject in these two sessions exhibited a negative and significant dummy variable or interaction term. In the specialized subject pool in the first price auction, there were also 2 out of 20 subjects who exhibited positive and significant coefficients on the interaction term between the charity dummy¹⁰ and the value. There was a third subject who technically had a negative and significant coefficient on value interacted with charity round, but this individual had a very high positive intercept shift during the charity rounds that counteracted the overall effect leaving the net impact on bidding positive.

While one might be tempted to interpret these results to suggest that the majority of subjects in these experiments did not possess “charitable preferences” or more formally that they possessed α and β close to or equal to 0, there is substantial evidence to suggest that this is not the only nor even the most plausible conclusion. First, it seems particularly difficult to claim that the subjects in the specialized pool did not possess reasonably strong charitable preferences. Many of those individuals had previously donated substantial amounts of time and effort on the behalf of the charity and they were participating in the experiment because they were told that they could raise money for the charity. Further, it is important to remember that the theoretical prediction is that there should be only a negligible effect on revenue if these subjects possess preferences such that they want the charity to raise money and are unconcerned about who pays it. So even if the subjects from the standard or specialized pool had very strong preferences for the charity then we would have expected

¹⁰Note that in the two general subject pool regressions, the Charity Round dummy variable corrects for those bidders who chose no charity by using a 0 for those bidders in all rounds. In the Specialized pool this dummy variable really does just indicate whether the round is a charity round or not.

little effect on revenue if they bid as the theory suggests. The only way we would expect a strong positive effect on revenue is if the subjects possessed SBS preferences or preferences with a high β and relatively low α . Due to the lack of a strong upward or downward shift we conclude that the preferences of the subjects did not correspond with the SBS or RRC specifications but their behavior is perfectly consistent with BC preferences.

This result should be important to those interested in charity auctions. Preferences for charitable giving which are of the form that the individuals just like the charity to raise more money regardless of the source (i.e. $\alpha = \beta > 0$) are unlikely to have a significant effect on revenue even if the bidders have what appear to be a high commitment to the organization. The theory, our induced value experiments and our homegrown preference experiments are all consistent with the idea that it is only with See and Be Seen preferences that charity auctions are likely to outperform their counterparts with bidders who have no charitable preferences.

5 Conclusion

The genesis of this study was to address what appears to be an open question in the literature which is whether or not the existence of charitable preferences leads to more revenue in an auction. To address this issue we present a set of hybrid lab-field experiments to carefully test the behavioral predictions of a model of charitable bidding and to test whether or not this model can describe behavior when real charities are involved. The general finding from the induced preferences section is that the model¹¹ is approximately as accurate in predicting behavior and revenue in the induced preferences environment as is the risk neutral model in standard auction environments. Of the three classes of preferences we examined, only the See and Be Seen preference specification in which individuals only gain utility if they win the auction led to substantial gains in revenue over auctions without bidders receiving price proportional benefits from auctioneer revenue suggesting that auctioneers can benefit from charitable preferences but only if they satisfy this property of $\beta > \alpha$ and that the difference is relatively large. The Basic Charity case, in which individuals gain utility from the auctioneer receiving more revenue regardless of the source, ends up generating only moderate revenue increases even for very strong charitable preferences which is consistent with the theory. The sessions in which the revenue from certain rounds was actually donated to charities on behalf of the auction winner showed little difference in bidding and revenue even for our subject pool with very strong preferences for the charity involved. This suggests that the preferences of even our strongly motivated subject pool did not satisfy the property of $\beta > \alpha$ strongly enough to lead to a benefit to the auctioneer from any charitable preferences.

