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30 Seconds or Free! Managing Quality Uncertainty Through Contingency Pricing

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30 Seconds or Free! Managing Quality Uncertainty Through Contingency Pricing*

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Abstract

This paper demonstrates that quality-contingent pricing is a useful mechanism for mitigating the negative effects of quality uncertainty in e-commerce and IT services. A contingency pricing contract specifies a sequence of possible quality levels and corresponding prices. The market estimates the firms performance at various quality levels based on historical statistics, and the firm may have additional private information with respect to its future, and true, probability distribution. Examining the monopoly case, we explicate the critical role of private information and differences in belief between the firm and market, in the choice of pricing scheme. Contingent pricing is useful when the market underestimates the firms performance; then it is optimal for the firm to offer a full-price rebate for mis-performance, with a correspondingly higher price for meeting the performance standard. In some cases, contingency pricing enables the existence of trade when standard pricing fails to bring the parties together when they have different beliefs about performance. We study the competitive value of contingency pricing in a duopoly setting where the firms differ in their probabilties of meeting the performance standard, but are identical in other respects. Each firms choice of contingency vs standard pricing scheme mirrors its choice in the monopoly setting, and each firm uses a full-price rebate when the market underestimates its own performance. However the superior firm achieves a greater increase in profits under contingency pricing. We show that contingency pricing is efficient as well, and consumer surplus increases because more consumers buy from the superior firm which uses contingency pricing to signal its superiority.

1 Introduction

Many IT-intensive commercial settings are characterized by quality uncertainty arising out of unobservability of quality or due to stochasticity in manufacturing or delivery processes. For digital goods, consumers must often make a purchase decision without inspecting the good. In e-commerce, consumers do not observe product quality till delivery and use (even for physical goods). Consumers interact with new firms, and vendors deal with wider range of consumers, creating greater uncertainty about the exchange. The IT sector has seen a large number of firms entering the market and also an explosion of new products and services, further increasing occurrence of quality uncertainty. For many IT services, consumers require end-to-end services and quality levels, even though in many cases firms control only a part of the service chain. For instance, in on-line trading, firms may guarantee overall trade execution times although they do not control market-clearing and communication functions. Hence there is inherent uncertainty in the quality of the delivered product. Moreover, the experience goods characteristics of many IT products and services also contributes to quality uncertainty.

How does this quality uncertainty affect commerce—for instance, the existence of markets or competition between firms? There are obvious negative effects. Firms must take quality uncertainty into account in setting prices, thereby putting a downward pressure on product prices. On the other hand, consumers also factor in quality uncertainty in determining their valuations for products, and often may balk from purchasing. **?** note that quality uncertainty deters trade, especially when firms and consumers differ in their expectations about product quality.

We examine the use of quality-contingent prices as a mechanism to mitigate the effects of quality uncertainty. In contingency pricing, the firm announces quality-price pairs for various levels of quality instead of a single price. Since quality is stochastic, any of the possible realizations of quality level may be achieved, and consumers pay depending upon the quality of the product or service received. Contingency pricing works well when quality is objectively verifiable and is unaffected by use, avoiding moral hazard. The framework assumes the availability of performance information—how well the firms are likely to deliver on quality. Modern IT infrastructure supports contingency pricing well in a setting where product quality is often digitally captured, continuously metered, and hence can be verified. The emergence of businesses such as `eTesting Labs` that specialize in collecting and reporting performance data for various products and firms in the IT sector (see Figure 1 for a specific example of such information) exemplifies this trend.

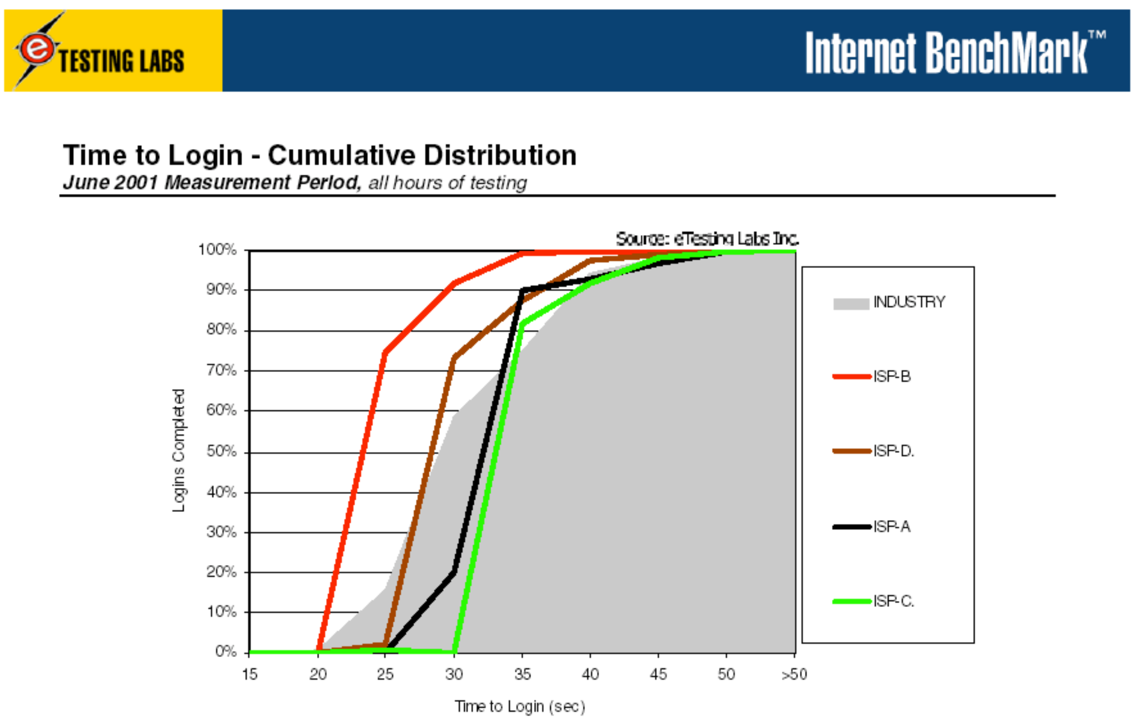

Figure 1: `Historical performance information about access delay for various dial-up internet service providers.`

Examples of contingent pricing structures in IT goods and services include download speeds, ISP uptime guarantees (MSN), trade execution times (AmeriTrade), and ASP service level guarantees (see e.g., ?). For a specific example of the use of contingency pricing as well as the availability of performance information under quality uncertainty, consider the market for carrying Internet data traffic. Many businesses today depend on full availability of Internet-based communication and demand performance guarantees from providers. ? gives examples of firms switching ISPs based on the type and availability of performance guarantees. In practice, network service providers cannot achieve 100% quality performance. In order to mitigate such quality concerns, a leading Internet backbone provider (Sprint[1]) announced a 100% end-to-end availability guarantee, with the following contingencies and rebates: outages of an hour or less lead to a three-day, pro-rated monthly port; longer outages lead to a day's credit for each hour the outage persists. The performance metric is quantified, continuously metered, and captured digitally. Customers can view performance statistics on the firm's web site, and are offered a suite of tools to monitor and validate network performance and service levels. For customers of its high-speed DSL service, the firm offers another contingent price contract with 99.9% network availability, with a rebate for each day of service interruption. Likewise, similar contingent price offerings are becoming common for network latency (Genuity guarantees a 55 msec round-trip latency and 0.5% packet loss) and managed hosting services (99.5% web server availability guarantees from Sprint). Contingency pricing works well when the information infrastructure enables easy quantification, capture, verification and dissemination of quality and performance information.

