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Auctions of Homogeneous Goods with Increasing Returns:

Experimental Comparison of Alternative "Dutch" Auctions

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Multi-unit auctions of goods that may have increasing returns to scale—i.e. goods such that bidders may value multiple units at a higher unit price than single units—present challenges for both auctioneers and bidders. We compare two commonly used auction formats for selling multiple homogeneous objects, both sometimes called "Dutch" auctions, in a set of value environments that potentially subject bidders to the "exposure" and "free riding" problems. We find that overall the descending price auction, best known for its use in the Dutch flower auctions, is robust and performs well in a variety of environments, although there are some situations in which the ascending uniform-price auction similar to the one used by internet auctions such as eBay, better avoids the free riding problem. We discuss the factors that influence each mechanism's performance in terms of the overall efficiency, the informational requirements, the seller's revenue, and the buyer's profit.

Keywords: Multi-Unit Auctions, Experimental Economics

JEL Classifications: D44, C91

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1. Introduction

Three mechanisms are commonly used to sell multiple homogeneous objects. Although all three are sometimes called "Dutch," the mechanisms are quite different. Perhaps the best known example of an institution specifically geared to selling homogeneous goods is the flower auction in Aalsmeer, Holland, where approximately 4 billion flowers and 400 million plants worth over 2 billion Dutch Guilders are sold by about 8000 different sellers annually. Close to 2000 buyers trade in the auction every day, completing about 50,000 transactions between 6:30 AM and 10:15 AM. (Gerard J. Van den Berg et al. 2001). The mechanism used is sometimes called a *descending*, or *reverse clock* auction because the price descends until a bidder decides to accept the current price by stopping the auction and buying all or part of the lot at that price per unit for as many units as desired.[1] If a part of the lot remains, the price goes back up, and the process is repeated until either the entire lot is sold or the price reaches its minimum level. If the price falls to the minimum the remainder of the lot is destroyed.

The Internet auction house eBay sells multiple units using a different mechanism it calls "Dutch." The eBay version is a uniform price ascending auction with a fixed end time, in which buyers bid by entering a price and a quantity, and the units are sold to the bidders with the highest prices whose quantities add up to the amount available. All bidders pay the price of the bidder who bid the lowest winning price, and that bidder is not guaranteed the entire quantity demanded. (In case of tie bids, priority is given to the bidder who demanded the greater quantity, and when bids are tied in both price and quantity, the earlier bid wins.)

The third "Dutch" mechanism is often used for common stock tender offers, in which a firm buys back its own shares by announcing a price range and the number of shares it wishes to repurchase (Ronald W. Best et al., 1998). Shareholders interested in selling back their shares specify the lowest price within the range at which they are willing to sell, as well as the quantity. The firm selects the price at which it can repurchase the required number of shares, and all transactions are conducted at this price.

The eBay and the tender offer mechanisms share two characteristics: they are both uniform price mechanisms, and neither guarantees a winning bidder the entire quantity on which

[1] Price is per stem, units are boxes of a certain number of stems, and there may be a minimum quantity of boxes required of any bid.

he bid[2]. In contrast, the Dutch descending auction does not yield a uniform price, but it does guarantee that the winning bidder receives the entire quantity demanded. That is, in the descending price auction, a bidder who stops the clock obtains the full quantity on which he bids, at the price at which he stopped the auction. Thus the descending Dutch auction is a simple version of a package-bidding (or "combinatorial") auction (see for example Lawrence M. Ausubel and Paul Milgrom 2002 for a discussion of combinatorial auctions and package bidding), in that, for homogeneous goods, it allows bidders to submit bids on any relevant package of items, i.e. on packages of any size, from a single unit to the entire quantity available at the time of the bid.

When bidders' preferences include complementarities such as increasing returns to scale, most simple mechanisms that do not allow package bidding subject bidders to the *exposure problem*. The exposure problem, in the case of homogeneous goods, occurs when a bidder bids on a specific (large) quantity at a price per unit that exceeds his unit value for a smaller number of units. In this case a mechanism, like the eBay and tender offer mechanisms, that does not guarantee a winning bidder the desired quantity, can leave a bidder "exposed" to potential losses.

While the descending price Dutch auction has received a good deal of attention in the economics literature, this has focused almost exclusively on auctions of a single item[3], and as far as we know its package bidding feature has not previously been studied[4]

As we will discuss below, the package bidding feature of the descending price auction (i.e. the fact that bidders can bid on any quantity) implies that bidders never suffer from the exposure problem in auctions of a homogeneous good. Part of the descending Dutch auction mechanism's success in the Aalsmeer Flower Auction may have to do with the fact that it protects buyers of large quantities from the exposure problem. The total number of buyers at

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[2] eBay has recently added a provision allowing the marginal bidder to withdraw a bid when the entire quantity desired is unavailable. Presumably the unsold units can then be sold off line, or in a subsequent auction.

[3] For auctions of a single item, William Vickrey (1961) shows that the descending price Dutch auction is strategically equivalent to the sealed bid auction. James Cox et al. (1982) report that in laboratory experiments the descending price Dutch auction generates lower revenues than the sealed bid auction. David H. Lucking-Reiley (1999) reports the opposite result in a field experiment conducted over the Internet (in this case the descending price Dutch auction generated higher revenues than the sealed bid auction). Elena Katok and Anthony Kwasnica (2002) demonstrate that the clock speed in a descending Dutch auction has an effect on revenue in laboratory experiments—fast clock mechanisms generate lower revenues than slow clock mechanisms do—and offer a theoretical explanation. Octavian Carare and Michael Rothkopf (2001) describe a decision theoretic model of a slow Dutch auction.

[4] Many studies of the descending price auction focus in fact on the case of a single unit for sale, or multiple units in an environment in which no bidder desires more than one (see for example Kevin McCabe et al. 1990).

Aalsmeer is close to 2,000 per year, but only approximately 50 buyers are responsible for about 50% of the total volume in guilders, while about 700 smallest buyers are responsible for less than 1% of the total volume (see Gerald J. van den Berg et al., 2001).

Other advantages of the descending auction mechanism include speed (over 50,000 transactions are completed in Aalsmeer daily in less than 4 hours), and the small amount of information it reveals[5]. The descending mechanism also allows bidders to get "immediate gratification," meaning that the winning bidder knows immediately that he won, and in the case of multi-unit auctions, he knows that he won exactly the number of units he wanted.

