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Efficiency with Linear Prices? The Combinatorial Clock Auction and its Extensions.

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The Combinatorial Clock (CC) design has a number of virtues, which made it the mechanism of choice in many practical combinatorial auctions. Ask prices are monotonously ascending, the ask price calculation is trivial, and the computationally hard winner determination is limited to the final stages of the auction. Unfortunately, linear-price combinatorial auctions cannot be fully efficient with best-response bidders and general valuations, and it is unclear which strategy a bidder should follow in such auctions. We show that the worst-case efficiency in the CC auction can actually be as low as 50%, if bidders follow the best-response strategy. We identify demand-masking valuations, i.e., the characteristics of valuation functions which can cause such low efficiency of the CC auction. Having analyzed the case of best-response bidding, which is typically assumed in theory, we focus also on powerset bidding strategies, in which bidders bid on all bundles with positive payoff. We introduce the CC+ auction, an extension of the CC auction with a modified price update rule, and show that powerset bidding always leads to efficient outcomes. We also show that with an even stronger price update rule and a Vickrey payment rule, powerset bidding is an ex-post Nash equilibrium. In computational experiments, we show that the CC+ auction achieves high levels of efficiency even if the bidders are restricted in the number of bids that they can submit in each round.

Key words: combinatorial clock auction, allocative efficiency, best-response bidding

1. Introduction

The single-item ascending Clock auction has a number of desirable economic properties. When all bidders know their private valuations, truthfully revealing one's demand is a dominant strategy (Isaac et al. 2007). The single-item Clock auction is individually rational, efficient, strategy-proof, and the payoff vector is in the core. It would be desirable to achieve such properties for combinatorial auction designs as well. It is well known, that the Vickrey-Clarke-Groves (VCG) design is the unique mechanism that satisfies individual rationality, efficiency, and strategy-proofness (Ausubel and Milgrom 2006b). However, its results can be outside the core, which leads to a number of problems in practical settings (Rothkopf 2007b).

1.1. Iterative Combinatorial Auctions

Researchers have been trying to find generalizations of the Clock auction for selling multiple items. For situations with multiple items, but unit demand (Demange et al. 1986) and for multiple homogeneous goods with marginal decreasing value (Ausubel 2004), it has been shown that there are generalizations of the Clock auction, which can be used to implement efficient, strategy-proof mechanisms. Finding efficient auctions with strong incentive properties turned out to be much harder for combinatorial auctions with general valuations.

The Ascending Proxy Auction (Ausubel and Milgrom 2006a), iBundle(3) (Parkes and Ungar 2000), and the dVSV auction (de Vries et al. 2007) achieve full efficiency with (myopic) best-response bidders, i.e., bidders only bid on those bundles that maximize their payoff in each round. If the coalitional value function satisfies the buyer submodularity condition (see Section 4.1), best-response bidding is an ex-post Nash equilibrium and the auction results in the VCG outcome (Ausubel and Milgrom 2002). These auction formats are based on non-linear and personalized ask prices and can be interpreted as an algorithm to solve the corresponding linear program. We refer to these auction formats as non-linear price personalized auctions (NLPPAs) in the following.

If the bidders' valuations in an NLPPA are not buyer submodular, bidders have an incentive to shade their bids and not follow the best-response strategy. Buyer submodularity is mostly not given for realistic value models. Lab experiments have shown that independent of the type of value model, bidders deviate significantly from the pure best-response strategy (Scheffel et al. 2008). Both computational and lab experiments have also illustrated the large number of auction rounds necessary for these non-linear price combinatorial auctions (Schneider et al. 2008).

As an alternative, linear-price combinatorial auctions have been suggested resembling the fictitious Walrasian tâtonnement. One line of research is based on a restricted dual of the relaxed winner determination problem, in which the pseudo-dual variables are used as ask prices in the auction (Rassenti et al. 1982, Kwasnica et al. 2005, Bichler et al. 2009). In contrast, Porter et al. (2003) suggested a simple mechanism with ascending linear ask prices, called the Combinatorial Clock (CC) auction. In every round the bidders, given the current ask prices, identify a package, or several packages, which they offer to buy. If two or more bidders demand an item then its price is increased for the next round. The bids which correspond to the current ask prices are called *standing*, and a bidder is standing if he has at least one standing bid. In a trivial case when at some point supply equals demand the auction is terminated and the goods are allocated according to the standing bids. If there is an excess supply for at least one item the auctioneer must determine the winners to find an allocation of items that would maximize his revenue. If the solution displaces a standing bidder, than prices of the corresponding standing bids tick upward. The auction ends when no prices are increased. The mechanism has achieved high levels of efficiency in the lab (Porter et al. 2003) and has a number of obvious advantages. It maintains strictly ascending, linear ask prices, and limits the computational burden on the auctioneer as he only has to solve the winner determination in the last rounds. Also, the information revelation between rounds makes it quite robust against collusion and limits the bidder's possibilities for signaling. Unfortunately, little theoretical research has focused on the CC auction. As Kagel et al. (2009) pointed out, experiments with combinatorial auctions are valuable, but they are also restricted to a limited set of bidder valuations.

There have been numerous fundamental contributions on combinatorial auctions in the areas of Computer Science, Economics, Information Systems, and Operations Research (Cramton et al. 2006). The Information Systems (IS) literature has been rich in proposals for bidder decision support, new designs and applications, and the analysis of information feedback in combinatorial auctions (see Banker and Kauffman (2004) for an overview of IS research streams and Kelly and Steinberg (2000), Fan et al. (2003), Xia et al. (2004), Adomavicius and Gupta (2005) and Bichler et al. (2009) for recent IS research on combinatorial auctions). This paper is in line with previous work on bidder decision support and information feedback, and tries to lay foundations for the design of efficient linear-price combinatorial auctions and respective software platforms.

1.2. Contribution of this Paper

We first provide analytical results on the worst-case efficiency of the CC auction. As it is usual in the related game-theoretical literature, we assume best-response strategies since they are straightforward to follow and limit the amount of information that needs to be revealed in each round. We show that the efficiency of the CC auction can be as low as 50% for general valuations.

Low efficiency with best-response bidding in the CC auction is a reason, why models, which assume a best-response strategy might be poor predictors for CC auctions in the field. As an alternative, we also evaluate a powerset strategy. Powerset bidders bid on all possible bundles with positive payoff in each round. We show, however, that with such a strategy the efficiency of the CC auction can even be close to 0%.

Next, we suggest a variation of the CC auction, the CC+ auction, and show that the powerset strategy leads to full efficiency in this format. We show, however, that bidders have incentives for demand reduction in the CC+ auction. Therefore, we propose additional payment rules that eliminate or minimize the bidders' incentives to deviate from the powerset strategy. The auctioneer can either calculate Vickrey payments or bidder-Pareto-optimal payments (Day and Raghavan 2007) from the bids elicited throughout the CC+ auction. We show that with a strengthened price update rule, powerset bidding is an ex-post Nash equilibrium.

Similarly to the NLPPAs, the exponential communication complexity remains a stumbling block (Nisan and Segal 2001). While in NLPPAs this leads to a huge number of auction rounds, CC+

auctions require bidders to submit a large number of bids in each round. However, while we show that winners need to reveal more information in the CC+ auction than in NLPPAs, the number of bids submitted by bidders in CC+ is much lower. For example, a particular instance of an efficient CC+ auction described in Section 5 required 4,128 bids with powerset bidders, while iBundle(3) with best response bidders required 7,741 bids per bidder, and the Credit-Debit auction even 14,895 bids per bidder. Note that the number of auction rounds in the CC+ auction is almost as low as in standard CC auctions. Interestingly, we show in computational experiments that the efficiency of the CC+ auction is still very high, even if bidders are severely restricted to the best six or ten bids sorted by payoff in each round. This suggests that it is a practically viable alternative to other auction formats. In contrast to these NLPPAs, the CC+ auction maintains simple linear ask prices throughout, bidders have an auction with only a single phase, and strong incentives to follow a powerset strategy for general valuations. From an auctioneer's perspective, the winner determination problem is avoided until there is excess supply on an item. In addition, he only has to communicate a linear number of ask prices in each round, the ask price determination is trivial, and the number of rounds is low.

2. Related Literature and Definitions

First we introduce the necessary notation and review the relevant theory on linear-price combinatorial auctions. The economic setup is formalized as follows: There is a set $\mathcal{K}$ of m indivisible items which are auctioned among n bidders. Let $\mathcal{I}$ denote the set of bidders, X an allocation of items, X_i the package of goods assigned to bidder i. Let v_i denote a value function of the bidder i, which assigns a real value to every subset $S \subseteq \mathcal{K}$ of items and indicates the value that bidder i obtains if he receives S. An allocation of the m items among bidders is $X = \{X_1, ..., X_n\}$, with $X_i \cap X_j = 0$ for every $i \neq j$. Then, the social welfare of an allocation X is $\sum_{i \in \mathcal{I}} v_i(X_i)$. The socially efficient allocation is an allocation with maximum social welfare among all allocations.