Other mechanisms for dealing with quality uncertainty include money-back guarantees (?, ?) warranties (?, ?) and limited-time trials. Although contingency pricing shares some features with these mechanisms, it differs significantly from each. In money-back guarantees,

[1] See http://www.sprintbiz.com/business/extranet/service.html

there is no notion of quality contingency. Consumers may return the product for any reason; hence transaction costs for the firm and consumer play a significant role, whereas with contingency pricing, products need not be returned; a simple monetary transaction completes the transaction. In the case of software goods, many firms offer 30-day trials to consumers to provide them a chance to observe quality, but this mechanism is essentially a money-back guarantee with a 30-day fuse. Warranties may be bundled with the product; or sold unbundled as extended warranties, often by a different firm. They involve the return and repair of the product to original specification, upon which the warranty may restart again, thereby involving the analysis of repeated gamble. Contingent pricing does not restore the product to original quality. IT goods and services differ from traditional goods in providing the ability to monitor the quality of the good or service as it is delivered and the ability to act on this performance information. For many IT-intensive goods and services, where the product delivery and consumption are simultaneous, these mechanism do not apply, whereas contingent prices work well.

? write about the potential value of contingency pricing under quality uncertainty, and note the lack of managerial understanding regarding how to design optimal contingent pricing schemes. **?** motivate contingency pricing and analyze its applicability and implementability in modern IT settings, using a monopoly profit maximization model. **?** study contingency pricing under demand uncertainty, and show that a firm may benefit by offering a low-type customer a contingency sale at a lower price, where the sale becomes invalid if the firm receives a high-type customer in a specified timeframe.

This article extends the study of contingency pricing to a competitive setting. We describe the nature of contingency price contracts and address how and when contingency pricing should be used and how to design optimal contingent prices. We discuss the framework for analyzing contingency pricing in §2, and analyze the use and design of contingency pricing in a monopoly setting in §3. §4 discusses the model in a duopolistic setting where the two firms differ in their probabilties of meeting the performance standard, but are identical in other respects. We study the choice and design of contingency pricing by each firm, and examine how contingency pricing may affect competitive advantage, its impact on relative market shares and profits, and its effect on consumer surplus and social welfare. §5 concludes the paper.

2 Framework for Contingency Pricing

We formally define a contingency pricing structure in the following way. Product quality q is stochastic and in some bounded continuous interval as given by a probability distribution function $G(q)$ which specifies the probability that quality is lower than q. The firm announces a price vector $(R_1, ..., R_N)$ corresponding to some N intervals on the quality scale, and the final price paid by the buyer is contingent on the quality level realized. The firm's objective is to choose the quality thresholds and the corresponding prices. This design appears similar to the quality differentiation problem (see e.g., (**?**), (**?**)), where the firm also offers a series of price-quality combinations. The crucial difference with the work on quality differentiation is that the literature has explored the case where the firm is able – with *certainty* – to offer each quality level on the menu: the intent is to let consumers *self-select* into segments. Contingency pricing, on the other hand, focuses on quality uncertainty: the multiple qualities are only possible realizations for product quality. The combination of the stochastic quality levels and associated prices (along with associated probabilities) defines a single choice item offered to all users: users do not have the option to select a particular quality. It would be valuable to examine the combination of these two mechanisms i.e., a menu of contingency prices; we defer this analysis to future work.

Information about the quality distribution is available to consumers, typically via historical information that is metered and disseminated by third-party firms, regulatory agencies, or the seller. The IT sector has a unique ability, due to the digital nature of many goods and services, to collect and disseminate such information at very low cost. For example, `eTesting Labs` provides performance information on dozens of IT products and firms (as illustrated in Figure 1), the FCC provides data on outages at Internet traffic carries, and Sprint provides detailed performance data on its website. Operationally, we represent this information as a probability distribution $F(q)$ representing the belief that quality level will fall below q. We note that this public historical information need not be entirely accurate or a perfect predictor of performance in the next period (i.e., $G(q)$).

Our framework assumes that the firm possesses private information with regard to the process that generates the quality distribution. The firm holds private information—about resource allocations, continuous improvements to technology, planned maintenance and breakdowns, process changes, and recent events—that allows it to measure the true probabilities at different quality levels. Hence we assume that the firm's information, the probability vector $G(q)$, reflects the true state of the world and it may differ from the publicly available performance information $F(q)$.

Following prior literature on quality uncertainty (see e.g., **?** and **?**), we focus on the case where the firm sets prices around just one quality threshold $\hat{q}$ so that there are two quality intervals, *standard* quality q_{High} and *inferior* quality q_{Low}. Often, the quality threshold emerges as an industry-wide performance standard, for example a 50 ms latency in data networks. Since we focus on a single quality threshold, we can simplify the price structure to a contingent pricing scheme $(R - r, R)$ where R is the price for the standard quality, and r is the rebate to the consumer if an inferior quality is realized. Similarly, it is useful to define the true and public probabilities $\mu = F(\hat{q})$ and $p = G(\hat{q})$ representing the probabilities that the realized quality is below the threshold $\hat{q}$. The main results generalize to multi-part contracts with multiple quality levels. The firm has constant expected marginal cost $\hat{C}$ which incorporates the production costs, costs due to customer service calls, error handling, or other processes required in dealing with inferior quality.

Let the index v represent consumer types, so that $U(v, q)$ is consumer type v's valuation for quality q. For ease of exposition, suppose that the types v are distributed uniformly in $[0, 1]$ (the results generalize easily to other distributions). Thus, consumers are heterogeneous in their valuation for the product and have different sensitivities to quality. Consumers are aware of the public performance distribution $F(q)$, and make decisions as expected value maximizers. Therefore, consumer type v perceives an expected benefit

$$U(v) = \int U(v, q)F'(q)dq$$

and all consumers see the same expected price

$$\bar{R} = R(1 - \mu) + (R - r)\mu = R - \mu r$$

We order the consumer types v so that $U(v)$ is increasing in v.