In spite of its many attractive properties, the descending auction is not very common in practice[6]. A reverse clock mechanism requires the knowledge of a good upper bound on the eventual purchase price, to set the starting price on the clock. If this price is too low, the seller runs the danger of selling the item immediately and foregoing potential revenues, but if this price is set too high, the auction may take an unnecessarily long time. This requirement makes descending auctions more appropriate for selling commodity-like objects (flowers) than unique objects (art). Another potential disadvantage of fast descending auctions is that they require bidders to assemble at a specific time and place.

The potential downside of the descending price Dutch auctions on which we specifically focus in this paper, is that it can leave the bidders open to a different problem that has been discussed in the context of package bidding—the *free rider* (or "threshold") problem. This problem occurs when the efficient allocation requires a group of (small) bidders to coordinate in order to beat a (big) bidder who has a disproportionately high value for a large quantity. If enough bidders purchase units early at high prices so that the remaining quantity is less than the large bidder requires, the large bidder stops being competitive, and any remaining bidders could potentially transact at a very low price. This situation may create an incentive among the small

[5] The only information it reveals is the winning price, and since bidders pay what they bid, this winning price is only a lower bound on the bidder's true valuation. The downside of this low information feature is that the mechanism may be more prone to errors, especially when bidders are unsure of their true valuation.

[6] Although a variation of the slow reverse clock descending mechanism may be an appropriate model for retailers and clearance sales (Edward P. Lazear, 1986). A well-known retail use of a slow Dutch auction explicitly is Filene's, which runs a slow reverse clock auction in the basement (called Filene's Basement). Items in the basement are discounted by 25% of the original price every two weeks. Other retailers use a form of a slow reverse clock auction also, and although they do not discount according to a pre-specified schedule, they do discount merchandise after it has not been sold for a period of time.

bidders to wait for other small bidders to buy at high prices, and if too many of them wait too long, the large bidder may purchase the units even when this results in an inefficient allocation.

In this paper, we investigate the performance of the descending price Dutch and eBay's uniform-price ascending "Dutch" auctions in situations in which bidder preferences include complementarities. We test the mechanisms in the laboratory on a set of environments that include the potential for the exposure and the free rider problems, thus creating a substantial challenge for both mechanisms. We find that the descending price Dutch mechanism is quite robust, and performs well even in situations in which the potential for free riding is present. The ascending mechanism performs less well in environments in which the exposure problem exists, and it also sometimes allows the exposure problem to appear in situations in which it would not be an issue if all players played equilibrium strategies of the isolated auction. However we also identify environments, in which the free rider problem is pronounced, in which the ascending auction produces efficient outcomes more often than the Dutch auction.

Related literature:

The Descending auction mechanism has not been explicitly studied in the context of selling multiple homogeneous items in environments with complementarities. David H. Lucking-Reiley 1999 conducted a field experiment comparing revenue in four auction mechanisms (English, First Price Sealed Bid, Second Price Sealed Bid, and Descending). The objects for sale in the field experiment were Magic™ Cards (which can be viewed as semi-homogeneous objects) and a large number of cards were auctioned off simultaneously.

Ascending and sealed bid auction mechanisms for selling homogeneous objects have been studied extensively. Lawrence Ausubel and Peter Crampton (1998) showed that the sealed bid uniform price auction mechanism is vulnerable to the "demand reduction" problem—potentially reducing competition and having a negative impact on seller's revenue. They also showed that often the uniform-price auction does not perform as well as a mechanism that permits price discrimination. Of course there may be practical reasons to prefer a uniform price mechanism, since price discrimination is often viewed as being unfair. Richard Engelbrecht-Wiggans and Charles M. Kahn (1998) characterize equilibria that involve demand reduction for the uniform price auction in the case that bidders demand at most two units.

A major focus of the empirical literature on auctions for multiple homogeneous units has been the so-called *declining price anomaly*. When identical units are sold sequentially in practice, units sold earlier tend to be purchased at higher prices. This phenomenon appears to be robust to the auction format or the type of object being sold. Orley Ashenfelter (1989) documents the declining price anomaly in ascending auctions for wine. Gerard J. van den Berg et al. (2001) document the same phenomenon in descending Dutch auctions for roses at Aalsmeer. They also report that "at any round, the decline is stronger if the number of remaining units is smaller." This finding is suggestive of increasing returns.

John A. List and David H. Lucking-Reiley (2000) use a field experiment to study demand reduction in uniform and Vickrey (sealed-bid second-price) auctions. They sell sports cards (football, basketball and baseball) in 2-person markets and find that, consistent with the theory, demand reduction is higher in the uniform price auctions relatively to the Vickrey auctions. They also find that, contrary to the theory, the bids on the first unit are higher in the uniform treatment than in the Vickrey treatment, and there are no significant differences in revenue between the two mechanisms. The field experiment setting does not provide a way for comparing efficiency. John H. Kagel and Dan Levin 2001 also find significant demand reduction in the laboratory, in both, sealed bid and ascending auctions.

The experimental study most closely related to ours is by John H. Kagel and Dan Levin 2000. They compare the performance of the sealed bid uniform price and the uniform price ascending multi-unit auctions when bidders have synergies (complementarities) between the units. They conduct the auctions in four different environments, three with the exposure problem. Two units are sold, and each market involves either four or six bidders: one human bidder with synergies (the *Big* bidder) faces three of the four environments in each auction, and either three or five computerized bidders who demand a single unit (*Small* bidders) only, have their valuations drawn from a uniform distribution, and always follow the strategy of bidding up to their values. Kagel and Levin find that the ascending mechanism generally works better than the sealed bid, in terms of the frequency of the optimal bidding behavior, and participants' profit, and consistently, the revenues are higher in the sealed bid auctions. A major finding is that subjects in ascending auctions tend to bid too timidly in response to the exposure problem, but this does not happen in sealed bid auctions. Bidders do learn to bid correctly in the environment without the exposure problem, and the learning is faster in ascending auctions than in the sealed bid.

Another related study, by Fevrier, Linnemer, and Visser (2002), compares sequential auctions of two items with and without a "buyers option" for the winner of the first auction to buy both units. They examine multiple auction formats and value environments, including increasing returns, in auctions with exactly two bidders, in which the free rider problem can therefore never arise.

2. Some simple theory of ascending and descending auctions for multiple units of a homogeneous good with increasing returns:

The problems arising from increasing returns can be illustrated when there are two units for sale, and at least three bidders, B, s_1 and s_2. (The generalization to more goods and bidders is straightforward because the good is homogeneous, so only the number of units is important in determining the value.)