We continue with the notion of a competitive equilibrium (CE) in Economics, i.e., a set of prices in which demand equals supply. We focus on linear-price combinatorial auctions, in which there is an ask price β_k for each of the m items available, and the price of a bundle S is the sum of the prices of the items in this bundle. Economic theory assumes that the *demand* of each bidder is a bundle that maximizes his utility.

Definition 1. (Blumrosen and Nisan 2007) For a given bidder valuation v_i and given item prices $\beta_1, ..., \beta_m$, a bundle $R \subseteq \mathcal{K}$ is called a *demand* of bidder i if for every other bundle $S \subseteq \mathcal{K}$ we have that $v_i(S) - \sum_{k \in S} \beta_k \leq v_i(R) - \sum_{k \in R} \beta_k$.

A *Walrasian equilibrium* can then be described as a vector of item prices for which each bidder receives a bundle in his demand set.

DEFINITION 2. A Walrasian equilibrium is a set of nonnegative prices $\beta_1^*, ..., \beta_m^*$ and an allocation X, if for every player i, S_i is the demand of bidder i at those competitive equilibrium prices and for any item k that is not allocated $\beta_k^* = 0$.

The winner determination problem (WDP) in combinatorial auctions can be described as a binary linear program. The decision variables $x_i(S)$ equals 1 if the bidder i gets bundle S allocated, and zero otherwise. If the integrality constraint is relaxed, the dual variables can be interpreted as linear ask prices for the individual items. Unfortunately, the WDP is a non-convex optimization problem, in which the dual prices overestimate the true item values. Simple examples illustrate that linear anonymous CE prices do not exist for a general *combinatorial auction* (CA) if goods are indivisible; in other words, for certain types of bidder valuations it is impossible to find linear prices which support the efficient allocation X^* (Pikovsky and Bichler 2005). Anonymous and linear competitive equilibrium prices only exist if goods are divisible, or the constraint matrix is totally unimodular.

The economic *goods are substitutes* property is a sufficient condition for the existence of Walrasian equilibrium prices (Kelso and Crawford 1982). Intuitively this property implies that every bidder will continue to demand the items which do not change in price, even if the prices on other items increase. Overall, the *goods are substitutes* condition is very restrictive as most known practical applications of combinatorial auctions rather deal with complementary goods. Ausubel (2006) has recently shown a way to maintain simple anonymous and linear prices, but there are multiple price paths and prices are non-monotonic. Bidders need to bid on price paths that are only used to calculate payment of a specific bidder.

This topic has strong ties to Welfare Economics. Blumrosen and Nisan (2007) extend the first welfare theorem to economies with indivisible items and show that if there is a Walrasian equilibrium, then the allocation X^* maximizes the social welfare over all fractional allocations. They also show that if an integral optimal solution exists for the WDP, then a Walrasian equilibrium exists.

Ascending combinatorial auctions are a subclass of iterative auctions in which ask prices can only increase. The bidders respond to the current prices by bidding on their demand under the current ask prices. Such a strategy is referred to as best-response bidding.

DEFINITION 3. The *best-response* bidder bids for all bundles that would maximize his payoff if he were to win any of them at current prices, and only for these bundles.

Note that a best-response bidder might bid on more than one bundle, given that there are multiple bundles that would maximize payoff at the current prices. Even if we know that best-response bidders arrive at an efficient solution, bidders do not necessarily have an incentive to do so. Actually, Gul and Stacchetti (2000) show that even if bidders' valuation functions satisfy the goods are substitutes condition, there exists no ascending combinatorial auction that uses anonymous linear prices and arrives at the VCG solution. This means that bidders may have an incentive to demand smaller bundles of items, in order to lower their payments.

Bikhchandani and Ostroy (2002) prove that only with personalized non-linear prices a combinatorial auction always achieves a competitive equilibrium. The proof is by strengthening the WDP so that the solution is always integral. The dual variables to this WDP can now be interpreted as non-linear and personalized ask prices. This formulation describes every feasible solution to an integer problem, and is solvable with linear programming. This may be viewed as the Second Welfare Theorem for this setting. A number of auctions have been designed using non-linear personalized prices (Parkes and Ungar 2000, Ausubel and Milgrom 2002, de Vries et al. 2007). Unfortunately, the number of rounds can be exponential in the number of items in these auctions. Also, the ask prices generated are not VCG prices for general valuations and thus players still might be incentivized to deflect from the best-response strategy.

3. Efficiency of the CC Auction

First we want to understand the efficiency of CC auctions under the assumption of best-response bidding, as it is common in Economic Theory. In the following, we show that if all bidders follow the best-response strategy, the efficiency can be as low as 50%. We draw on a recent Theorem by Kagel et al. (2009) on the efficiency of auctions which maximize the seller's revenue based on bid prices.

We use β to denote both ask prices and bid prices of the buyers. A class A of auctions is defined such that it selects an allocation $\overline{X}$ to maximize the seller's revenue $\overline{X} \in \arg\max_X \sum_{i \in \mathcal{I}} \beta_i(X_i)$ and has bidder i pay $\beta_i(\overline{X}_i)$. $\beta_i(X_i)$ denotes the highest price that i bids for a package X_i during the course of the auction, and X_i^* is the bundle bidder i gets in the efficient allocation. Finally, $\overline{r}_i = v_i(\overline{X}_i) - \beta_i(\overline{X}_i)$ is bidder i's payoff from the allocation.

THEOREM 1. *(Kagel et al. 2009) Consider any auction in class A. If, for all bidders i,* $v_i(X_i^*) - \beta_i(X_i^*) \leq \overline{r}_i$, *then the goods assignment* $\overline{X}$ *is efficient.*

To promote these results, the auction mechanism must encourage bidders to bid all the way up to their full values ($\beta_i(X_i) = v_i(X_i)$) for *relevant packages*, i.e., packages that may become winning but are not included in the efficient allocation. If a bidder follows the best-response strategy in the CC auction, he will not bid on all relevant packages in the course of the auction. In order to show this, we introduce the notion of a demand masking set of valuations.

3.1. Worst-case Efficiency of the CC Auction with Best-Response Bidders

Definition 4. A *demand masking* set of bidder valuations is given, if the following properties are fulfilled. There is a set of bidders $\mathcal{I} = \{1, ..., n\}$ with $n \geq 3$, a set of items $S = \{1, ..., s\}$ with $A, B \subset S$, $B \cap A = 0$, and $B \cup A = S$. The valuations v_i for bidders i are $v_1(S) = v_2(S) = v_3(A) = \pi$ and $v_1(B) = \delta(s)\pi - \rho$, with $\rho > 0$ and $\delta(s) \leq (s - |A|)/s$.

The first statement $v_1(S) = v_2(S) = v_3(A) = \pi$ ensures a constant raise of the ask price of all items from round to round. The second statement $v_1(B) = \delta(s)\pi - \rho$ leads to the fact that during the course of the auction bidder 1 is unable to reveal the value of the relevant package B.

	$\overbrace{X}^{A}$	Y	Z	XY	XZ	$\overbrace{YZ}^{B}$	$\overbrace{XYZ}^{S}$
v_1						$\delta\pi - \rho$	π
v_2							π
v_3	π						

Table 1 **Example of a demand masking set of bidder valuations.**

Note, that instead of $v_1(S) = \pi$ and/or $v_2(S) = \pi$, there can also be one or multiple bidders (in case of a XOR bidding language) with valuations $v_i(A \cup C_l) = \frac{\pi}{|A \cup C_l|}$ $\forall j \in \mathcal{I} \backslash \{3\}$ with $\bigcup_l C_l = B$. The same holds if the valuation $v_1(B) = \delta(s)\pi - \rho$ is replaced by $v_i(C_l) = v_1(B)\frac{|C_l|}{|B|} \forall j \in \left\{\mathcal{I} \backslash \{3\} : v_i(A \cup C_l) = \frac{\pi}{|A \cup C_l|}\right\}$.

Lemma 1. *If bidder valuations are demand masking and all bidders follow the best-response strategy in the CC auction, then the efficiency converges to 50% with an increasing number of items in the auction in the worst case.*

Proof: The following proof is provided for three or more bidders, as these are practically relevant cases. For restricted special situations with only two bidders, the demand set needs to be redefined, but a similar proof can be made so that the result holds for all auctions with $n = 2$ (see Appendix A).

Without loss of generality, we assume item-level bid increments of $\epsilon = 1$ in each round $t \in \mathcal{T} \subset \mathbb{N}$. We also assume that $|A| = 1$ and show later that if $|A|$ increases, efficiency can only increase. Obviously, the efficient allocation with a demand masking set of valuations is to allocate B to bidder 1 and A to bidder 3.