3 Contingency Pricing by Monopoly

To isolate the impact of contingency pricing, we consider a monopolistic firm that offers a single product. Setting $U(v) = 0$ yields the marginal customer v_m who is indifferent towards buying the product, so that $U(v_m) = \bar{R}$. All consumers with valuation greater than v_m buy the product, so the fraction of buyers is $1 - v_m$. Hence the demand function depends only on the market price expectation $\bar{R}$ rather than the actual prices $(R - r, R)$. Rewriting v_m as $U^{-1}(\bar{R})$, the demand function is $D(\bar{R}) = 1 - U^{-1}(\bar{R})$. We make the usual assumption that the demand function satisfies non-decreasing price elasticity i.e., that the proportional fall in demand caused by a proportional change in expected price weakly increases with price. Formally, we assume that the term $\frac{-D'(\bar{R})}{D(\bar{R})}\bar{R}$ increases in $\bar{R}$.

3.1 Role of Private Information

The firm's expected profit under quality uncertainty and contingency pricing is

$$\pi = (\hat{R} - \hat{C})D(\bar{R})$$

where $\hat{R}$ is the firm's price expectation per sale. By definition, $\hat{R} = R - pr$, so that $\hat{R} = \bar{R} + r(\mu - p)$. Notice that the standard pricing scheme with a single price and no rebate is just a special case of the contingency pricing scheme with $r = 0$. A second special case of interest is the *full rebate* solution $r = R$ where the firm offers a full-price rebate for inferior quality. The optimal pricing scheme is

$$(R^*, r^*) = \arg\max_{(R,r)} \pi = \arg\max_{(R,r)} (\bar{R} + r(\mu - p) - \hat{C})D(\bar{R}) \tag{1}$$

Solving for optimal prices, we find that

Proposition 1 *It is never optimal for the firm to offer less than full-price rebate ($r < R$) except when the market has perfect information about the probability at quality level $\hat{q}$ (i.e., $\mu = p$). When public information is perfect, a convex combination of prices is optimal, including a full-rebate contingent price contract and a standard single price contract, indicating that contingency pricing offers no additional value in this case.*

Proof. See Appendix. ■

The proposition formalizes the intuition that when there is any benefit from offering a certain rebate r, then a higher rebate (and a smaller increase in the price R) will confer an even greater benefit (note that a unit increase in R requires a greater increase in rebate because the probability of rebate is less than 1). If $\bar{R}^*$ is the expected price that induces the optimal number of consumers to buy, the firm may achieve $\bar{R}^*$ by a number of price-rebate combinations $(\bar{R}^* + r\mu, r)$. Since the firm's profits increase with the offered rebate (and corresponding higher price), it will offer the maximum possible rebate, equal to price. When the market underestimates the firm's performance, the firm is able to play this game. When public information is perfect, however, the optimal expected price (hence market share) can be achieved by infinite combinations of price and rebate, including the zero-rebate standard pricing scheme.

3.2 Optimal Contingency Pricing under Private Information

Proposition 1 indicates that the optimal pricing scheme under private information is either a standard single price R^s (no penalty for performance failure) or a contingency pricing scheme with price R^c and full-price rebate for inferior performance. Now we compute and compare the optimal prices and profits under each case, and determine conditions under which each pricing scheme is optimal.

Lemma 1 *The optimal single price R^s is the unique solution to the equation*

$$\frac{-D(R)}{R \cdot D'(R)} = \frac{R - \hat{C}}{R}$$

For contingency pricing, the optimal price is $R^c = \frac{\bar{R}^c}{1-\mu}$, where the market price expectation $\bar{R}^c$ is the unique solution to the equation

$$\frac{-D(\bar{R})}{\bar{R} \cdot D'(\bar{R})} = \left(\frac{1-p}{1-\mu}\right)\left(\frac{\bar{R} - \left(\frac{1-\mu}{1-p}\right)\hat{C}}{\bar{R}}\right) = \left(\frac{\hat{R}^c - \hat{C}}{\hat{R}^c}\right)$$

Proof. See Appendix. ■

For the standard pricing scheme, the optimal price represents the usual condition that the inverse elasticity of demand equals the firm's market power (margin relative to price). The

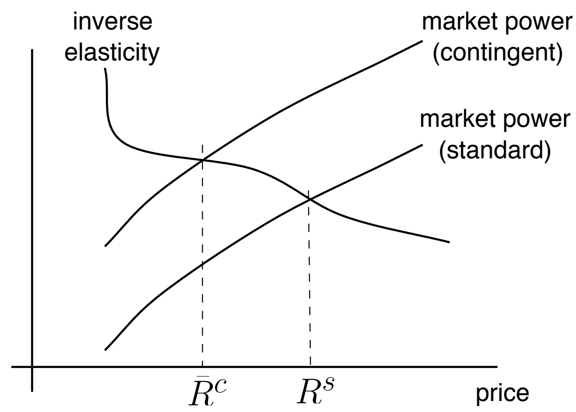

Figure 2: **`Optimal prices under standard pricing and full-rebate contingency pricing, when the market underestimates firm performance.`**

optimal contingent price follows a similar structure, except that inverse elasticity of demand now equals the firm's *effective* market power. Figure 2 illustrates the optimal solutions under the standard and contingency pricing schemes for the case where $\mu > p$ i.e., the market underestimates the firm's ability to deliver quality level $\hat{q}$ ($\frac{1-\mu}{1-p} < 1$).

It is clear from the two expressions (and the figure) that when $\frac{1-\mu}{1-p} < 1$, consumers see a lower expected price under the optimal contingency pricing scheme than the optimal standard price ($\bar{R}^c < R^s$), thus the firm expands its market coverage with contingency pricing. Further, since the inverse elasticity of demand is downward sloping, the firm's effective market power is greater when it employs contingency pricing. The optimal profits for the two cases are $\pi^s = D(R^s)(R^s - \hat{C})$ and $\pi^c = D(\bar{R}^c)(\hat{R}^c - \hat{C})$, respectively.