Suppose that bidder B has increasing returns to scale; i.e. $v_B(2) > 2v_B(1)$, where $v_B(k)$ is the total value to B of k units. And suppose that Bidders s_1 and s_2 want only a single unit: $v_{si}(2) = v_{si}(1)$ $i = 1,2$.

Bidder B faces the *exposure problem* when, in attempting to win two units at a unit price greater than $v_B(1)$, he runs the risk of acquiring only one unit at that price, and consequently making a loss. If bidder B is deterred by this risk from bidding up to a unit price of $v_B(2)/2$ for two units, then the auction may result in an inefficient outcome whenever $v_{s1}(1) + v_{s2}(1) < v_B(2)$ but the small bidders nevertheless win the auction due to B's reluctance to bid up to $v_B(2)$.

Bidders s_1 and s_2 suffer from the *free rider* or *threshold* problem whenever there is an opportunity to make a profit by bidding on one unit and winning it, but there is a possibility of making a larger profit by bidding less, and hoping that the other small bidder wins one unit at a high price, thus reducing the unit value of the remaining unit to the big bidder, and allowing the second small bidder to win a unit at a lower price. If either small bidder is deterred from bidding near $v_{si}(1)$ for this reason, then the descending price auction may result in an inefficient outcome whenever $v_{s1}(1) + v_{s2}(1) > v_B(2)$, but the big bidder nevertheless wins both units, because each small bidder hoped that the other would win one of the units first.

In the uniform-price ascending auction, B suffers from the exposure problem when the current bid price per unit exceeds $v_B(1)$, but is less than $v_B(2)/2$, and 2 units are available. If B makes a bid of e per unit for 2 units, such that $v_B(1) < e < v_B(2)/2$, then if another bidder makes a bid for 1 unit at a price $f > [v_B(2) + (e\text{-}v_B(1)/2]$, B's best response will be not to bid any further. In this case, if no more bids are made, the auction will end with B winning 1 unit at a price of e, and making a loss of $v_B(1) - e$. That is, in making the attempt to win 2 units at the potentially profitable price of e such that $v_B(1) < e < v_B(2)/2$, bidder B is exposed to the risk of winning only 1 unit, and making a loss.

Proposition 1 (Exposure problem): In a descending price Dutch auction for multiple units of a homogeneous good, a bidder B who wishes to buy more than one unit never faces the exposure problem.

Proof: At any moment in the descending price auction when more than one unit is still available, B can purchase up to the number available at the current price. So B never wins fewer units than he bid for. ■

Proposition 2 (No Strategic Equivalence): The descending price Dutch auction for multiple units is not strategically equivalent to the uniform-price sealed-bid auction in which the lowest winning bidder may win only part of the quantity he bid on. Neither is it equivalent to the first-price sealed bid auction (despite their strategic equivalence when only one unit is to be purchased).

Proof: A bidder who bids on more than one unit can suffer the exposure problem in the uniform-price sealed bid auction, but not in the descending Dutch auction. And once e.g. bidder s_1 buys a unit in the descending price auction, s_2 can wait to bid until the clock nears the big bidder's reservation price for one unit, whereas in the sealed bid first price auction he cannot condition his bid on that of the other small bidder. ■

However, while the descending price Dutch auction eliminates one problem faced by bidders in uniform price auctions, it raises another.

In the descending price Dutch auction, bidders s_1 and s_2 face the *free rider* problem whenever $v_{si}(1) > v_B(2)/2$ for both $i = 1,2$. For prices p such that $v_B(2)/2 < p < v_{si}(1)$, either bidder s_i could profitably stop the clock just before the price reaches $v_B(2)/2$, and win one unit at a profit of $v_{si}(1) - v_B(2)/2$. If one of the small bidders does so, then the unit value to the large bidder is immediately reduced to $v_B(1)$, allowing the second small bidder *j* to guarantee himself a profit of $v_{sj}(1) - v_B(1) > v_{sj}(1) - v_B(2)/2$. Consequently it is more profitable to let the other small bidder stop the clock. If both bidders delay until the price has dropped below $v_B(2)/2$, then bidder *B* may profitably bid on both units. The outcome of the auction will be inefficient if the free rider problem causes the small bidders to defer bidding until the large bidder jumps in, when $v_B(2) < v_{s1}(1) + v_{s2}(1)$.

Proposition 3 (Free rider problem): The free rider problem cannot occur in the uniform price ascending auction (or in any uniform price auction).

Proof: Since the auction is uniform price, no small bidder can obtain a unit at a lower price than any other small bidder. ■

So far we have discussed only the *structural* properties of the auctions, i.e. the properties concerning which outcomes and strategies are feasible. After introducing the particular auction environments used in the experiment, we'll consider, in section 4, the equilibrium predictions.

3. Design of the Experiment

Our experimental comparison of descending and ascending auctions is similar to Kagel and Levin's comparison of uniform price sealed bid and ascending auctions, except that, to investigate behavior of small bidders, we use all human bidders. We also manipulate the *Small* bidders' values to create environments with the free riding problem.

Our design manipulates two factors: the auction mechanism and the distribution and correlation of bidder values. In all auctions three bidders compete for two units of an artificial asset. The *Big* bidder always has a high value for both units, but only if he wins both, and a low value for one unit. The two *Small* bidders each want one unit of the asset and their values can be either high or low.

- In the *Exposure* environment the Big bidder's value for one unit is 20 with probability .75, and is 40 with probability .25. His value for two units is 100 or 120 (unit value of 50 or 60) with equal probability. The Small bidders' values are correlated: 33% of the time they are both low (either 35 or 45 with equal probability, independently drawn), and 67%

of the time one is high (65 or 75 with equal probability) and the other is low (35 or 45 with equal probability).

- The only difference between the Exposure and the *Free Riding* environments is that in the Free Riding environment the two small bidders' values are either both low or both high—they are both low 33% of the time and are both high 67% of the time—conditional on being low or high, each bidder has an equal probability of having a value of 35 or 45 (if low) or 65 or 75 (if high). This implies that at equilibrium there is no danger of the exposure problem in the free riding environment, since a big bidder who is outbid on one unit can expect to be outbid on both.
- The only difference between the Free Riding and the *Super Free Riding* environments is that the Big bidder has a value of zero for one unit in the Super Free Riding environment. This makes the free riding problem for Small bidders worse because the potential benefit from free riding successfully is higher.

All information about value distributions is public, and the roles, Big or Small bidder, do not change for the duration of the session. Figure 1 summarizes the three environments in our experiment.