The proof is by showing that a pure best-response bidder 1 cannot bid on B throughout the auction in a demand masking set of valuations. For this, the payoff $r(S)$ must be higher than $r(B)$ for bidder 1 in each round of the auction $t \in \mathcal{T}$:

$$v_1(S) - \beta_{1,t}(S) > v_1(B) - \beta_{1,t}(B) \quad \forall t \in \mathcal{T} \tag{1}$$

Since we know that $v_1(S) = v_2(S) = \pi$, and all bidders bid best-response, we know that the price for all items rise in each round by ϵ. Therefore, inequality (1) can be rewritten as

$$\pi - ts\epsilon > \delta\pi - \rho - (s-1)t\epsilon \overset{\epsilon=1}{\Longrightarrow} (1-\delta)\pi + \rho > t \tag{2}$$

Inequality (2) shows that as long as t is smaller than the left-hand side, a best-response bidder always bids on the bundle S. We can now determine a round $t_{min} = \min\{t | t \geq (1-\delta)\pi + \rho\}$, in which the payoff $r_1(S)$ is for the first time smaller or equal to the payoff $r_1(B)$. We call t_{min} the *decisive round*. If either the right side or both sides of inequality (1) become negative in round t_{min}, bidder 1 cannot bid on B or the auction ends for bidder 1 as also for S the ask price is higher than $v_1(S)$. If best-response bidder 1 does not reveal his preferences for B throughout the auction, then the auctioneer in a class A auction selects any of the other bids with a revenue of π, resulting in an efficiency of $\pi/((1+\delta)\pi - \rho)$.

We determine $\delta(s)$ such that in round t_{min} the payoff of bidder 1 on bundle B is negative. We know that as long as bidder 1's payoff is negative in the decisive round t_{min}, i.e., $\delta\pi - \rho - (s-1)t_{min} < 0$, then bidder 1 does not bid on B. We also know that $t_{min} = \lceil (1-\delta)\pi + \rho \rceil$ is the decisive round . We can now maximize δ such that $\delta\pi - \rho - (s-1)\lceil (1-\delta)\pi + \rho \rceil < 0$, resulting in $\delta(s) = \max\{\delta | s > 0, \rho \to 0, \delta \leq (s-1)/s\}$. Since $\lim_{s\to\infty,} \delta(s) = 1$ and with $\rho \to 0$, efficiency can be as low as 50%. □

For example, if $\pi = 100$, $\delta(3) = 2/3$ then $t_{min} = 34$, i.e., round 34 is decisive. The right-hand side of inequality (1) in round 34 is negative $(66.\overline{6} - \rho - 68)$, as is the left hand side $(100 - 102)$. In other words, the auction ends for bidder 1, he could not bid on bundle B throughout the auction, and the efficiency is $\pi/((5/3)\pi - \rho) \approx 3/5$. Now, consider the same example with $v_1(B) = 67$. The decisive round will be $100 - 67 = 33$ and the right-hand side (payoff on bundle B) is 1, while it is also 1 on the left-hand side of the inequality (1). Now, a best-response bidder would bid on bundle B and S. The auction would be fully efficient.

We show now that no other set of valuations can be worse than a demand masking set, by showing that every deviation of the conditions from a demand masking set does have a positive or no effect on efficiency.

LEMMA 2. *If bidders follow the best-response strategy in the CC auction, no set of valuations can have a lower efficiency than a demand masking set of valuations.*

Proof: The proof is by showing that any deviation from the properties of a demand masking set of valuations can only increase efficiency.

In a demand masking set there is a set of items $S = \{1, ..., s\}$ with $A, B \subset S$, $B \cap A = 0$, and $B \cup A = S$. The valuations v_j for bidders j are $v_1(S) = v_2(S) = v_3(A) = \pi$ and $v_1(B) = \delta(s)\pi - \rho$, with $\delta(s) = (s - |A|)/s$. We have also assumed $|A| = 1$.

- If $v_1(B) > \delta\pi - \rho$ we have already shown in lemma 1 that a best-response bidder would bid on B and the auction would be efficient. If $v_1(B) < \delta\pi - \rho$, then the efficiency $E = \pi/(\pi + v_1(B))$ increases. If $v_1(B)$ is missing, the auction would find an efficient solution.
- If $v_1(S) > \pi$ and/or $v_2(S) > \pi$, then the second best allocation (the actual outcome of the CC auction) would increase by a constant c and efficiency $E = (\pi + c)/(1 + \delta)\pi$ increases. If either of these valuations is lower than π, then inequation (2) changes, and bidder 1 bids on B, which increases efficiency. If $v_1(S)$ is missing, bidder 1 would only bid on B and the CC auction selects the efficient allocation. If $v_2(S)$ is missing, the prices on B would not rise, and bidder 1 bids on B in the auction, since the payoff at S decreases with rising prices on A. This would reduce the number of bidders involved in the auction to $n = 2$.
- If $v_3(A) > \pi$ by a constant c, then this constant would be added to the numerator and the denominator in the efficiency calculation $E = (\pi + c)/((1 + \delta)\pi) + c)$, i.e. the efficiency will increase. If $v_3(A) < \pi$, then the denominator will decrease and efficiency will increase. If $v_3(A)$ was missing, the CC auction would find the best allocation, by assigning S either to bidder 1 or 2.
- If $|A| > 1$, inequality (2) would change to $\pi - ts\epsilon > \delta\pi - \rho - (s - |A|)t\epsilon$ and the decisive round would be earlier in the auction, i.e. δ would decrease to $(\delta(s) = \max\{\delta | s > 0, \rho \to 0, \delta \leq (s - |A|)/(s + 1 - |A|)\})$ and the level of efficiency increases. In other words, $|A| = 1$ is the worst case concerning efficiency.
- If there were additional bidders with additional valuations in a demand masking set, the efficiency of the CC auction cannot be worse. We have already discussed bids on A, B and S by bidders 1, 2, and 3. Bids by additional bidders on the same bundles would lead to similar effects.

□

THEOREM 2. *If bidders follow a best-response strategy in the CC auction, the worst case efficiency is* 50%.

Proof: This Theorem follows directly from lemmas 1 and 2. □

In addition to these fundamental problems, we discuss a few additional sources of inefficiency in the CC auction. One of these is the termination rule. If supply exceeds demand and the auctioneer runs the winner determination, it guarantees that every standing bidder gets a bundle allocated. This is not necessarily an efficient allocation.

Another source of inefficiencies is in subadditive valuations. The CC auction has originally been defined with the OR bidding language (Porter et al. (2003)). Note, that with subadditive valuations or budget constraints and OR bidding language the CC auction might lose individual rationality, as bidders can have a negative expected payoff due to overflow risks. While the OR bidding language has its advantages in experimental settings, we assume the XOR bidding language in the rest of this paper.

3.2. Worst-Case Efficiency of the CC Auction with Powerset Bidders

One of the reasons for the popularity of ascending auctions is that they require only partial revelation of the private information (Blumrosen and Nisan 2007). In a combinatorial auction this might be less of an advantage, as it is still necessary to elicit all valuations, except those of the winning bids in the efficient allocation in the worst case. This means that if there are z winning bundle bids in an efficient allocation, $n2^m - z$ valuations need to be elicited by the auctioneer to guarantee full efficiency. For example, ascending auctions with non-linear personalized prices such as iBundle (Parkes and Ungar 2000), the Ascending Proxy Auction (Ausubel and Milgrom 2002), or dVSV (de Vries et al. 2007) are protocols that in each round elicit the demand set of each bidder and provably find an efficient solution at the expense of an exponential number of auction rounds (Blumrosen and Nisan 2007). In such an NLPPA with best response bidders at least all valuations of all losing bidders get elicited.

As an alternative to best response bidding, the auctioneer can try to encourage bidders to bid on many bundles from the start.

Definition 5. The *powerset* bidder evaluates all possible bundles in each round, and submits bids for all bundles with a positive payoff given his valuations and the current ask prices.

The question is, whether the CC auction achieves full efficiency if all bidders follow the powerset strategy.

Theorem 3. *If all bidders follow the powerset strategy, the efficiency of the CC auction converges to 0% in the worst case.*

Proof: In the following example, the final prices would be $p(X) = 2, p(Y) = 2$, and $p(Z) = 1$, and the final allocation would be to sell bundle XY to bidder 1, which is inefficient if $\pi > 4$. Efficiency decreases to 0% if $\pi \to \infty$.

	X	Y	Z	XY	XZ	YZ	XYZ
v_1				4		π	
v_2				2			

Table 2 **Example of a CC auction with a powerset strategy.**

□

We assume no free disposal in the example. Otherwise, bidder 1 would have a valuation of π also for bundle XYZ, and this would get sold to bidder 1 for a price of \$5. The payoff for bidder 1 in this allocation would be $\pi - 5$, which would be efficient, as the sum of the bidders' payoffs and the auctioneer revenue gets maximized.

The inefficiency described in the example in Table 2 can only happen, if there are two overlapping bundles by the winning bidder, and there is only competition on the bundle with the lower valuation. This drives up the prices on the latter one and this bundle gets sold, although the bidder had a much higher valuation for the first bundle. In the next section, we discuss possibilities to avoid also this type of inefficiency.

4. Extensions of the CC Auction

Given the above problems of the CC auction, it is natural to ask, whether there are any auction rules that make this auction design fully efficient.