Proposition 2 *The firm will apply contingency pricing if and only if $\mu > p$ i.e., when the market underestimates the firm's performance. When $\mu > p$, the firm offers a contingent contract with price R^c (and a full-price rebate); when $\mu < p$ the firm should offer a standard single-price contract with price R^s.*

Proof. We construct a full-rebate contingent price contract that outperforms the optimal standard price when $\mu > p$. Given the optimal standard price R^s with profit π^s, define a contingency pricing scheme with price R such that the consumers' price expectation $R(1-\mu) = R^s$ i.e., $R = \frac{R^s}{1-\mu}$. Hence the firm gets the same market coverage as under the single price. However, its true expected price $\hat{R} = R^s\left(\frac{1-p}{1-\mu}\right)$ is greater than R^s, hence the firm earns a higher profit with this contingent price contract. ■

To understand this result, let us interpret a contingent contract as a lottery. When the market overestimates the firm's inability to deliver promised quality—i.e., consumers overestimate the probability of receiving a rebate—a contingent contract represents a lottery biased in favor of the firm. Since the bias increases with the size of the rebate, the firm offers the maximum (full-price) rebate. Conversely, when the firm realizes that its performance will be poorer than what the market believes, a contingent contract represents a lottery biased against the firm. Hence, the firm does not employ this mechanism. It is interesting to note that the literature on money back guarantees **?** and **?** often assumes that the price will be fully refunded when consumers return the product. Here, we formally show that such "full-rebate" policy is indeed optimal under market over-estimation. Propositions 1 and 2 also highlight a rather simple *All-or-Nothing* contingency pricing strategy for managers when they have good reason to believe that they can beat market expectations. The result also mirrors the spirit of the full-insurance conditions in insurance markets under similar conditions.

3.3 Market Existence under Contingency Pricing

Examining each of the two equations in Lemma 1, we see that there is a feasible price (i.e., the market exists) when the LHS and RHS terms of the equation are equal somewhere. Since the LHS (inverse elasticity) is decreasing in R and the RHS (market power) is increasing in R, the market fails to exist when the RHS term is always smaller than the LHS. For any demand and marginal cost functions, the RHS term is greater under contingency pricing than under the standard price whenever $\mu > p$. Thus, a feasible contingent price (that generates positive profit) can be offered even when there is no feasible single price. This argument may also be understood by examining Figure 2, since there is no feasible solution when the *market power* curve is always below the inverse elasticity curve. Therefore, we see that *contingent contracts enhance the likelihood of market existence*. This expansion occurs when consumers underestimate firm performance.

How does contingency pricing enable the creation of trade when standard pricing cannot? In one-to-one bargaining **?**, it is well known contingent contracts can create trading opportunities when buyer and seller disagree on performance probabilities. While the buyer and seller may not be able to agree on a single price, they may be able to—because of their different probability assessments—to agree in an expected value sense. We generalize this result in the context of contingent price contracts offered to a heterogeneous population of consumers. **?** note that quality uncertainty deters trade when firms and consumers differ in their expectations about product quality. What contingency pricing does is to enable firms to signal their higher quality. In markets where buyers underestimate product performance, they underestimate their expected valuation hence have a lower willingness to pay. With standard pricing, there is no way for the firm to attract those consumers whose (false) expected valuation is below the expected marginal cost. With contingency pricing, the firm is able to give these buyers a chance to increase their expected valuation to higher values and thereby participate in the market. The firm increases its price but also offers a high rebate, on which the marginal non-buyer puts greater weight and hence expects a positive surplus. As noted earlier, considering contingency pricing as a biased lottery is a useful tool to understand how it can expand markets. Consumers who previously would not buy due to a mis-estimate of firm performance are induced to buy because of contingency pricing. They may not get as much surplus as they expected, but many will derive some surplus while some may not. The balance of the trade-off between the costs of inferior quality and the pricing levels and actual mismatch in performance levels determine the eventual winners and losers.

3.4 Illustration

We illustrate the effect and design of contingency pricing with a linear form for consumer valuations. Specifically, consumers discern only two quality levels — standard and inferior. Let $U(v, q_{\text{High}}) = v$ and $U(v, q_{\text{Low}}) = v - (\alpha + \delta v)$ where $(\alpha + \delta v)$ is consumer type v's disutility for inferior quality q_{Low} so that disutility is proportionally increasing with valuation. The variables $\hat{C}, \mu, p$ represent marginal costs, public and private performance probabilities respectively as before.

We investigate the existence of the market by inspecting participation constraints for the consumer as well as the firm. For the market to exist under contingency pricing, the expected value for the highest value consumer should exceed the expected marginal costs to the firm giving us the constraint

$$\hat{C} - (1 - (\alpha + \delta\mu)) < r(\mu - p) \tag{2}$$

which is looser than the corresponding existence constraint under standard pricing (which requires $r = 0$) when $\mu - p > 0$. Hence, contingency pricing expands the likelihood of market existence.

The shape of the feasible regions for both types of contracts depends on the relationship between the disutility and the firm's expected cost $\hat{C}$, as illustrated in Figure 3. The feasibility region is the region below the respective market participation constraints. Standard pricing is feasible only for certain combinations of p and μ (the shaded region), while contingency pricing can yield a feasible solution for additional combinations (the region with the x's).

The relative outcomes for the optimal standard and optimal contingency pricing schemes

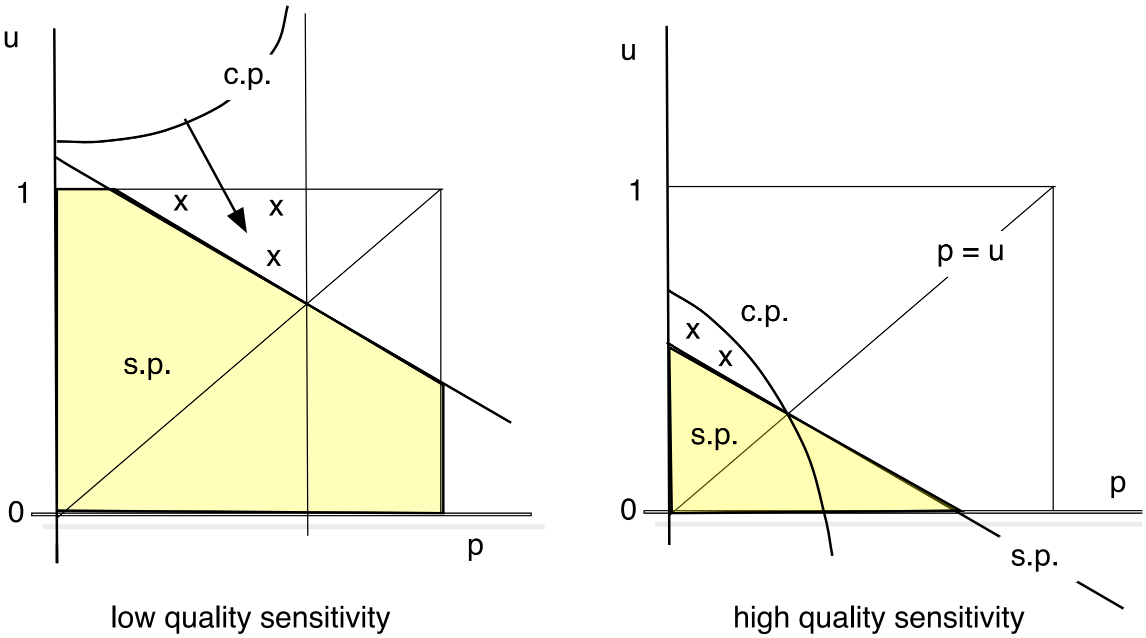

Figure 3: Feasible regions for single price and contingent price contracts when disutility from inferior quality is low (left) and high (right). The region marked with the x's indicates the combinations of p and μ values under which contingency pricing expands the likelihood of market existence.