During each session participants are matched in groups of 6 (we call each group of 6 a *cohort*). Each cohort participates in a sequence of 20 auctions, with two separate groups of three bidders bidding in each round. After each round the participants are randomly re-matched within the cohort, in a way that no participant is matched with the same two participants for two consecutive auctions. The participants are told this[7]. The two auction mechanisms we study are the descending auction and an adaptation of the eBay's ascending auction.

- In the *Descending* mechanism the price starts at 85 and goes down by 0.5 every second. To purchase one or two units a participant clicks on a button. Big bidders have two buttons (for 1 and for 2 units) and small bidders have only one button. If a big bidder buys two units the auction ends. If a small bidder buys a unit (or if a big bidder buys one unit) the price of the remaining unit goes back up to 85[8] and the process repeats. Once a bidder stops the clock (wins a unit) he cannot stop the clock again, and if only one unit is available because one unit was already purchased, the big bidder received an error message if he tries to purchase two units.

[7] See complete instructions, including screen shots, at http://lema.smeal.psu.edu/katok/katok_roth_instructions.pdf.

[8] In the *Super Free Riding* environment the price did not go back up to 85 but instead it continued to go down. This was done, following a pilot experiment, to save time, since the remaining unit usually was sold for 0.5 or 1 token.

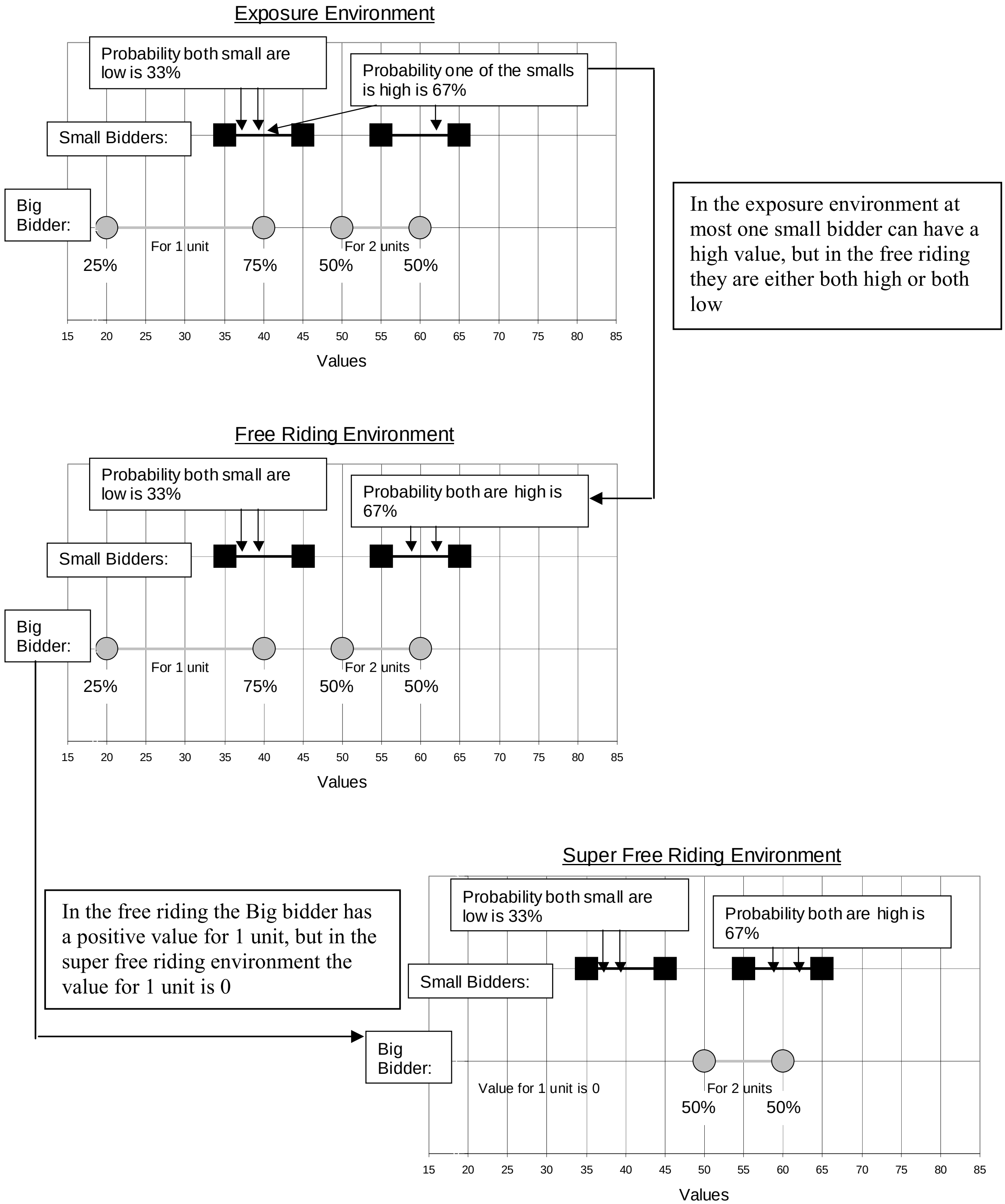


Figure 1: Distribution of bidder values in the three environments.

- In the *Ascending* mechanism the price starts at 10 and increases by 0.5 every second. To bid in the auction the bidders do nothing, but to stop bidding a bidder clicks a button. Once a bidder clicks a button he becomes ineligible for the duration of the auction. The auction ends when the total number of units demanded by the eligible bidders falls to (or below) two. When this happens the eligible bidder(s) win(s) units at the highest price that was reached before the auction ended, i.e. at the last price before he dropped out. The remaining unit (if any) goes to the bidder who became ineligible last, at a price 0.5 below the selling price for the winning bidder. As in eBay, ties are broken based on quantity. The exposure problem is realized when one of the small bidders becomes ineligible early, and the Big bidder becomes ineligible next. In this case, the small bidder who did nothing wins one unit, and the big bidder wins the other unit and possibly loses money. Figure 2 summarizes our experimental design.

	Mechanism	
Environment	Ascending	Descending
Exposure	5 cohorts = 30 subjects	5 cohorts = 30 subjects
Free Riding	4 cohorts = 24 subjects	5 cohorts = 30 subjects
Super Free Riding	4 cohorts = 24 subjects	4 cohorts = 24 subjects

The study contained 162 subjects. Each cohort (6 subjects) participated in 20 rounds. Every round contained two separate auctions with three subjects in each auctions. Each subject had the same role (Big or Small bidder) in every auction. No subject was matched with the same 2 subjects for two consecutive periods.