4.1. The Clock-Proxy Auction

The Clock-Proxy auction is one of the most discussed designs for combinatorial auctions. In a first phase, the CC auction is conducted with a revealed preference activity rule (cf. Ausubel et al. (2006)), until clearing is attained. Consequently an ascending proxy auction is conducted as a "final round". The bids submitted by the proxy agents are restricted to satisfy the relaxed revealed-preference activity rule. All package bids of a bidder are valid throughout and treated as mutually exclusive in the proxy phase, when the auctioneer selects the bids that maximize revenue. The advantage of the first phase is the price discovery using linear prices. The proxy phase enables bidders to fine-tune the allocation based on price information from the clock phase. With adequate activity rules, the auction reduces to an ascending proxy auction, which has shown to be fully efficient and in the core.

Theorem 4. *(Ausubel and Milgrom 2002, Parkes and Ungar 2000) The payoff vector resulting from the Ascending Proxy Auction or iBundle(3) with best-response bidders is a core imputation relative to the reported preferences.*

Under buyer submodularity conditions (BSM) an NLPPA yields VCG payments. However, if BSM fails, a bidder can pay more than the VCG payment by following the best-response bidding strategy.

Definition 6 (Buyer Submodularity Condition, BSM). BSM requires that for all $M \subseteq M' \subseteq \mathcal{I}$ and all $i \in \mathcal{I}$ there is

$$w(M \cup \{i\}) - w(M) \geq w(M' \cup \{i\}) - w(M')$$

The activity rule is essential for the Clock-Proxy auction. A common example is the monotonicity rule, which means bidders are not allowed to bid on more items in the current round than they did in the previous rounds. This activity rule makes it difficult to follow the best-response strategy. The revealed-preference activity rule allows for a best-response strategy. The transition from the Clock to the Proxy phase can, however, still create problems. Ausubel et al. (2006) discuss a relaxed revealed preference activity rule, when bidders enter their values to a proxy agent, in order to address possible demand reduction in the Clock phase.

4.2. The CC+ Auction

In the following, we propose the CC+ auction which uses a modified price update rule in order to elicit all losing bundle valuations of all bidders. CC+ uses an XOR bidding language and has two phases, which are however transparent for the bidders. In the first phase item prices are only increased if two different bidders demand the same item or a standing bidder is displaced by the winner determination, as is usually the case for the CC auction. We refer to this as the *standard Clock price update rule*. Phase II starts, when the original CC auction would terminate. In phase II, the prices are increased when any standing bid is displaced by the winner determination, even by a bid of the same bidder. The prices are increased for all items of such displaced bids. Alternatively, the price update rule of Phase II can be implemented from the start. Then, the CC+ auction has only a single phase, just like a CC auction but with a different price update rule. We refer to this price update rule as a *partial revelation price update rule*, because the full valuation of the winning bids is not revealed.

Definition 7. A *partial revelation price update rule* in the CC+ auction increases prices for each item of a standing bid which is displaced by the winner determination.

We suggest using a strong activity rule to encourage powerset bidding, which is necessary to achieve an efficient outcome (Theorem 6). For example, a strong activity rule might ask bidders to bid on all bundles with positive payoff in the first round and does not allow bidders to bid on bundles that they have not bid on in the previous rounds. In addition, we allow bidders to submit last-and-final bids on a bundle (Parkes and Ungar 2000). This means, bidders can submit a last bid on a bundle that is higher than the ask price for this bundle in the previous round, but lower than the ask price in the current round.

Initialization

For every item $k \in \mathcal{K}$, set $\beta_k \leftarrow \emptyset$.

For every bidder i let $X_i \leftarrow \emptyset$

Repeat

For each i, submit a bid on each bundle when $v_i(S) - \beta_i(S) \geq 0$

IF there is excess demand on an item

For all items with excess demand: $\beta_k \leftarrow \beta_k + \epsilon$

ELSE IF there is no excess supply and all standing bids

are submitted by different bidders

allocate goods to the standing bidders

exit the loop

ELSE

execute winner determination with all collected bids

IF any standing bid is displaced (is not winning)

For all items included in displaced bids: $\beta_k \leftarrow \beta_k + \epsilon$

ELSE

results of the winner determination is the final allocation $\overline{X}$

exit the loop

Finally

Output the allocation $\overline{X}$

Table 3 CC+ auction with a single phase, but without last-and-final bids

4.2.1. Efficiency. We now demonstrate that the CC+ auction is inefficient with best-response bidders and then that it becomes fully efficient with powerset bidders.

THEOREM 5. *If all bidders follow the best-response strategy, the efficiency of the CC+ auction can be as low as 50%.*

Proof: In a demand masking set of valuations (Definition 4), bidder 1 is losing. Phase 2 would not help in eliciting the valuation of bidder 1 on bundle B, and the auction would remain inefficient. □

Theorem 6. *The CC+ auction with powerset bidders terminates at an efficient outcome and, what is more, at a core allocation, with respect to the reported preferences.*

Proof: Let $\overline{X}$ be the allocation determined by the CC+ auction in the final round. $\prec_i$ denotes a preference ordering of bidder i over all allocations. Note that we assume a class A auction in which the final payoffs result from the final bid prices of the auction. $\overline{X}$ is individually rational for bidders ($\overline{X_i} = \emptyset$ or $\emptyset \prec_i \overline{X}$). Only coalitions of bidders and the seller 0 are possible, and we show that no coalition of the seller and one or more bidders can block the allocation $\overline{X}$ in the CC+ auction.

Suppose that a coalition consisting of the seller and a nonempty set of bidders $D \in \mathcal{I}$ blocks $\overline{X}$ with X, i.e., $\overline{X} \prec_0 X$ and for all $i \in D$, $\overline{X_i} = X_i$ or $\overline{X_i} \prec_i X_i$. We next show that the powerset bidders in the CC+ auction reveal their valuation on all relevant packages throughout the auction. By construction of the CC+ auction, if all bidders follow a powerset strategy, all losing bidders of the first phase in the CC+ auction have revealed their valuations on all their bundles that they weakly prefer to $\overline{X_i}$. Thus losing bidders do not need to be considered in the second phase.

The partial revelation price update rule in phase II ensures that also a powerset bidder i, who is winning in phase I, reveals all valuations on bundles in the CC+ auction that he weakly prefers to $\overline{X_i}$ at least to a level so that the bid is not displaced any more by the winner determination, i.e., it is the only remaining bid on this bundle at the end of the auction. Therefore, it cannot be that $\overline{X} \prec_0 X$. Every core allocation is also an efficient allocation. □

Note, there is an equivalence between the core of the coalitional game and the set of competitive equilibrium (CE) prices in case of a single auctioneer (Bikhchandani and Ostroy 2002). Non-core means not competitive equilibrium, and CE outcomes are always in the core (Milgrom 2004).

In the example in Table 2 the prices in the final round of phase I are $p(X) = 2, p(Y) = 2$, and $p(Z) = 1$. In phase II, the auctioneer would raise the prices on $p(Y) = 3$, $p(Z) = 2$ and allocate bundle YZ to bidder 1 at a price of 5, which is the efficient allocation. If there was free disposal, and bidder 1 had a valuation of $\pi = 101$ for bundle XYZ as well, then phase II would end up in prices of \$3, \$50, and \$49, allocating YZ to bidder 1 for a bundle price of \$99, which is efficient.

4.2.2. Incentives for Powerset Strategies without Payment Rules. Now we consider incentive properties of the CC+ auction. Given that the CC+ auction with powerset bidders is always in the core, it cannot always achieve Vickrey outcomes. In Table 4 the Vickrey result would be zero and outside the core, while the prices in the CC+ auction would be $p(X) = 2$, and $p(Y) = 2$.

	X	Y	XY
v_1	0	0	2
v_2	0	2	2
v_3	2	0	2

Table 4 Example of a CC+ auction, which does not result in Vickrey prices.

A desirable property would be a profile of strategies with an ex-post Nash equilibrium, in which a bidder would not regret his bid even when he is told what everyone's type was after the auction. Note that we are not focusing on a dominant strategy equilibria in iterative combinatorial auctions, as preference elicitation can invalidate dominant strategy equilibria existing in the single-step version of a mechanism if the queries asked to an agent depend on other agents' preceding answers (Conitzer and Sandholm 2002). When iterative preference elicitation is used to implement a mechanism which would be a dominant-strategy direct-revelation mechanism in a sealed-bid version, then each agent's best (even in hindsight) strategy is to act truthfully if the other agents act truthfully (Conen and Sandholm 2001). In other words, truthful strategies form an ex-post Nash equilibrium. Ex-post equilibria are not as robust as dominant strategy equilibria, but are more robust than Bayesian Nash equilibria in that they are prior-free.

Definition 8. Truthful bidding in every round of an auction is an ex-post Nash equilibrium if, for every bidder $i \in \mathcal{I}$, if bidders in $\mathcal{I}_{-i}$ follow the truthful bidding strategy, then bidder i maximizes his payoff in the auction by following the truthful bidding strategy (Mishra and Parkes 2007).