These findings from the experiment help in interpreting the field results used to motivate this investigation. Because we find significant differences in bidding behavior between charity and non-charity rounds for only 10-15% of subjects and no overall revenue difference we conclude that the behavior of our subjects with naturally occurring or homegrown preferences for real charities is consistent with the predictions of our Basic Charity specification of preferences. This conclusion is also consistent with the field results of Elfenbein and McManus (2007) and Isaac and Schnier (2005b) and therefore it seems reasonable to

¹¹Specialization of the model from Salmon and Isaac (2006) .

conclude that the model can provide a suitable explanation for the results in those studies. The results from Popkowski and Rothkopf (2006) which show a substantial difference in revenue between charity and non-charity auctions do not seem consistent with our results or our theory unless one assumes those bidders possessed See and Be Seen style preferences in which they receive a utility bonus only from winning or at least that the utility bonus from winning is substantially greater than that from losing. Since that study was conducted under very different circumstances though, there could certainly have been other institutional differences which could have led to different underlying preferences or to a different interaction of the preferences with the institution. For the former two studies though, we appear to have a satisfactory explanation as to why revenues and behavior do not appear to shift in those studies between charity and non-charity auctions.

The practical implications from these results for the conduct of auctions in which bidders may have price proportional benefits are complex. One key point is that the nature of the preferences themselves may interact with the exact manner in which the auctions are conducted. For example, bidders who possess preferences roughly in-line with our See and Be Seen specification may bid quite differently in auctions that are public and whose results have high public visibility than auctions in which winners are officially anonymous or just not publicized. This can be thought of as the institutional design actually altering the nature of the preference parameters or perhaps as different designs triggering different values of an individual's parameters. A relatively small change in the nature of how a charity auction is conducted in this regard might therefore generate a substantial increase in revenue. The other key insight for the conduct of charity auctions is that such auctioneers should not count heavily on the charitable preferences of their bidders delivering substantial extra revenue if those bidders possess preferences of the Basic Charity nature. This is perhaps the most surprising finding of the paper and one that charity auctioneers should certainly pay careful attention to. This result re-emphasizes a point made in Isaac and Schnier (2005b) which is that, in charity auctions, the main aspect of charitable donation is from those donating items to the organization to be auctioned rather than in the bids of the individuals showing up to bid on them. The indication from our results is that the two primary things a charity auctioneer should likely focus on is giving substantial publicity to donors of items to encourage their donation and also to giving substantial publicity to those winning items in the auction as this may lead to a greater manifestation of See and Be Seen preferences.

We also note that the sessions with the induced price proportional benefits, in particular the See and Be Seen treatment, constitute a direct test of how bidders respond to subsidies. There are many cases of auctioneers subsidizing bidders such as when the Federal Communications Commission awards what they refer to as bidding credits to small businesses, see Salmon (2004), that work in a mathematically identical nature to how we induced the price proportional preferences here. It is also worth noting that the FCC has awarded bidding credits as high as 45% which means that winning bidders with these bidding credits get a 45% discount off of any winning bid. This is of the same magnitude of the 50% subsidy we tested. The experimental results show that bidders respond more or less in-line with the predicted shift in bidding for first price auctions by bidding in a much more aggressive fashion. In the second price auctions, bidders failed to respond as aggressively as theory predicts. These results suggest that subsidies to benefit disadvantaged bidders may be more effective in first price rather than second price auctions. One should be careful with this

conclusion though because typically in cases involving subsidized bidders, those bidders are being subsidized because they are disadvantaged in the sense that they have tighter budget constraints or that their value was drawn from a stochastically dominated distribution. That is not the case in our experiments and it is possible that results could change if there were a correlation between a bidder receiving a bonus/subsidy and their value. Our results are still suggestive about this issue.