Figure 2: Experimental design.

All sessions were conducted at Harvard Business School's Computer Laboratory for Experimental Research (CLER) between August 2001 and October 2001. Participants were recruited through flyers posted on billboards. Cash was the only incentive offered. Participants were paid their total individual earnings from the 20 auctions plus a $10 show-up fee at the end of the session[9]. The software was built using the zTree system (Fischbacher, 1999). Sessions lasted about 90 minutes and average earnings were $25.

4. Equilibrium Analysis

Let the price increment in an auction be δ. Consider a setting with three bidders, *B* with the value of $v_B(2)$ for two units, and two small bidders. Let Pr(H) be the probability that at least one small bidder is a high type, so 1-Pr(H) is the probability that both are low types. Suppose high type small bidders always have per-unit valuations above the Big bidders, $v_{iH}(1) > v_B(2)/2 + \delta$, and low type small bidders always have per-unit valuations below the Big bidder's, $v_{iL}(1) < v_B(2)/2$. Suppose the Big bidder's value for one unit is $v_B(1) < v_{iL}(1)$. (For most of our analysis it

[9] An additional $5 payment was added to the Big bidders' earnings in the Ascending/Exposure treatments and to all participants' earnings in the Descending/Exposure treatment. Participants in all treatments were informed that this payment will exist in some treatments (see instructions for exact wording).

will be sufficient to speak about the players' realized values; but in proposition 6 we will have to pay a bit closer attention to the fact that our experimental environment is a game of incomplete information with two potential high and low values for each player.)

4.1 Descending Mechanism

We provide some general results about equilibrium bidding behavior in descending auctions with heterogeneous bidders[10].

Proposition 4 (Dominated strategies): For a risk neutral bidder *B*, there is no equilibrium in undominated strategies at which *B* bids above $\bar{P} = v_B(2)/2 - \left(v_B(2)/2 - \max_i v_{iL}(1)\right)\left(1 - \Pr(H)\right)$, and a high type small bidder never bids above $\bar{P} + \delta$. If bidders are risk averse then

$$\bar{P} > v_B(2)/2 - \left(v_B(2)/2 - \max_i v_{iL}(1)\right)\left(1 - \Pr(H)\right).$$

Proof: If *B* bids at $\max_i v_{iL}(1)$ then his expected profit is at least $\left(1 - \Pr(H)\right)\left(v_B(2)/2 - \max_i v_{iL}(1)\right)$, so if *B* is risk neutral, he must earn at least as much as the expected value of bidding at $\max_i v_{iL}(1)$ whenever he bids. Therefore, any price *P* at which *B* might bid must satisfy $v_B(2)/2 - P \geq \left(1 - \Pr(H)\right)\left(v_B(2)/2 - \max_i v_{iL}(1)\right)$. So $\bar{P} = v_B(2)/2 - \left(v_B(2)/2 - \max_i v_{iL}(1)\right)\left(1 - \Pr(H)\right)$ if *B* is risk neutral and $\bar{P} > v_B(2)/2 - \left(v_B(2)/2 - \max_i v_{iL}(1)\right)\left(1 - \Pr(H)\right)$ if *B* is risk averse. ■

Proposition 5 (Lower Bound): there exists a price $\underline{P} \geq \min_i v_{iL}(1) - \delta$ such that, at any equilibrium of the descending price auction, the clock is stopped for the first time at some $P' > \underline{P}$.

Proof: We prove this by induction. At $P_t = \delta$ big and small bidders stop the clock with certainty, because if they wait the auction ends and they earn 0, so $p_t^B = p_t^s = 1$. Since *B* wins in

[10] The computations of some specific examples of mixed strategy equilibria is straightforward, but tedious and we make them available for interested readers at: http://lema.smeal.psu.edu/katok/dutchexample.pdf.

case of ties, B always wins at $P_t = \delta$. Now, suppose there is some price $P_t < \min_i v_{iL}(1)$ at which a small bidder stops the clock with certainty. In this case, B stops the clock with certainty at P_t also, because he makes a positive profit at the price P_t, since $v_B(2)/2 > \min_i v_{iL}(1) > P_t$. But if B stops the clock with certainty at P_t, then the small bidder will stop the clock with certainty at $P_{t-1} = P_t + \delta$ if he makes a positive profit at P_{t-1}, $v_{is}(1) - P_t - \delta > 0$, which is always true if $P_t < \min_i v_{iL}(1)$ -δ. ■

Proposition 6 (pure strategy equilibrium): Pure strategy equilibrium can only exist if the proportion of high type small bidders is sufficiently small so as to make the big bidder's strategy of never bidding above $\max_i v_{iL}(1)$ profitable. If

$$v_B(2)/2 - \max_i v_{iL}(1) - 2\delta > (1 - \Pr(H))\left(v_B(2)/2 - \max_i v_{iL}(1)\right),$$

no pure strategy equilibrium exists.

Proof: For simplicity, we first prove the proposition for the slightly simpler game in which the big bidder's realized value $v_B(2)$ is common knowledge, and then extend the argument to our experimental environment in which it is not. Suppose there exists a price P^* at which B bids with certainty. Then, a high type small bidder with a value $v_{iH}(1)$ bids with certainty at $P^* + \delta$ if this is profitable (which it always would be at equilibrium, since a high type's valuation is always above B's valuation). Therefore at P^*, if the auction has not ended, B knows with certainty that the small bidder has a low value, and therefore B has no reason to bid above $\max_i v_{iL}(1)$, the maximum value a low type may have. So it cannot be an equilibrium for B to stop the clock for the first time with certainty at any $P^* > \max_i v_{iL}(1)$. Hence one candidate for P^* is $\max_i v_{iL}(1)$. If $P^* = \max_i v_{iL}(1)$, then the high type small bidder bids with certainty at $\max_i v_{iL}(1) + \delta$ (which is always profitable for the high type). But if

$$v_B(2)/2 - \max_i v_{iL}(1) - 2\delta > (1 - \Pr(H))\left(v_B(2)/2 - \max_i v_{iL}(1)\right),$$

B prefers to bid at $\max_i v_{iL}(1)+2\delta$ and win for certain instead of waiting until $\max_i v_{iL}(1)$, and therefore bidding at $P^*=\max_i v_{iL}(1)$ is not a pure strategy equilibrium strategy. If, on the other hand

$$v_B(2)/2-\max_i v_{iL}(1)-2\delta \le (1-\Pr(H))\left(v_B(2)/2-\max_i v_{iL}(1)\right),$$

then any $P_t \in \left[\min_i v_{iL}(1), \max_i v_{iL}(1)\right]$ is a candidate for a pure strategy P^* at which the big bidder stops the clock for the first time. Whether a pure strategy equilibrium exists, and if so the value of P^*, depends on the distribution of the low type small bidders.