In other words, in a Bayesian game, a profile of strategies is an ex-post Nash equilibrium if for each agent, following the strategy is optimal for every vector of types (given the others' strategies). Ex-post Nash equilibrium has been adopted as a solution concept in other ascending price auction models (de Vries et al. 2007, Mishra and Parkes 2007). Unfortunately, powerset bidding is not an ex-post Nash equilibrium in the CC+ auction (without an additional payment rule), as it is illustrated in Table 5.

	X	Y	XY	XZ	YZ
v_1	10^*	30	40		
v_2					50^*
v_3				25	

Table 5 Example for demand reduction in the CC+ auction.

Bidder 1 would win item X for a price of 10 using the powerset strategy. He could increase his revenue by deviating from this strategy and bidding only on item Y instead. This results in an inefficient allocation in which bidder 1 receives item Y for a price of 26. This means bidders can increase their revenue by *demand reduction* and not following the powerset strategy in the full information game. This is a well-known problem in linear-price combinatorial auctions. The example shows that even a strong activity rule cannot avoid the incentives for demand reduction.

There is yet another reason for demand reduction in the CC+ auction. Although bidders in the CC+ auction do not know in which phase of the auction they are, they might have an incentive to reduce their demand in later stages of the auction. For example, if the valuation of bidder 1 for bundle XY in Table 2 was \$10, then the bidder would have to pay a price of \$10 for YZ, although the only bid of bidder 2 had a price of \$2. This might seem excessive and creates a strategically complex situation for bidders, as they must speculate on other bidders and decide how much to shade their demand, which is also a source of inefficiency.

PROPOSITION 1. *A powerset strategy is not ex-post incentive compatible in the CC+ auction in which bidders must pay the full bid price of their winning bids.*

It is still possible that weaker solution concepts exist, such as Bayes-Nash equilibria. The choice that each player makes in such a strategy depends on his beliefs about distribution of the types in the set of bidders and is, therefore, quite demanding for bidders in a combinatorial auction in which the types are described by a vector of exponential size in the number of items.

4.2.3. The CC+ Auction with Additional Payment Rules. In order to reduce incentives for demand reduction, the auctioneer can introduce a different from "pay-what-you-bid" payment rule. One possibility is to calculate Vickrey prices, which are equal to the opportunity costs of winning bids. We have shown that the CC+ auction is fully efficient, if all bidders follow the powerset strategy with a partial revelation price update rule. Here, the auctioneer does not learn valuations of the winning bundle bids to a full extent.

THEOREM 7. *A CC+ auction with a partial revelation price update rule and a VCG payment rule does not have an ex-post Nash equilibrium.*

Proof: In the example in Table 6 the CC+ auction with a partial revelation price update rule ends up with ask prices of 3 and 4 for items X and Y resp., before the Vickrey payment is calculated. If the auctioneer calculates Vickrey prices based on these winning bids, then bidder 2 pays $3-(7-5)=1$ for the item X. If bidder 2 knew $v_3(Y)$, he could have bid up to 6 on item Y. This would increase the final ask price for Y to 7, and lead to a new Vickrey price of $3-(10-7)=0$ for X for bidder 2. In a VCG mechanism, bidder 2 could not influence the bid submission of bidder 3 in a similar way, which is why the VCG mechanism has a dominant strategy. Therefore, in the

CC+ auction with a partial revelation price update rule, the strategy of bidder 2 is not independent of other bidders' types. Even if the other bidders bid truthfully, a bidder could improve his payoff by deviating from a truth revealing powerset strategy, if he knew the other bidders' types and the other bidders truthfully follow the powerset strategy.

	X	Y
v_1	0	3
v_2	3*	0
v_3	2	7*

Table 6 Example.

$\square$

A possibility to achieve a strong solution concept such as an ex-post Nash equilibrium in the CC+ auction is to raise the prices on items until even the last and winning bidder drops out, i.e., also the valuations of the winning bids in the efficient allocation get elicited. The final price will then be determined as the last price with at least one active bidder. We refer to this as a *full revelation price update rule.*

Definition 9. A *full revelation price update rule* in the CC+ auction increases prices on items as long as at least a single bidder bids on the item.

Let us consider how the information set in such a CC+ auction differs from a direct revelation mechanism, such as the VCG mechanism, and from other iterative CAs. In a CC+ auction with a full revelation price update rule a bidder sees the price clock increase on various items and learns the highest valuation on every bundle $S \subseteq \mathcal{K}$. In a VCG auction, he only learns that a bundle bid has lost. Similar to the VCG mechanism, a bidder in a CC+ auction reveals all valuations. Prices increase as long as a bidder keeps bidding in the CC+ auction. This might be because he is the only bidder on an item or there are multiple bidders. This is different to other ascending CAs such as iBundle, in which the prices do not increase any more if there is only one winning bidder left. This means, winners do not need to reveal their full valuation on winning bundle bids in these iterative CAs.

Given the volume of valuations that a bidder needs to reveal in any efficient combinatorial auction, this difference might be less of a concern. We have already discussed that if there are z winning bundle bids in the efficient allocation, $n2^m - z$ valuations need to be elicited to the full extent in an efficient CC+ auction. With a full revelation price update rule also the remaining z winning bundles get elicited to the full extent and not only partially.

This is a difference to NLPPAs, in which in the worst case $n2^m - z$ preferences get elicited, and in the best case not all valuations on losing bundles need to be elicited. With a full revelation price update rule all $n2^m$ valuations are revealed, with a partial revelation price update rule only $n2^m - z$. Table 2 provides a good example with $\pi > 4$. Here, an NLPPA would only reveal a partial revelation of 3 on bundle YZ. A CC+ auction with a partial revelation update rule would elicit also the winner's valuation on XY, a full revelation price update rule would elicit all valuations of the winner to a full extent.

THEOREM 8. *A powerset strategy is an ex-post Nash equilibrium in the CC+ auction with a full revelation price update rule and a VCG payment rule.*

Proof: The proof is from the VCG mechanism. We look at the bidder j and assume all other bidders follow the truth revealing powerset strategy. Bidder j receives a payment of $\sum_{i \neq j} u_i(t'_i, X) - \sum_{i \neq j} u_i(t'_i, X_{-j})$ from the center. The final payoff to bidder j reporting type t' and an allocation X and a VCG payment rule is $u_j(t_j, X) + \sum_{i \neq j} u_i(t'_i, X) - \sum_{i \neq j} u_i(t'_i, X_{-j})$. A bidder in this payment rule cannot affect the choice of X_{-j}. Hence, j can focus on maximizing $u_j(t_j, X) + \sum_{i \neq j} u_i(t'_i, X)$, i.e., his utility and the sum of the other's utilities. As the auction will maximize $\sum_i u_i(t'_i, X)$, j's utility will be maximized, if $t'_j = t_j$. $\square$

As all bidders reveal all valuations, a bidder cannot improve his payoff by unilaterally deviating from the truthful powerset strategy in a respective CC+ auction, or influence whether the other bidders reveal their valuations truthfully. Therefore, the bidder's truthful powerset strategy is independent of the other bidders' types. The discussion shows, what types of price update and payment rules are necessary and sufficient for a powerset strategy to be an ex-post Nash equilibrium. Appendix B provides a discussion on why the CC+ auction with a full revelation price update rule and a Vickrey payment rule satisfies the ex-post Nash equilibrium rather than the dominant strategy equilibrium.

The suggested approach, however, suffers from some of the problems of the VCG design, in particular that the outcome might not be in the core (Ausubel and Milgrom 2006b, Rothkopf 2007a). In other words, there are some bidders who could make a counteroffer to the seller that both sides would prefer to the Vickrey outcome. In such situations, the auctioneer can increase his sales revenue by excluding certain bidders, which is also referred to as revenue non-monotonicity. The bidders too could increase their payoff through shill bidding. These vulnerabilities render VCG mechanisms impractical, and an auctioneer will have to relax on the assumption of incentive-compatibility. In practical settings, it might be sufficient to have a mechanism, in which it is hard for bidders to misrepresent their valuations in order to improve their payoff.

Alternatively, the auctioneer might aim for bidder-Pareto-optimal payments in the core. An outcome of an auction is bidder-Pareto-optimal in the core if no Pareto improvement is possible within the core. This means, if we lower one bidder's payment, some other bidder's payment must increase to remain in the core. In other words, such an auction minimizes the total payments within the core.

Definition 10. (Day and Raghavan 2007) An outcome is *bidder-Pareto-optimal* if there is no other core outcome weakly preferred by every bidder in the winning coalition.

This has also been referred to as minimal competitive equilibrium prices.

Definition 11. (Parkes 2006) Minimal competitive equilibrium prices minimize the auctioneer's total revenue on the efficient allocation across all competitive equilibrium prices.