are

	standard pricing	contingency pricing
expected price	$\frac{1}{2}(1+c-(\alpha+\delta)\mu)$	$\frac{1}{2}\left(1+c\frac{1-\mu}{1-p}-(\alpha+\delta)\mu\right)$
market coverage	$\frac{1}{2}-\frac{1}{2(1-\delta\mu)}(c+\alpha\mu)$	$\frac{1}{2}-\frac{1}{2(1-\delta\mu)}\left(c\frac{1-\mu}{1-p}+\alpha\mu\right)$

It can be seen that when $q > p$, then contingency pricing yields a lower expected price and increases market coverage. The increase in profit due to contingency pricing is

$$\frac{\mu-p}{4(1-\delta\mu)}\left(\frac{(1-(\alpha+\delta)\mu)^2}{1-q}-\frac{c^2}{1-p}\right)$$

which is positive (when $q > p$) so long as the market participation constraint is satisfied. Moreover, contingency pricing improves the *ex-post* total consumer surplus in the amount

$$\frac{(\mu-p)^2c^2}{8(1-p)^2(1-\delta\mu)}$$

demonstrating that contingency pricing improves efficiency, raising both consumer surplus and firm profits.

In the region where contingent pricing is preferred, it is interesting to examine how the fraction of buyers, firm's margin, and profit change as the public probability μ changes. We state the results below, omitting the proof.

Proposition 3 *When customers are highly quality sensitive ($\alpha + \delta > 1$), then the firm's optimal price, rebate, expected revenue, and margin decrease with an increase in μ, the market's estimate of firm mis-performance. When quality sensitivity is low (i.e., $\alpha + \delta < 1$), these outcomes increase with an increase in μ*

These results indicate that the firm does not always benefit or lose when the market's underestimate deviates further from the actual performance. When customers are highly sensitive to quality, then an increase in the belief about mis-performance hurts the firm, because customers willingness to pay is reduced. However, when they are less sensitive to quality, the firm can overcome the reduction in willingness to pay by offering a larger rebate: consumers have a greater expected value from this rebate because of the higher probability of mis-performance.

4 Contingency Pricing under Duopolistic Competition

We have demonstrated the value of contingency pricing for a monopoly firm, showing that it expands market share and leads to greater profits when the market underestimates the firm's performance. In this section, we discuss how contingency pricing affects competitive outcomes in a duopolistic setting. Suppose there are two firms of low and high quality L and H respectively. The higher quality firm has a lower probability of failing to meet any level of specified performance, thus $G_H(q) \leq G_L(q)$. The firms are identical in other respects such as consumer preferences, marginal costs and market estimation of their performance, which as before, we represent by the distribution $F(q)$. This framework applies to a scenario where two firms are employing a new technology into an established product; hence the market estimates reflect the historical common performance of the established product, but the high quality firm has a superior adaptation of the technology. As a specific example, consider competition between backbone internet service providers where latency is one of the measures of quality (the industry standard today is between 50 and 70ms roundtrip delays), and most firms have similar technologies; however, different adaptations of a new algorithm or faster switching hardware can cause a short-term difference in performance levels between competing firms. The market's best estimate may be identical for all firms, due to lack of distinguishing information.

As before, consumers have heterogeneous valuations, high-value consumers have a greater disutility for quality failure, and the pricing schemes are designed around an exogenously specified industry quality threshold $\hat{q}$. Publicly available information about performance probability is denoted by a probability of failure μ that is identical for both firms. Each firm i possesses private information on its own probability of failure p_i. Each firm chooses its pricing scheme with price R_i and rebate r_i to maximize its profits. As before, we write $\bar{R}_i$ and $\hat{R}_i$ to denote the consumers' and firm's price expectations. Consumers' purchase decision is based on both expected prices and additional attributes of each firm with respect to their preferences. Specifically, the firms would split the market equally if they were to offer the same expected price $\bar{R}$; in other words, the firms are, on the aggregate, identically situated with respect to consumer preferences. To make the problem interesting, we assume that the two firms cover the market. We seek to investigate optimal pricing strategy (whether to offer contingency pricing and how to set prices) for each firm and the impact of contingency pricing on market shares, profits and social welfare.

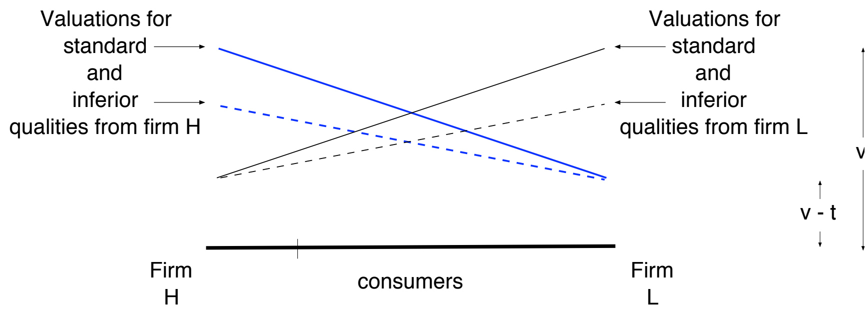

Figure 4: `Duopoly competition under quality uncertainty`

To model this sort of competition under quality uncertainty, imagine that the firms are engaged in spatial competition **?**. The standard-quality levels from the two firms are positioned on either end of the attribute space, whereas the customers are located uniformly along the segment. For either firm, customers incur misfit cost xt proportional to their distance x from the firm's product, yielding a net valuation $v - xt$ for standard quality. There is an additional misfit cost for inferior quality, given by $a + d(v - xt)$ so that the loss is monotonically increasing in proximity to the product, hence higher-value customers suffer a greater loss in valuation on receiving inferior quality. Here, a may be interpreted as a constant loss for all consumers, while d is the percentage loss in net valuation for customer at distance x. Figure 4 provides a graphical illustration.