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APPENDIX A - DERIVATION

Bidding Strategy in Symmetric First Price Auctions

If we hypothesize the existence of the bid function $b_f^*(v_i, \beta, \alpha)$ that is used by the other $n - 1$ bidders each of whom possess the same parameters for β and α , we can find what its form must be by determining what value bidder i would choose to submit to such a bid function. Since in this case, the bid function will be symmetric, we can see that the probability of winning if i bids as if their value were r is $\Pr(b_f^*(r, \beta, \alpha) > b_f^*(v_j, \beta, \alpha) \text{ for all } j) = \Pr(r > v_j \text{ for all } j) = F(r)^{n-1}$. Letting $\rho(r) = F(r)^{n-1}$ define the probability of winning given that bidder i has bid according to value r , we have:

$$\max_r S(v_i, r) = (v_i - (1 - \beta)b_f^*(r, \beta, \alpha))\rho(r) + \alpha \int_r^{\bar{V}} b_f^*(t, \beta, \alpha)d\rho(t) \quad (16)$$

Equilibrium condition is

$$\frac{\partial(S(v_i, r))}{\partial r} \Big|_{r=v_i} = 0 \quad (17)$$

FOC:

noting that $\frac{\partial\left(\int_p^y f(x)dx\right)}{\partial p} = -f(p)$

$$v_i\rho'(r) - (1 - \beta)b_f^*(r, \beta, \alpha)\rho'(r) - (1 - \beta)\rho(r)b_f^{*\prime}(r, \beta, \alpha) - \alpha b_f^*(r, \beta, \alpha)\rho(r) = 0 \quad (18)$$

$$(1 - \beta + \alpha)b_f^*(r, \beta, \alpha)\rho'(r) + (1 - \beta)\rho(r)b_f^{*\prime}(r, \beta, \alpha) = v_i\rho'(r) \quad (19)$$

If we multiply both sides by $\rho(r)^{\frac{\alpha}{1-\beta}}$ we get:

$$(1 - \beta + \alpha)b_f^*(r, \beta, \alpha)\rho(r)^{\frac{\alpha}{1-\beta}}\rho'(r) + (1 - \beta)\rho(r)^{\frac{1-\beta+\alpha}{1-\beta}}b_f^{*\prime}(r, \beta, \alpha) = v_i\rho'(r)\rho(r)^{\frac{\alpha}{1-\beta}} \quad (20)$$

Which allows us to write the left side as $\frac{\partial\left((1-\beta)b_f^*(r, \beta, \alpha)\rho(r)^{\frac{1-\beta+\alpha}{1-\beta}}\right)}{\partial r}$ and also noting that in equilibrium $r = v_i$ we have

$$\frac{\partial\left((1 - \beta)b_f^*(r, \beta, \alpha)\rho(r)^{\frac{1-\beta+\alpha}{1-\beta}}\right)}{\partial r} = v_i\rho'(v_i)\rho(v_i)^{\frac{\alpha}{1-\beta}} \quad (21)$$

Since this condition must hold for all values, we can integrate both sides between 0 and v_i :

$$\int_0^{v_i} \frac{\partial\left((1 - \beta)b_f^*(t, \beta, \alpha)\rho(t)^{\frac{1-\beta+\alpha}{1-\beta}}\right)}{\partial t} dt = \int_0^{v_i} t\rho'(t)\rho(t)^{\frac{\alpha}{1-\beta}} dt \quad (22)$$

Using boundary condition that $b_f^*(0, \beta, \alpha) = 0$, we get

$$(1 - \beta)b_f^*(v_i, \beta, \alpha)\rho(v_i)^{\frac{1-\beta+\alpha}{1-\beta}} = \int_0^{v_i} t\rho'(t)\rho(t)^{\frac{\alpha}{1-\beta}} dt \quad (23)$$

$$b_f^*(v_i, \beta, \alpha) = \frac{\int_0^{v_i} t\rho'(t)\rho(t)^{\frac{\alpha}{1-\beta}} dt}{(1 - \beta)\rho(v_i)^{\frac{1-\beta+\alpha}{1-\beta}}} \quad (24)$$

since $\rho(x) = F(x)^{n-1}$ then $\rho'(x) = (n-1)f(x)F(x)^{n-2}$ so

$$b_f^*(v_i, \beta, \alpha) = \frac{\int_0^{v_i} t(n-1)f(t)F(t)^{\frac{(n+\beta(2-n)+\alpha(n-1)-2)}{1-\beta}} dt}{(1-\beta)(F(v_i)^{n-1})^{\frac{1-\beta+\alpha}{1-\beta}}} \quad (25)$$

This will be a valid equilibrium bid function so long as it is differentiable and monotonically increasing which can be shown by extension of Engelbrecht-Wiggans (1994) or as in Engers and McManus (2007).