Note that if items are complements, core prices may need to strictly exceed Vickrey prices. Any auction that minimizes the seller's revenue among core allocations results in seller revenue monotonicity with an increasing number of bids (Day and Milgrom 2007). The Ascending Proxy Auction by Ausubel and Milgrom (2002) results in bidder-Pareto-optimal outcomes. Although it is not strategy-proof, it is ex-post incentive-compatible for bidders with buyer submodular valuations. Day and Milgrom (2007) show that a core-selecting auction provides minimal incentives for bidders to deviate from truthful reporting, if it chooses a bidder-Pareto-optimal allocation.

Recently, Day and Raghavan (2007) described a constraint generation approach that generates bidder-Pareto-optimal core prices rapidly for sealed bid auctions. The payment scheme minimizes the total availability of gains from unilateral strategic manipulation. The final bid prices of each bidder on all bundles in a CC+ auction can then be used to calculate bidder-Pareto-optimal core prices. In contrast to the Clock-Proxy auction bidders do not need to type in valuations to a proxy agent after the Clock auction has finished, and the bidder-Pareto-optimal prices get calculated right away.

As with the Ascending Proxy Auction, communication complexity remains a stumbling block. Clearly, already in auctions in which bidders have valuations for more than just a few items, they will not be able to follow the powerset strategy. Actually, in the lab Scheffel et al. (2008) have found that bidders only bid on 6 to 10 bundles in an auction. Segal (2006) showed that the communication cannot be much reduced without severely limiting the efficiency of a combinatorial auction. In some practical settings, however, bidders have only a rather small set of bundle valuations.

5. Computational Experiments

This Section describes results of computational experiments and analyzes efficiency and the number of auction rounds with artificial bidders in the CC and the CC+ auction. Our bidders follow either the best-response or the powerset strategy, plus we also implement agents with restrictions on the number of bundles submitted in each round. The experiments should help us understand, how restricted communication impacts efficiency in the CC+ auction. In contrast to the worst-case analysis that we provide in the first sections of this paper, this should provide more of an average case analysis for different bidder types, based on realistic value models.

5.1. Experimental Setup

The experimental setup is based on two treatment variables, the bidding strategies and the types of valuations.

5.1.1. Bidder Valuations. Since there are hardly any real-world CA data sets available, we have based our experiments on synthetic valuations generated with the Combinatorial Auctions Test Suite (CATS) (Leyton-Brown et al. 2000) and some that have been proposed by An et al. (2005).

The ***Transportation*** value model uses the *Paths in Space* model from the CATS. It models a nearly planar transportation graph in Cartesian coordinates, in which each bidder is interested in securing a path between two randomly selected vertices (cities). The items traded are edges (routes) of the graph. Parameters for the Transportation value model are the number of items (edges) m and graph density η, which defines an average number of edges per city, and is used to calculate the number of vertices as $(m * 2)/\eta$. The bidder's valuation for a path is defined by the Euclidean distance between two nodes multiplied by a random number, drawn from a uniform distribution. Consequently only a limited number of bundles, which represent paths between both selected cities, are valuable for the bidder. This allows to consider even larger transportation networks in a reasonable time. In this work we use the ***Transportation Small*** value model with 25 items and 15 bidders and the ***Transportation Large*** value model with 50 items and 30 bidders. Every bidder has interest in 16 different bundles on average.

The ***Real Estate 3x3*** value model is based on the *Proximity in Space* model from the CATS. Items sold in the auction are the real estate lots k, which have valuations v_k drawn from the same normal distribution for each bidder. Adjacency relationships between two pieces of land p and q (e_{pq}) are created randomly for all bidders. Edge weights $r_{pq} \in [0, 1]$ are then generated for each bidder, and they are used to determine bundle valuations of adjacent pieces of land:

$$v(S) = (1 + \sum_{e_{pq}: p,q \in S} r_{pq}) \sum_{k \in S} v_k$$

In this work we use the *Real Estate 3x3* value model with 9 lots for sale. Individual item valuations have a normal distribution with a mean of 10 and a variance of 2. There is a 90% probability of a vertical or horizontal edge, and an 80% probability of a diagonal edge. Edge weights have a mean of 0.5 and a variance of 0.3. All experiments with the Real Estate value model are conducted with 5 bidders.

The ***Airports*** value model is an implementation of the *matching* scenario from CATS. It models the four largest USA airports, each having a predefined number of departure and arrival time slots. For simplicity there is only one slot for each time unit and airport available. Each bidder is interested in obtaining one departure and one arrival slot (i.e., item) in two randomly selected airports. His valuation is proportional to the distance between the airports and reaches maximum when the arrival time matches a certain randomly selected value. The valuation is reduced if the arrival time deviates from this ideal value, or if the time between departure and arrival slots is longer than necessary.

The ***Pairwise Synergy*** value model from An et al. (2005) is defined by a set of valuations of individual items $\{v_k\}$ with $k \in \mathcal{K}$ and a matrix of pairwise item synergies $\{syn_{k,l} : k, l \in \mathcal{K}, syn_{k,l} = syn_{l,k}, syn_{k,k} = 0\}$. The valuation of a bundle S is then calculated as

$$v(S) = \sum_{k=1}^{|S|} v_k + \frac{1}{|S|-1} \sum_{k=1}^{|S|} \sum_{l=k+1}^{|S|} syn_{k,l}(v_k + v_l)$$

A synergy value of 0 corresponds to completely independent items, and the synergy value of 1 means that the bundle valuation is twice as high as the sum of the individual item valuations. The model is very generic, as it allows different types of synergistic valuations, but it was also used to model valuations in transportation auctions (An et al. 2005). We use the Pairwise Synergy value model with 7 items, item valuations are drawn for each auction independently from a uniform distribution between 4 and 12. The synergy values are drawn from a uniform distribution between 1.5 and 2.0. The auctions with the Pairwise Synergy value model have 5 bidders.

In the *Real Estate* and *Pairwise Synergy* value models bidders were interested in a maximum bundle size of 3, because in these value models large bundles are always valued over small ones. This is also motivated by real-world observations An et al. (2005), in which bidders typically have an upper limit on the number of items they are interested in. Without this limitation, the auction easily degenerates into a scenario with a single winner for the bundle containing all items.

5.1.2. Bidding Agents. In our theoretical analysis, we have already introduced best-response and powerset strategies. The ***Powerset*** bidder evaluates all possible bundles in each round, and submits bids for all bundles which are profitable given current prices. In addition to the pure powerset bidder, we tested limited versions of this bidder who bid only on the best six or best ten bundles in each round, i.e., those six or ten bundles with the highest payoff. This restriction is based on the observation that bidders typically do not bid on many bundles in an auction round (Scheffel et al. 2008). In contrast, the ***BestResponse*** bidder only bids on his demand in each round, i.e., on those bundle(s) that maximize his payoff given current prices.

Inspired by observations in the lab, we also modeled a ***Heuristic 5of20*** bidder. This agent randomly selects 5 out of his 20 best bundles based on his payoff in a round. This bidder allows to evaluate the robustness of the auction against imprecise bidding strategies.

Finally, we also analyzed the ***Preselect 10*** bidder. If a bidder is restricted in time during the auction, he might select his most valuable bundles a priori, and stick to this selection throughout the auction. This might be a strategy by some bidders in auctions with a large number of items. The Preselect agent selects his 10 most valuable bundles before the auction. During the auction, the bidder follows the best-response strategy but bids only on the preselected bundles.

5.2. Experimental Results

We used a $5*6$ factorial design, in which all value models are analyzed with all of the above bidding strategies. Each treatment was repeated 50 times with different random seeds for value models and bidder strategies (if applicable), and the XOR bidding language. The auctions use the minimum increment of 1. All value models are tailored to have the value of efficient allocation around 200. For comparability with the CC design which does not use last-and-final bids, we test two versions of the CC+ auction, with and without last-and-final bids.

Format \ Bidder Type		Best-Response	Preselect10	5of20	Powerset6	Powerset10	Powerset Pure
CC	Mean Efficiency in %	99.48	99.43	97.02	97.39	96.96	96.83
	Min. Efficiency in %	94.81	94.81	84.65	86.71	83.22	83.15
	Mean Rounds	29.10	29.42	25.36	25.28	25.10	24.96
CC+ no L&F Bids	Mean Efficiency in %	99.48	99.43	99.87	99.87	99.86	99.93
	Min. Efficiency in %	94.81	94.81	96.69	96.69	96.69	98.60
	Mean Rounds	29.94	30.02	31.78	31.94	31.66	31.50
CC+ with L&F Bids	Mean Efficiency in %	99.78	99.64	100.00	99.90	99.93	100.00
	Min. Efficiency in %	94.81	94.81	100.00	96.69	96.69	100.00
	Mean Rounds	30.50	30.16	32.02	32.02	31.52	31.32