Combining the valuations for standard and inferior quality, under a mis-performance estimate μ, the consumer at distance x from firm i has an expected surplus $(v-(a+dv)\mu) - t(1-\mu d)x) - \bar{R}_i$ on purchasing from firm i. We derive the indifferent consumer x by equating the surplus for the two firms. This generates the demand functions D_H and D_L (as a function of expected prices), where

$$D_i = D_i(\bar{R}_i, \bar{R}_j) = \frac{1}{2} - \frac{\bar{R}_i}{2t(1-\mu d)} + \frac{\bar{R}_j}{2t(1-\mu d)}$$

Firm i's profit function is $\pi_i = D_i(\hat{R}_i - \hat{C})$. It is easy to see that the firms capture equal

Firm L \ Firm H	**market overestimates performance**	**market *underestimates* performance**
market overestimates performance	L: standard **H: *standard***	L: standard **H: *contingent***
market *underestimates* performance	L: contingent **H: *standard***	L: contingent **H: *contingent***

Figure 5: `Optimal pricing strategies under competition`

market share if they offer identical expected prices; otherwise, the higher priced firm captures a smaller share of the market. Furthermore, we note that were the two firms both constrained to use only standard pricing, they would be equally competitive and capture equal market share and profits, even though firm H is, unknown to consumers, superior.

4.1 Standard or Contingency Pricing?

Let $\bar{R}_j$ denote the expected price of firm j. Firm i's pricing scheme is a best response to $\bar{R}_j$, regardless of whether firm j employs standard or contingency pricing. This follows easily from the nature of the demand function, since consumer reaction (and firm i's profit function) is on the competitor's expected price.

Let $\bar{R}_i$ denote any standard price chosen by firm i given $\bar{R}_j$, and let π_i be the corresponding profit. We construct a contingent price solution that generates the same expected price $\bar{R}^c_i = \bar{R}_i$ and show that the firm earns a greater margin, hence greater profits, under this solution. Specifically, let $R_i = \frac{\bar{R}_i}{1-\mu}$, which yields the same expected price to consumers. However, the firm's true expected price $\hat{R}_i = R_i \left(\frac{1-p_i}{1-\mu}\right)$ is greater than R_i when $\mu > p_i$, hence the firm earns a higher profit with contingency pricing when the market underestimates its performance. We have proved that

Proposition 4 *Contingency pricing is a dominant strategy for the firm when the market underestimates its performance.*

Each firm chooses contingency pricing when the market underestimates its own performance, and standard pricing when the market overestimates performance. Combining the behavior of both firms, Figure 5 displays the four possible scenarios regarding the optimal pricing schemes.

4.2 Price Response Functions

Firm i's prices are a best response to firm j's expected price $\bar{R}_j$, but its pricing strategy depends only on the market's under- or over-estimation of its own performance. We first determine the best-response pricing functions in each case and then compare the respective optimal prices.

Consider how the demand function for a firm changes with respect to its own price. Let D' denote the partial derivative of firm i's demand D_i with respect to its expected price $\bar{R}_i$, so that $D' = \frac{-1}{2t(1-\mu d)}$. Examining first-order conditions (see Appendix), the optimal lies on the boundary $r_i = 0$ or $r_i = R_i$, except when public information is perfect ($p = \mu$) in which case several combinations of price and rebate are optimal. Thus, *neither firm will offer less than full price rebate* should it choose contingency pricing, analogous to the result of Proposition 1.

Therefore we study the optimal pricing of each firm under the two special cases of standard pricing and full-rebate contingency pricing. Let $\Omega_i^c(\bar{R}_j)$ be firm i's best full-rebate contingency price response function to firm j's expected price. Note that the function returns the expected price seen by consumers, given the full-rebate pricing scheme employed by the firm. Similarly, $\Omega_i^s(\bar{R}_j)$ is firm i's best standard price response to $\bar{R}_j$.

For the case of standard pricing, let $\bar{R}_i^s$ be firm i's single price, and let π_i^s be the corresponding profit. As shown in the Appendix, the first-order condition yields the best response function $\Omega_i^s(\bar{R}_j)$ for firm i's price as

$$\Omega_i^s(\bar{R}_j) = \frac{\hat{C}}{2} + \frac{1}{2}(\bar{R}_j + (1 - \delta\mu)t) \tag{3}$$

To examine the best contingency price response, let R_i^c be the contingent price (and rebate) for firm i, so that the market expectation of the firm's price is $\bar{R}_i^c = R_i^c(1 - \mu)$ while its true expected price is $\hat{R}_i^c = R_i^c(1 - p)$. The first-order condition yields the price response function (see Appendix)

$$\Omega_i^c(\bar{R}_j) = \frac{\hat{C}}{2}\frac{1 - \mu}{1 - p_i} + \frac{1}{2}(\bar{R}_j + (1 - \delta\mu)t) \tag{4}$$

Since $q > p$ we can see by comparing the RHS of Equations 3 and 4, that

Proposition 5 *Firm i's contingent price response (to any expected price $\bar{R}_j$ of the competing firm) always yields a lower expected price to the consumer than its best response under single pricing, when the market underestimates firm i's performance. Formally, $\Omega_i^c < \Omega_i^s$ when $\mu > p_i$*

Figure 6 provides a graphical illustration. This result generalizes the monopoly result that the optimal expected price is lower under contingency pricing when the market underestimates the firm's performance.

Equilibrium Prices and Outcomes under Market Underestimation We focus now on the case where the market underestimates both firms' performance, ignoring the less interesting cases where the market overestimates performance (where the firm chooses standard pricing). Under this case, both firms will independently prefer to offer full-rebate contingency pricing. The equilibrium prices are obtained at the intersection of the two firms'

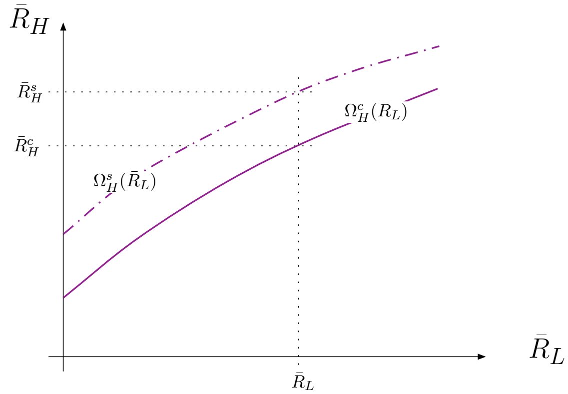

Figure 6: Price response functions under standard and contingency pricing.