Now consider the case of our experimental environment, in which $v_B(2)/2$ may equal either 50 or 60 with equal probability. The proof above has to be modified to consider a big bidder strategy of bids at $P^*(50)$ and $P^*(60)$. A high value small bidder's best response will be a bid with certainty at δ over one of these prices. If the small bidder's strategy were to bid above the higher of these two prices, then if no such bid were forthcoming, the big bidder would know that the small bidder was a low type, and so neither of the two prices can be above max v_{iL}, from the argument above. If the small bidder's strategy were to bid just above the lower of the two prices, then a best response by the big bidder would be to set the higher price to coincide with the small bidder's bid, which is again a contradiction. So the only pure strategy equilibrium strategies for the big bidder to stop the clock for the first time must be in the interval $P_t \in \left[\min_i v_{iL}(1), \max_i v_{iL}(1)\right]$, and cannot exist except as described above. In particular, no pure strategy equilibria exist in our experimental environment. ■

Proposition 7 (Free Rider problem in descending auctions): In a descending auction and free riding environment (i.e. in which small bidders are always of the same type) there are efficiency losses due to free riding.

Proof: In settings where there is no pure strategy equilibrium (Proposition 6), B wins with some positive probability when two high type small bidders are present. ■

To summarize in connection with our particular experimental environments, $v_B(2)/2$ = either 50 or 60; v_{iH} = either 65 or 75; v_{iL} = either 35 or 45; Pr(H) = 2/3; and δ = ½, so $\min_i v_{iL}(1)=35$ and

$\max_i v_{iL}(1) = 45$. There is no pure strategy equilibrium: By Proposition 6 we know that there is no pure strategy equilibrium because for big bidders with the value of 60,

$$v_B(2)/2 - \max_i v_{iL}(1) - 2\delta = 60 - 45 - 1 = 14 >$$
$$(1 - \Pr(H))\left(v_B(2)/2 - \max_i v_{iL}(1)\right) = \tfrac{1}{3}(60 - 45) = 5.$$

For a big bidder with a value of 50,

$$v_B(2)/2 - \max_i v_{iL}(1) - 2\delta = 50 - 45 - 1 = 4 >$$
$$1 - \Pr(H)\left(v_B(2)/2 - \max_i v_{iL}(1)\right) = \tfrac{1}{3}(50 - 45) = \tfrac{5}{3}$$

- There is no bidding in undominated strategies above 55 by a risk neutral big bidder and above 55.5 by a small bidder. By Proposition 4,

$$\bar{P} = v_B(2) - \frac{\left(v_B(2) - \max_i v_{iL}(1)\right)}{(1 - \Pr(H))} = 60 - \frac{60 - 45}{3} = 55.$$

- There is no bidding at prices below 35, by proposition 5.
- In our free riding and super free riding environments, equilibrium play does lead to some efficiency losses due to free riding.
- In our exposure environment, there are potentially much smaller (but still positive) efficiency losses due simply to the absence of a pure strategy equilibrium in a descending mechanism.

4.2 Ascending Mechanism

Obtaining equilibrium benchmarks in ascending auctions is much easier than in descending because pure strategy equilibria always exist.

- In the free riding and the super free riding environments, it is a dominant strategy for small bidders is to bid up to their values. The best reply for big bidders in these experimental conditions is to bid up to $\max_i v_{iL}(1) + \delta$. This weakly dominates the strategy of bidding up to $v_{iB}(2)/2$, in which case *B* also earns 0 if high types are present, but high types earn less. If high types are not present, bidding in undominated strategies never continues beyond $\max_i v_{iL}(1) + \delta$.

- In the exposure environment, it is still the dominant strategy for small bidders to bid up to their value, and the optimal strategy of big bidders depends on their values[11]:
    - Big bidders with the value of 20 for 1 unit and 50 per unit for two units should bid up to 20 (or equivalently not bid at all).
    - Big bidders with the value of 20 for 1 unit and 60 per unit for two units should bid up to 35, and if neither of the two small bidders drops out at 35, drop out at 35.5. However, if one of the small bidders does drop out at 35, the big bidder should fully go for the two units, and not drop out at all.
    - Big bidders with the value of 40 for 1 unit and 50 per unit for two units should bid up to 35.5.
    - Big bidders with the value of 40 for 1 unit and 60 per unit for two units should bid up to 45.5.

Proposition 8 (Exposure problem in ascending auctions): Big bidders sometimes suffer from the exposure problem when they follow the equilibrium strategy in ascending auctions.

Proof: In our setting, big bidders with the value of 60 per unit for two units make a higher expected profit if they bid above their value for one unit, $v_{iB}(1)$ than if they bid up to $v_{iB}(1)$. Big bidders have a positive probability of being forced to purchase one unit at a loss when they bid above $v_{iB}(1)$ in our exposure environment. ■

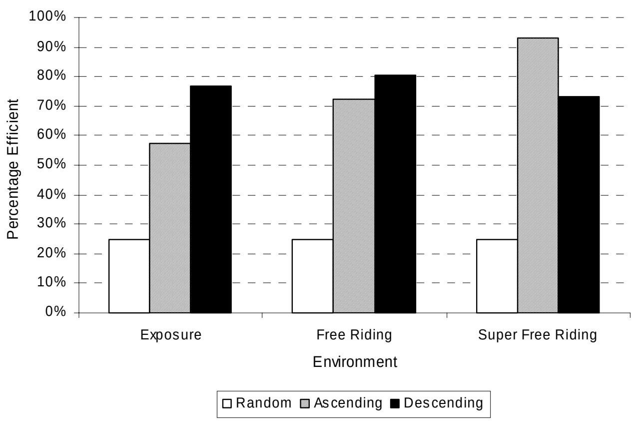
5. Data Analysis

Recall from the theoretical discussion that, if players make equilibrium bids, we expect the ascending auction to be vulnerable to efficiency losses in the exposure condition, and the descending auction to be vulnerable to losses of efficiency in the free riding and super free riding conditions.