Table 7 Transportation Small with 25 items and 15 bidders

Format \ Bidder Type		Best-Response	Preselect10	5of20	Powerset6	Powerset10	Powerset Pure
CC	Mean Efficiency in %	98.48	97.60	97.94	97.85	97.86	98.07
	Min. Efficiency in %	90.74	90.09	85.00	85.00	85.00	85.00
	Mean Rounds	17.80	17.08	14.14	14.08	13.82	13.62
CC+ no L&F Bids	Mean Efficiency in %	98.44	97.60	99.12	99.17	99.25	99.34
	Min. Efficiency in %	90.74	90.09	95.29	95.29	95.35	95.29
	Mean Rounds	18.18	17.44	15.82	15.78	15.46	15.22
CC+ with L&F Bids	Mean Efficiency in %	98.95	98.17	99.87	99.87	99.91	100.00
	Min. Efficiency in %	93.21	90.09	97.78	97.78	98.38	100.00
	Mean Rounds	15.94	16.32	15.70	15.72	15.64	15.38

Table 8 Transportation Large with 50 items and 30 bidders

Format \ Bidder Type		Best-Response	Preselect10	5of20	Powerset6	Powerset10	Powerset Pure
CC	Mean Efficiency in %	98.60	98.60	97.02	97.26	97.20	97.28
	Min. Efficiency in %	95.39	95.74	93.16	93.17	93.97	93.86
	Mean Rounds	10.88	10.86	8.50	8.32	8.22	8.20
CC+ no L&F Bids	Mean Efficiency in %	98.57	98.58	98.84	98.84	98.81	98.78
	Min. Efficiency in %	95.39	95.74	96.57	96.45	97.01	97.03
	Mean Rounds	13.72	13.70	11.98	11.62	11.46	11.48
CC+ with L&F Bids	Mean Efficiency in %	99.22	99.20	99.88	99.76	99.97	100.00
	Min. Efficiency in %	96.49	96.49	98.94	98.58	99.47	100.00
	Mean Rounds	15.46	15.56	12.48	12.04	11.90	11.82

Table 9 Airports with 84 items and 40 bidders

Format \ Bidder Type		Best-Response	Preselect10	5of20	Powerset6	Powerset10	Powerset Pure
CC	Mean Efficiency in %	95.63	92.47	97.18	95.95	97.19	98.74
	Min. Efficiency in %	74.74	69.73	82.05	75.84	82.05	91.11
	Mean Rounds	29.54	29.02	26.10	26.08	25.84	25.70
CC+ no L&F Bids	Mean Efficiency in %	95.52	92.47	97.79	96.74	98.00	99.85
	Min. Efficiency in %	74.74	69.73	82.05	75.84	82.05	98.45
	Mean Rounds	30.62	29.62	29.32	29.20	29.10	28.54
CC+ with L&F Bids	Mean Efficiency in %	94.82	92.48	98.10	96.99	98.01	100.00
	Min. Efficiency in %	70.81	72.51	82.05	75.84	82.05	100.00
	Mean Rounds	30.82	29.82	29.90	29.80	29.70	29.02

Table 10 Real Estate 3x3 with 9 items and 5 bidders

Format \ Bidder Type		Best-Response	Preselect10	5of20	Powerset6	Powerset10	Powerset Pure
CC	Mean Efficiency in %	99.62	82.53	99.00	99.10	99.20	99.26
	Min. Efficiency in %	91.48	43.98	94.32	91.48	91.48	94.51
	Mean Rounds	31.08	30.00	30.92	31.18	30.96	30.72
CC+ no L&F Bids	Mean Efficiency in %	99.21	82.53	99.51	99.37	99.56	99.84
	Min. Efficiency in %	91.48	43.98	94.32	91.48	91.48	98.45
	Mean Rounds	35.14	30.40	34.08	34.36	33.96	33.70
CC+ with L&F Bids	Mean Efficiency in %	98.84	83.64	99.81	99.38	99.72	100.00
	Min. Efficiency in %	89.58	44.90	96.70	91.48	91.48	100.00
	Mean Rounds	35.94	30.88	34.34	34.86	34.30	34.10

Table 11 Pairwise Synergy with 7 items and 5 bidders

Tables 7 to 11 and Figures 1 to 3 provide an overview of the efficiency and the number of auction rounds of the CC auction and both versions of the CC+ auction. The efficiency of the CC+ auction with the Pure Powerset bidder was almost always 100% even without last-and-final bids. The worst efficiency of 95.29% with Pure Powerset bidders was obtained in one of the Transportation Large samples. The lack of full efficiency in some cases is due to the minimum bid increment. With an ϵ bid increment and m items, the outcome of a CC+ auction without last-and-final bids can be $(m-1)\epsilon$ away from full efficiency. With last-and-final bids the CC+ auction is always 100% efficient with Pure Powerset bidders. They also bring a marginal improvement in efficiency for powerset bidders with limits on a number of bids per round. Interestingly, the outcome efficiency of the CC+ auction with last-and-final bids and best-response bidders even deteriorates.

Another possibility to address the remaining inefficiencies of the CC+ auction without last-and-final bids is to reduce the minimum increment, but at the expense of an increase of auction rounds. For example, with minimum increment of 0.1 Pure Powerset bidders achieve 100% efficiency in 49 samples of the Transportation Large value model, and 99,38% efficiency in the last sample. Dynamic bid increments that adapt to the level of competition might provide a middle ground.

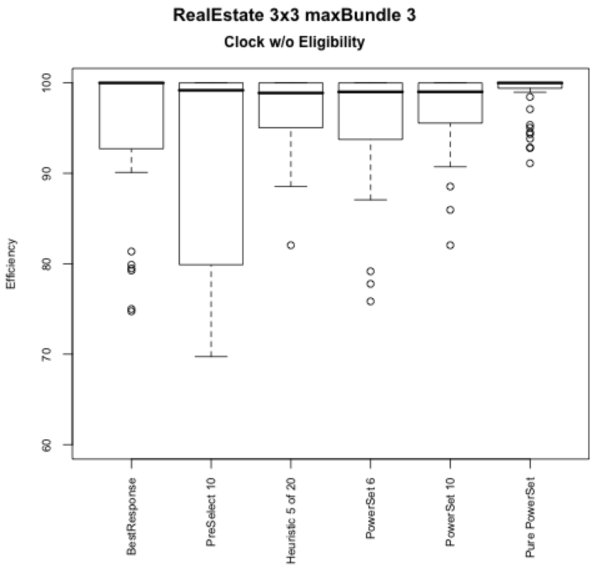

Figure 1 Efficiency of the CC auction in the Real Estate value model (9 items, 5 bidders).

The efficiency with Powerset6, Powerset10, and Heuristic bidders was quite high in the Pairwise Synergy value model. In contrast, these strategies led to efficiencies as low as 75% in the Real Estate value model in CC+. All variants of the powerset bidder outperform the best-response bidders in the CC+ auction.

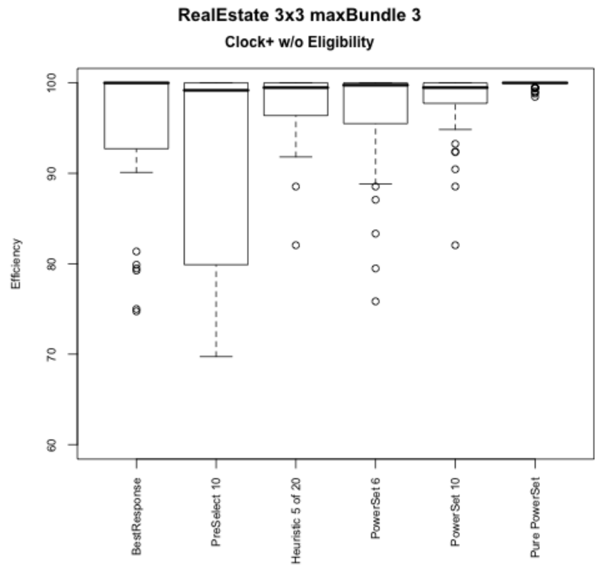

Figure 2 **Efficiency of the CC+ auction without L&F bids in the Real Estate value model (9 items, 5 bidders).**

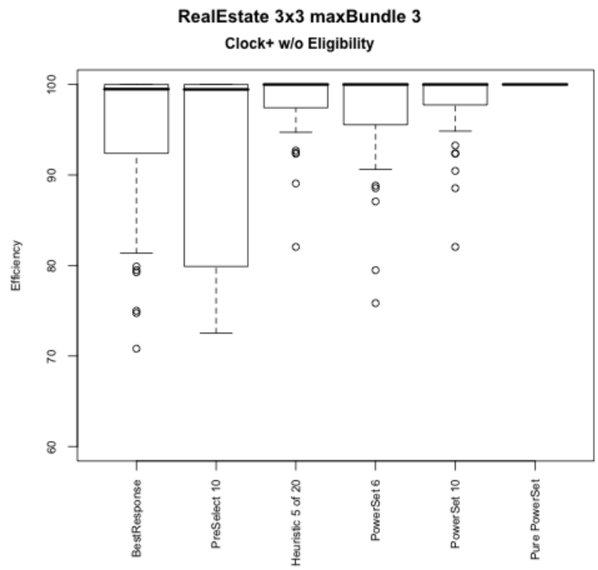

Figure 3 **Efficiency of the CC+ auction with L&F bids in the Real Estate value model (9 items, 5 bidders).**

Interestingly, the average efficiency of the CC+ auction with the Poweset10 bidder, restricted to 10 bundle bids in each round was 99.72% in the Pairwise Synergy and 98.01% in the Real Estate value model. Even a Powerset6 bidder restricted to 6 bids in each round led to very high levels of efficiency, which suggests that the CC+ auction is fairly robust against restrictions in the number of bids submitted in each round. Throughout, also Heuristic (5 of 20 best bids) bidders achieved very high levels of efficiency in the CC+ auction. Actually, Scheffel et al. (2008) found in their laboratory experiments, that a bidder only submits 6 to 10 bundle bids in each round.