best-response functions $\Omega_i^c(\bar{R}_j)$ given by the respective realizations of Eq. 4. A unique intersection is ensured under non-decreasing price elasticity, so that the optimal values of $\bar{R}_H^c$ and $\bar{R}_L^c$ are

$$\bar{R}_H^c = \frac{c(1-\mu)}{3(1-p_L)} + \frac{2c(1-\mu)}{3(1-p_H)} - (t - \mu d) \quad (5)$$

$$\bar{R}_L^c = \frac{c(1-\mu)}{3(1-p_H)} + \frac{2c(1-\mu)}{3(1-p_L)} - (t - \mu d) \quad (6)$$

Under the monopoly analysis (§3) we demonstrated that contingency pricing is a useful pricing instrument for the firm, and helps it improve both profits and market coverage. Now we examine a similar question when contingency pricing is employed by two competing firms. Proposition 4 demonstrated that each firm benefits from contingency pricing when the market underestimates its own performance. But, when the firms are heterogeneous, which one does better under contingency pricing? Obviously, both firms cannot improve market coverage with contingency pricing, since this is now a zero-sum game. Will firm H gain market share at the expense of L? Or will it prefer to improve its margin instead? How do their prices compare? Does contingency pricing improve both firm's profits?

Proposition 6 *Under contingency pricing, the optimal expected price $\bar{R}_H^c$ of the higher quality firm is lower than the optimal expected price $\bar{R}_L^c$ of the lower quality firm. Hence firm H*

captures a greater market share under contingency pricing.

In examining its optimal contingency prices under market underestimation, each firm sees an advantage in luring customers with a lower expected price since it is aware that the true expected cost of rebate is lower. Since firm H is superior in this respect, it is better able to deploy this tool and therefore captures (at the expense of firm L) a greater market share in equilibrium. Formally, the result follows quite simply from Eq. 5 and Eq. 6, since $p_H < p_L$.

Next we see that, despite offering the lower expected price, the superior firm H makes greater profit than firm L.

Proposition 7 *The superior firm H makes greater profit than firm L when $\mu > p_L > p_H$.*

Proof. Let $\bar{R}^c$ be the expected price of firm L under any full-rebate contingent price solution. Then its true expected margin is $\bar{R}^c \frac{1-p_L}{1-\mu} - \hat{C}$. For firm H, construct a full-rebate solution with the same expected price $\bar{R}^c$. Hence the two firms gain equal market share under these prices. However, firm H has a greater expected margin ($\bar{R}^c \frac{1-p_H}{1-\mu} - \hat{C}$) since $1 - p_H > 1 - p_L$. Since this is true for any solution, it also holds for the equilibrium prices. ■

Contingency pricing confers competitive advantage to the superior firm. While both firms benefit from the use of contingency pricing, the superior firm does it better. This contrasts with the benchmark case of standard pricing where both firms would earn equal profit, since the better performance would make no difference under standard pricing (when the market estimation of their performance is identical). Finally, the gain in consumer surplus when both firms optimally use contingency pricing, compared with if they were constrained to use standard pricing, is positive

$$\Delta(\text{consumer surplus}) = \frac{c}{2}\left(\frac{\mu - p_H}{1 - p_H} + \frac{\mu - p_L}{1 - p_L}\right) + \frac{c^2(p_L - p_H)^2(1-\mu)^2}{36(1-\delta\mu)t(1-p_H)^2(1-p_L)^2}$$

again demonstrating that contingency pricing is not only profit-maximizing but also efficient, a better way to trade when there are differences in belief.

5 Conclusion

We have presented in this paper the value of contingent price contracts in expanding commerce by mitigating the effects of quality uncertainty. When quality is realized only upon delivery or use, and is objectively measurable, contingent contracts become possible. Our analysis demonstrates the critical role of performance probabilities and private information in this framework. Contingency pricing is attractive when the firm has private information about performance (specifically when it expects to perform better than perceived by the market), both increasing profits and enabling market existence in situations where no single-price contract could provide positive social welfare. Interestingly, in this case, the firm offers a full-price rebate for inferior quality. When public performance information mirrors the firm's actual performance, the firm derives no direct economic benefit from offering contingent contracts, but may benefit due to perceptions of fairness.

Under duopolistic competition, each firm's optimal choice regarding the use of contingency pricing depends only on whether the market underestimates its own performance. When the market underestimates both firms—the scenario examined in this article—then both employ contingency pricing. The superior firm, however, derives a competitive advantage, gaining market share and a greater increases in profits due to contingency pricing. Compared to the case of standard pricing, where both firms are equally competitive in the market, contigency pricing therefore decreases competitive intensity. Despite the decrease, contingency pricing improves consumer surplus, because the superior firm can signal its better performance thereby attracting more buyers.

These results provide actionable recommendations to managers, especially since the information infrastructure and information-content of the relevant quality metrics allows implementation of the contingency pricing framework. We find that contingency pricing is relevant, applicable and implementable in many IT-intensive business contexts such as electronic commerce, information goods, online transactional services, telecommunications, and IT infrastructure services. Private information has a crucial role in making contingency pricing attractive, and the value of contingency pricing increases when the firm is more confident, relative to the market, about its performance. The differences in public and private information can be very marked for the IT sectors where new technologies are deployed at a rapid pace and new firms are constantly emerging with new products. Since buyers are unfamiliar with these new offerings and new firms, contingency pricing can serve as effective signalling mechanism for new entrants.

We have not investigated the case where the firm's information itself may be inaccurate, but consider it a useful avenue for future research. A related topic for future work concerns the detailed analysis of signalling that a firm may send out by offering different types of contracts. Other topics for future research include the role of contracts under competition, multi-part contingent contracts, menu of contingent contracts (contingency pricing and price discrimination), and empirical studies of consumer response to contingency pricing.