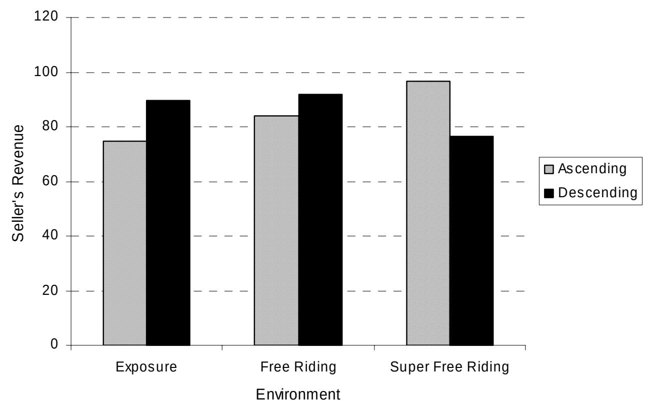
[11] We make these computations available at http://lema.smeal.psu.edu/katok/auction_strategy_EXP.xls

5.1 Mechanism Comparison: Efficiency, Revenue and Profit

Figure 3(a) compares the percentage of efficient allocations over all 20 rounds. The Descending mechanism is more efficient in the Exposure environment, and the Ascending mechanism is more efficient in the Super Free Riding environment, but the two mechanisms are equally efficient in the simple Free Riding environment. The differences in the Exposure and the Super Free Riding environments are highly significant (p-values are 0.0011 and 0.0039) and the difference in the Free Riding environment is not significant (p-value is 0.1515). The efficiency is fairly constant over time in the Exposure and the Free Riding environments. In the Super Free Riding environment the Ascending mechanism becomes nearly 100% efficient after several initial periods. We will say more about this observation in the next section when we discuss individual bidder behavior.



(b) Percentage of efficient allocations over all rounds



(b) Average revenue per auction (from the sale of two units) over all rounds

Figure 3. Overall efficiency comparisons.

The relative performance of the two auction mechanisms in terms of efficiency is as predicted in the exposure and super free riding environments. But there a small anomaly in the free rider condition, which we examine more closely below, and to which we will return in detail in the next section when we discuss individual bidding behavior.

Figure 3(b) compares seller's revenue in periods 1 – 20. As predicted, the descending auction yields higher revenue in the exposure environment (p-value is 0.0222) and the Ascending revenues are higher in the Super Free Riding environment (p-value is 0.0022). There is no significant difference in the Free Riding environment (p-value is 0.1567).

Treatment	Small bidder 1	Small bidder 2	Big Bidder (1 unit)	Big Bidder (2 units)
Ascending				
Exposure	23.68 (4.98) 80 (40%)	23.68 (4.98) 80 (40%)	44.60 (3.86) 63 (31.5%)	48.64 (7.52) 57 (28.5%)
Free Riding	42.74 (7.94) 111 (55.5%)	42.74 (7.94) 111 (55.5%)	43.62 (6.56) 40 (20%)	39.15 (4.07) 49 (24.5%)
Super Free Riding	50.49 (4.52) 96 (50%)	50.49 (4.52) 96 (60%)	48.43 (4.97) 7 (4.4%)	44.07 (5.25) 57 (35.6%)
Descending				
Exposure	52.30 (2.86) 63 (31.5%)	24.43 (3.68) 63 (31.5%)	30.54 (5.92) 30 (15%)	46.68 (2.93) 107 (53.5%)
Free Riding	56.21 (5.06) 83 (51.9%)	42.66 (7.11) 83 (51.9%)	38.13 (8.99) 11 (6.8%)	47.51 (3.87) 66 (41.3%)
Super Free Riding	50.32 (4.63) 72 (45%)	8.06 (8.42) 72 (45%)	N/A	46.81 (2.54) 88 (55%)

Table 1. Average selling prices, standard errors, and the number of occurrences in the six treatments.

Table 1 summarizes average unit selling prices (top), their standard errors (second number, in parenthesis), the number of occurrences (third number), and the percentage of occurrences (bottom number in parenthesis) in the six treatments. In all the Ascending treatments when the two small bidders win they pay the same price. In the Exposure environment, the average purchase price of the small bidders when they both win is 23.68 tokens. This low price means that the Big bidder often decides to leave the auction very close to the value for 1 unit (which is 20 75% of the time and 40 25% of the time). In other words, the Big bidder cannot successfully compete in the Exposure environment due to the exposure problem (the Big bidders in the Kagel and Levin 2001 study behave similarly). When the Big bidder wins only one unit in this treatment (this happened 63 times—31.5% of the time), the average selling price is 44.6 tokens—the Big bidder loses money. Since the Big bidder is unable to compete successfully, the small bidders often win at very low prices, hence the low revenues in the Ascending/Exposure treatment.

The Descending mechanism eliminates the exposure problem for Big bidders and allows them to compete successfully. The average price Big bidders pay for two units in the Exposure environment is very similar in the Ascending and Descending treatments (48.64 tokens vs. 46.68 tokens). However, big bidders win two units more often in the Descending/Exposure treatment (107 times—53.5% of the time vs. 57 times—28.5% of the time under Ascending), hence higher efficiency. When Big bidders only win one unit in the Descending/Exposure treatment, the average purchase price is 30.54, and the Big bidders never lose money. They win one unit when one of the small bidders has purchased a unit at a high price, and the Big bidder is able to successfully compete with the second small bidder[12].

An important difference between the Ascending and the Descending mechanisms is the ability of the Descending mechanism to price discriminate. When the small bidders win in the Descending/Exposure treatments they on average pay radically different prices (52.30 vs. 24.43). They also have different values (one always has a high value of 65 or 75, while the other always has a low value of 35 or 45). So in this case the seller is able to capture a great amount of the consumer surplus from the small bidder with a high value. This is in contrast to the Ascending/Exposure treatment when both small bidders end up paying 23.68 tokens on average, meaning that the small bidder with the high value gets to keep much of the consumer surplus.

Now let us examine the Free Riding and the Super Free Riding environments. First, note that the efficient allocation is always the same in these two environments and never includes the Big bidder winning one unit. In fact, the Big bidder's value for one unit should be irrelevant. Nevertheless, we do see Big bidders winning one unit under the Ascending mechanism. This happens more often in the Free Riding (40 times—20% of the time) than in the Super Free Riding (7 times—4.4% of the time) environment, and we will discuss this further in the next section. However, note that when Big bidders win one unit in these environments, the average price is 43.62 in the Free Riding environment and 48.43 in the Super Free Riding. This means that the Big bidders do occasionally suffer from the exposure problem even in the environments in which this problem would not arise at equilibrium.