Note that there was only a modest increase in the number of auction rounds in the CC+ auction compared to the CC auction. Apparently, the increased efficiency in the CC+ auction over the CC auction comes at almost no cost in terms of an increase in the number of auction rounds. The number of bids that need to be submitted in the CC+ auction is significantly lower than in NLPPAs. This is due to the bid increments of bundles. If the prices for 5 items increase by ϵ, then the price for the bundle of these 5 items increases by $5 * \epsilon$. For example, in our Real Estate (3x3) value model a bidder had 130 valuations. In the CC+ auction (with last-and-final bids) 4,128 bids were submitted in 32 rounds by Pure Powerset bidders, and only 419 bids were submitted by Powerset10 bidders. Even in the Real Estate value models the average efficiency with Powerset10 bidders was 98%, in all other value models beyond 99.7%. In contrast, the same auction in iBundle(3) (Parkes and Ungar 2000) elicited 7,741 bids per bidder in 150 rounds, and in the Credit-Debit auction even 14,895 bids in 266 rounds.

6. Conclusions

Several authors analyzed iterative auction designs for markets allowing for bundle bids. Unfortunately, Walrasian equilibria with linear prices were only found for restricted settings. Already, Kelso and Crawford (1982) showed that the goods are substitutes property (aka gross substitutes) is a sufficient and an almost necessary condition for the existence of linear competitive equilibrium prices. Later, Gul and Stacchetti (2000) found that even if bidders' valuation functions satisfy the restrictive goods are substitutes condition, there exists no ascending VCG auction that uses anonymous linear prices. Finally, Bikhchandani and Ostroy (2002) showed that there always exist personalized non-linear competitive equilibrium prices. The implicit assumption in all of these models is best-response bidding. In other words, bidders only bid on those bundles that maximize their payoff in each round. The auction terminates if demand equals supply. Several auction designs are based on this theory and use non-linear personalized prices, unfortunately, at the expense of an exponential number of auction rounds as all losing bundles need to be elicited from all bidders for full efficiency.

In this paper, we analyzed the worst case efficiency of the CC auction with best-response bidders and found a lower bound of 50%. Then we deviate from the assumption of the best-response bidding strategy and analyze powerset strategies. We show that with a partial revelation price update rule the CC+ auction leads to full efficiency, if all bidders follow the powerset strategy.

We proof that with a full revelation price update rule, the CC+ auction even satisfies an ex-post Nash equilibrium. Clearly, a pure powerset strategy is prohibitive for any but small combinatorial auctions. Interestingly, the auction shows to be robust against deviations from pure powerset bidding strategies. Even if the number of bids submitted is severely restricted or bidders heuristically select some of their best bids in each round, the auction achieves very high efficiency levels. The volume of ask prices that need to be communicated by the auctioneer, as well as the number of bids required by bidders is drastically reduced compared to NLPPAs.

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Appendix A: Additional Proofs

LEMMA 3. *If bidders follow the best-response strategy in the combinatorial clock auction, the worst case efficiency is* 50% *also in case of only two bidders* ($n=2$).

Proof: The proof is analog to the case in which $n \geq 3$. Consider the following valuation set:

	$\overbrace{X}^{A}$	$\overbrace{XYZ}^{S}$
v_1	π	$\delta\pi$
v_2		π

Table 12 Example of a demand masking set for two bidders only.

The efficient solution is to sell bundle S to bidder 1 if $\delta > 1$. In that case it is clear that a best response bidder starts bidding on bundle S. So for a worst case analysis bidder 1 must switch to bundle A at a point in the auction process and must outbid his own bid on S. For this, the payoff $r_1(A)$ must be higher than $r_1(S)$ in a round of the auction $t \in \mathcal{T}$:

$$v_1(A) - \beta_{1,t}(A) \geq v_1(S) - \beta_{1,t}(S) \quad \exists t \in \mathcal{T} \tag{3}$$

Since $v_2(S) = \pi$ and all bidders bid best-response, we know that the "switch" of bidder 1 to bundle A must have happen while bidder 2 is still bidding, otherwise the auction would terminate. That means the price for all items will rise in each round by ϵ. Therefore, inequality (3) can be rewritten as

$$\pi - |A|t\epsilon \geq \delta\pi - |S|t\epsilon \Leftrightarrow \delta \leq -\frac{(|A| - |S|)\epsilon t}{\pi} + 1 \tag{4}$$

Since bidder 1 must place his first bid before the auction ends, this has to be done in a round $t \leq \frac{\pi}{|S|\epsilon}$. In order to determine the worst case the inequality can be replaced by an equality and t can be inserted in (4) resulting in

$$\delta \leq 2 - \frac{|A|}{|S|} \tag{5}$$

If inequation (5) holds, bidder 1 outbids his bid on S and receives bundle A as single winner. This results in an inefficient outcome with $\frac{\pi}{\delta\pi}$%. In order to determine the worst case we have to maximize δ. Clearly with $|A| = 1$ and $|S| \to \infty$ we achieve the worst case efficiency of 50%. Any deviation from the treated valuations would increase efficiency.

□

Appendix B: Ex-Post Nash Equilibrium of the CC+ Auction with A Full Revelation Price Update Rule and VCG Payments

Does the CC+ auction with a full revelation price update rule satisfy a dominant strategy or an ex-post Nash equilibirum? In the single-unit case, there has been an interesting recent discussion on the types of ascending auctions that actually satisfy a dominant strategy equilibrium. Isaac et al. (2007) have shown that while the clock version of an ascending single-item auction has a dominant strategy, the wide-spread English auction, which allows for jump bids, has not.

The CC+ auction can be seen as a multi-item generalization of the ascending clock auction. Also, the VCG auction can be thought of a single-round version of the CC+ auction, in which the bidder's dominant strategy is to bid truthfully on all possible bundles, similar to a powerset strategy. Both auctions satisfy a dominant strategy equilibrium. Does also the CC+ auction satisfy a dominant strategy, or is it restricted to an ex-post equilibrium? In the following, we provide an example, in which signals revealed throughout the CC+ auction can make it beneficial for a bidder to deviate from his truth telling powerset strategy, when also others deviate from this strategy.

	X	Y	XY
v_1	2*	0	0
v_2	0	3*	0
v_3	0	0	4

Table 13 Example.

The valuations for three bidders and three items are given in Table 13. The Vickrey price of bidder 1 is $2-(5-4)=1$ for item X, and his payoff is 1. Now, lets assume that bidder 1 knows that bidder 2 will increase his bid on Y to 4, if the ask price for X was 3. At round 2 the price clock ticks to 2 for each item and all three bidders signal demand at these prices. At round 3 prices are 3 for both items and again bidders 1 and 2 will signal demand. This will encourage bidder 2 to signal demand even in round 4 for item Y, when bidder 1 drops out. Now, bidder 1 gets a Vickrey price of $3+(7-4)=0$ and consequently increased his true payoff from 1 to 2. Bidder 2 learns through the course of the CC+ auction that there is a demand for X at a price of 3, which would not be possible in a direct revelation VCG auction.

This cannot happen in a clock auction with only a single item, as the bidders can only drop out or continue to signal demand on a single item. This illustrates that the dominant strategy equilibrium does not extend from the single-item clock auction to its multi-item generalization. The powerset strategy in a multi-item CC+ auction is therefore an ex-post Nash equilibrium and no dominant strategy equilibrium.

Appendix C: List of Symbols

$\mathcal{K}$	set of items
$S, R \subseteq \mathcal{K}$	subset of items
k	item index $k \in \{1, \ldots, m\}$
$\mathcal{I}$	set of bidders (including the auctioneer)
i, j	bidder index $i, j \in \{1, \ldots, n\}$
$v_i(S)$	private valuation of bidder i for the bundle S
X	allocation $X = (X_1, \ldots, X_n)$ (in some examples X denotes an item, which is mentioned in the text explicitly)
X^*	efficient allocation $X^* = (X_1^*, \ldots, X_n^*)$
$x_i(S) \in 0, 1$	binary variable which determines, whether the bidder i becomes the bundle S allocated
β_k	ask price for item k
$\beta_i(S)$	bid price of bidder i for bundle S
$r_i(S)$	bidder i's payoff/revenue if he were to win bundle S
ϵ	minimum increment
η	graph density in the Transportation value model
ρ	marginal value used for the definition of a demand masking set
$\mathcal{T}$	set of auction rounds
t	round index $t \in \{1, \ldots, T\}$
s_i	strategy profile of bidder i
t_i	type of bidder i

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