Bid Function in Symmetric Second Price Auctions

For this case we must consider three possibilities. First is what I expect to get if I win, second is what I get if I come in second and third is what I expect if I come in less than second. We again assume that some monotonic and differentiable bid function $b_s^*(v)$ exists and we wish to check to see if i wants to bid as if they possess some value r instead of v_i .

$$S(v_i, r) = \int_0^r (v_i - (1-\beta)b_s^*(t, \beta, \alpha))dF(t)^{n-1} + \alpha b_s^*(r, \beta, \alpha)(n-1)F(r)^{n-2}(1-F(r)) + \alpha \left(\int_r^{\bar{V}} b_s^*(t, \beta, \alpha)(n-2)(n-1)F(t)^{n-3}(1-F(t))dF(t) \right) \quad (26)$$

Equilibrium condition is again that $\frac{\partial(S(v_i, r))}{\partial r} \Big|_{r=v_i} = 0$

Taking the derivative we get:

$$v \frac{dF(r)^{n-1}}{dr} + b_s^*(r)(\beta-1) \frac{dF(r)^{n-1}}{dr} - b_s^*(r, \beta, \alpha) \alpha (1-F(r)) F(r) (n-2) \frac{dF(r)^{n-1}}{dr} + b_s^*(r, \beta, \alpha) \alpha F^{n-2}(r) (n-1) (1-F(r)) - b_s^*(r) \alpha \frac{\partial F(r)^{n-1}}{\partial r} + b_s^*(r, \beta, \alpha) \alpha F(r) (1-F(r)) (n-2) \frac{\partial F(r)^{n-1}}{\partial r} = 0 \quad (27)$$

This will simplify to

$$vf(r) = b_s^*(r, \beta, \alpha)(1-\beta+\alpha)f(r) - b_s^*(r, \beta, \alpha)\alpha(1-F(r)) \quad (28)$$

multiply everything by $-\frac{(1-F(r))^{\frac{1-\beta}{\alpha}}}{\alpha_i}$ and get

$$-vf(r) \frac{(1-F(r))^{\frac{1-\beta}{\alpha}}}{\alpha} = -b_s^*(r, \beta, \alpha)(1-\beta+\alpha)f(r) \frac{(1-F(r))^{\frac{1-\beta}{\alpha}}}{\alpha} + b_s^*(r, \beta, \alpha)(1-F(r))^{1+\frac{1-\beta}{\alpha}} \quad (29)$$

Notice that $(1-F(\bar{V})) = 0$ if \bar{V} is the max of the distribution. Also, since this condition must hold for all v , we can integrate both sides:

$$\int_r^{\bar{V}} \frac{\partial(b_s^*(t, \beta, \alpha)(1-F(t))^{\frac{1+\alpha-\beta}{\alpha}})}{dt} dt = \int_r^{\bar{V}} \frac{t}{1+\alpha-\beta} \frac{\partial((1-F(t))^{\frac{1+\alpha-\beta}{\alpha}})}{\partial t} dt \quad (30)$$

In equilibrium $r = v$, and we also need an obvious correction for the case $\alpha_i = 0$.

$$b_s^*(v, \beta, \alpha) = \begin{cases} \frac{1}{\alpha} \frac{\int_v^{\bar{v}} t(1-F(t))^{\frac{(1-\beta)}{\alpha}} dF(t)}{(1-F(v))^{\frac{1+\alpha-\beta}{\alpha}}} & \text{if } \alpha > 0 \\ \frac{v}{1-\beta} & \text{if } \alpha = 0 \end{cases} \quad (31)$$