A Detailed Derivations and Proofs: Monopoly

Proof of Proposition 1. The firm's optimization problem is to choose price and rebate (R, r) to maximize $\pi = D(\bar{R}).(\hat{R} - \hat{C})$ Recall that $\hat{R} = R - rp$ and $\bar{R} = R - r\mu$. First order conditions with respect to R and r yield

$$\begin{aligned} \frac{\partial \pi}{\partial R} &= (\hat{R} - \hat{C})D'(\bar{R}) + D(\bar{R}) \\ \frac{\partial \pi}{\partial r} &= -\mu(\hat{R} - \hat{C})D'(\bar{R}) - pD(\bar{R}) \end{aligned}$$

When $p \neq \mu$ the stationary point yields a zero-profit solution (with $D = 0$ or $\hat{R} = \hat{C}$), hence the optimal must lie at one of the boundaries in this case. For the public information problem where $p = \mu$, the two first order conditions reduce to a single relation

$$(\bar{R} - \hat{C})D'(\bar{R}) + D(\bar{R} = 0$$

The above equation can be rewritten such that the optimal single price $\bar{R}$ satisfies the condition

$$\frac{-D(R)}{R \cdot D'(R)} = \frac{R - \hat{C}}{R} \tag{7}$$

This condition guarantees a unique solution for $\bar{R}$ under non-decreasing price elasticity of demand. However, as indicated above, this effective expected price can be achieved by many price-rebate combinations including the two extreme cases of $r = 0$ and $r = R$, and all convex combinations of these two solutions. ■

A.1 Candidate Optima under Private Information

Proof of Lemma 1.

We examine the two candidate optima at the boundaries $r = 0$ and $r = R$

No Rebate Profit Function Setting $r = 0$, the firm's problem is to choose R to maximize

$$\pi^s = (R - \hat{C})D(R)$$

The first order condition is

$$(R - \hat{C})D'(R) + D(R) = 0$$

Rewriting this equation, the optimal single price R^s is the unique solution to the equation

$$\frac{-D(R)}{R \cdot D'(R)} = \frac{R - \hat{C}}{R} \tag{8}$$

We will show below that the profit function is concave in R and hence this solution is optimal.

Concavity of profit function under non-decreasing price elasticity. The assumption of non-decreasing price elasticity of demand, formally, is $\frac{\partial}{\partial x} \frac{-xD'(x)}{D(x)} > 0$ which, after computing and rearranging the terms, simplies to

$$xD''(x) < \left[\frac{-xD'(x)}{D(x)}\right](-D'(x)) - D'(x) \tag{9}$$

Substituting for the price elasticity term in the square brackets on the right hand side of Eq. 9 from the first order condition in Eq. 8 and expressing in terms of R, we get

$$RD''(R) < \left(\frac{R}{R - \hat{C}}\right)(-D'(R)) - D'(R)$$

which can be rearranged as

$$(R - \hat{C})D''(R) + 2D'(R) < \frac{\hat{C}}{R}D'(R)) \tag{10}$$

Concavity of the profit function requires that the second derivative $\frac{\partial^2 \pi}{\partial R^2} = (\hat{R} - \hat{C})D''(R) + 2D'(R)$ is negative. Since we know that $D'(R)$ is negative, Eq. 10 guarantees that the second derivative is indeed negative.

Full Rebate Profit Function With $r = R$, the firm's problem is to choose R to maximize

$$\pi^c = (R(1-p) - \hat{C})D(\hat{R})$$

The first order condition

$$\frac{\partial \pi}{\partial R} = (1-\mu)(R(1-p) - \hat{C})D'(\bar{R}) + (1-p)D(\bar{R}) = 0$$

and rewriting this equation, the optimal single price R^c is the unique solution to the equation

$$\frac{-D(\bar{R})}{\bar{R} \cdot D'(\bar{R})} = \left(\frac{1-p}{1-\mu}\right)\left(\frac{\bar{R} - \left(\frac{1-\mu}{1-p}\right)\hat{C}}{\bar{R}}\right) = \left(\frac{\hat{R}^c - \hat{C}}{\hat{R}^c}\right) \tag{11}$$

Once again, we will show that the second derivative $\frac{\partial^2 \pi}{\partial R^2} = (1-\mu)^2 \left[(\hat{R} - \hat{C})D''(\bar{R}) + 2\frac{1-p}{1-\mu}D'(\bar{R})\right]$ is negative to verify the concavity of the profit function. As before, substituting the expression for the inverse elasticity of demand from the first order condition 11 in Eq. 9, expressing in terms of R and rearranging terms, we get

$$(\hat{R} - \hat{C})D''(\bar{R}) + 2\frac{1-p}{1-\mu}D'(\bar{R}) < \frac{\hat{C}}{\bar{R}}D'(\bar{R})) \tag{12}$$

which implies the second-order condition required since $D'(\bar{R})$ is negative.

■

A.2 Duopoly

Price Response Functions The derivatives with respect to a firm's announced price and rebates are $\frac{\partial D_i}{\partial R_i} = D'$ and $\frac{\partial D_i}{\partial r_i} = -\mu D'$. Computing first derivatives for the profit function of firm i, with respect to its announced price and rebate, given an expected price $\bar{R}_j$ from firm j,

$$\frac{\partial \pi_i}{\partial R_i} = D'(\hat{R}_i - \hat{C}) + D_i \tag{13}$$

$$\frac{\partial \pi_i}{\partial r_i} = -\mu D'(\hat{R}_i - \hat{C}) - p_i D_i \tag{14}$$

Comparing the two first order conditions, we see that the optimal lies on the boundary $r_i = 0$ or $r_i = R_i$, except when public information is perfect ($p = \mu$) in which case several combinations of price and rebate are optimal.

Price response functions under standard pricing Under standard pricing with r=0, firm i's problem is to choose R_i to maximize $\pi_i = (R_i - \hat{C})D_i$. After substituting for the demand function, the first order condition is

$$\frac{\partial \pi_i}{\partial R_i} = \frac{1}{2(1-\delta\mu)t}\left[\hat{C} - \frac{1-p_i}{1-\mu}(2\bar{R}_i - \bar{R}_j - (1-\delta\mu)t)\right] = 0$$

and it is clear that the second derivative is negative. Solving the FOC for $\bar{R}_i$, the optimal single best response price Ω_i^s is

$$\Omega_i^s(\bar{R}_j) = \frac{\hat{C}}{2} + \frac{1}{2}(\bar{R}_j + (1-\delta\mu)t) \tag{15}$$

Price response functions under contingency pricing Under full rebate, firm i's problem is to choose R_i to maximize $\pi_i = (\hat{R}_i - \hat{C})D_i$. The first order condition is

$$\frac{\partial \pi_i}{\partial R_i} = \frac{1}{2(1-\delta\mu)t}\left[\hat{C} - \frac{1-p_i}{1-\mu}(2\bar{R}_i - \bar{R}_j - (1-\delta\mu)t)\right] = 0$$

and again the second derivative is negative. Solving the FOC for $\bar{R}_i$, the optimal single best response price Ω_i^c is

$$\Omega_i^c(\bar{R}_j) = \frac{\hat{C}}{2}\frac{1-\mu}{1-p_i} + \frac{1}{2}(\bar{R}_j + (1-\delta\mu)t) \tag{16}$$