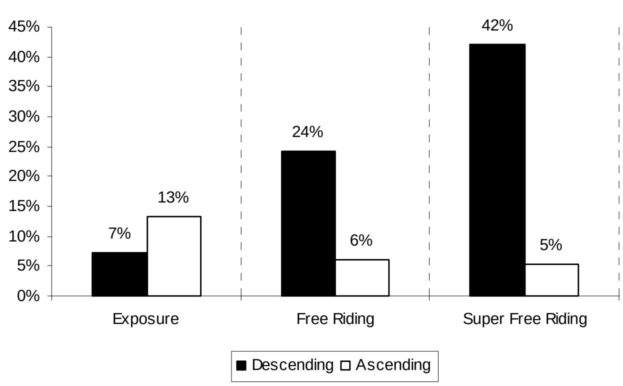
The seller's ability to price discriminate actually has a negative impact on revenues in the Super Free Riding environment. This is because after one small bidder wins one unit, the Big

[12] The second small bidder in the Exposure environment has a low value (35 or 45) and the Big bidder's value for one unit is either 20 (75% of the time) or 40 (25% of the time).

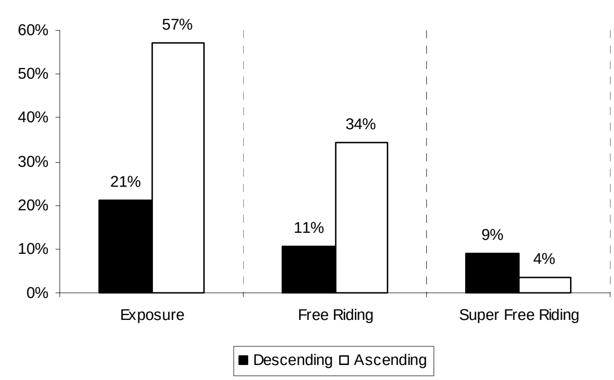
bidder is completely out of the auction, so the second small bidder needs only to wait for the price to reach it's minimum. We see this happening in the Descending/Super Free Riding treatment—one of the small bidders pays 50.32 tokens on average, while the second pays 8.06. We also see price discrimination in the Descending/ Free Riding treatment, where one small bidder pays 56.21 tokens on average, while the other pays 42.66. However, in this case the Big bidder is still somewhat competitive for one unit (with the value of 20 or 40), and this insures that the second small bidder pays a fairly high price.

5.2 Individual Bidding Behavior and the Role of Information

In all three environments under the descending mechanism, high type small bidders always stop the clock above 35 (consistent with Proposition 5). The winning price is below 55.5 74% of the time in the Exposure environment (65 out of 88 times), 68% of the time in the free riding environment (60 out of 88 times), and 81% of the time in the super free riding environment (54 out of 67 times). For big bidders, the winning bid is between 35 and 55 89% of the time in the exposure environment (95 out of 107 times), 94% of the time in the free riding environment (62 out of 66 times) and 100% of the time in the super free riding environment (88 times). There are three instances of the winning price being below 35 in the free riding environment, and the rest of the deviations are above 55. It is not surprising that Proposition 4 has somewhat more bite for big bidders than for small bidders, in the laboratory setting, because small bidders have to do one extra step of dominance reasoning.



(a) The big bidder wins when he should have lost.



(b) The big bidder loses when he should have won.

Figure 4: The frequency of inefficient outcomes.

Figure 4 classifies the causes of inefficient outcomes. Figure 4(a) confirms that the free riding problem exists under the descending mechanism, and is significantly higher in the environment in which the rewards from the free riding are high (the super free riding environment), but is also present, but to a smaller extent, in the free riding environment. Figure 4(b) shows the extent of the exposure problem under the ascending mechanism. Big bidders are excessively conservative in the exposure environment, and also, interestingly, in the free riding environment. That is, the results for the Free Rider condition in Figure 4(b) suggest that big bidders may have sometimes suffered from the exposure problem even though this wouldn't happen if small bidders always bid up to their values.

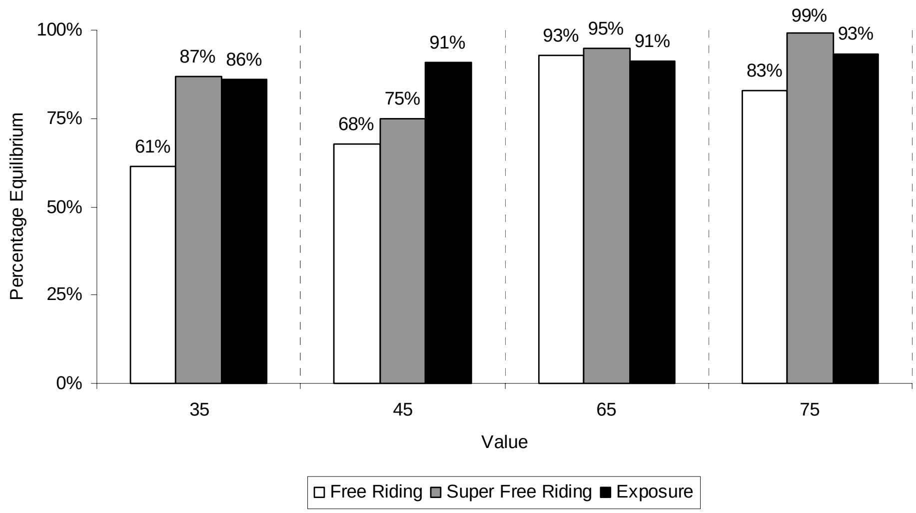


Figure 5: Percentage of small bidders who bid up to within 1 token of their values in ascending auctions.

At equilibrium big bidders in the free riding environment under the Ascending mechanism should bid up to 45.5. Under this strategy they always win two units when the two small bidders are low type and nothing otherwise. But the success of this strategy rests heavily on the assumption that small bidders bid up to their value. Figure 5 summarizes the percentage of small bidders who either win the auction or bid up to within 1 token of their values in ascending auctions. Of the high type small bidders in the free riding environment, about 88% either win the auction or bid up to their value—this number is substantially higher (97.5%) in the super free riding environment.

Figure 6 focuses on small bidders with high values who lose the auction, and displays the distribution of their highest bids. In general, small bidders with high values should never lose the auction under the Ascending mechanism, but in some cases this happens because Big bidders continue to bid above their value and eventually win two units at a loss. We see this happen in both Ascending/ Free Riding and Ascending/Super Free Riding treatments, with the same frequency of about 17%.

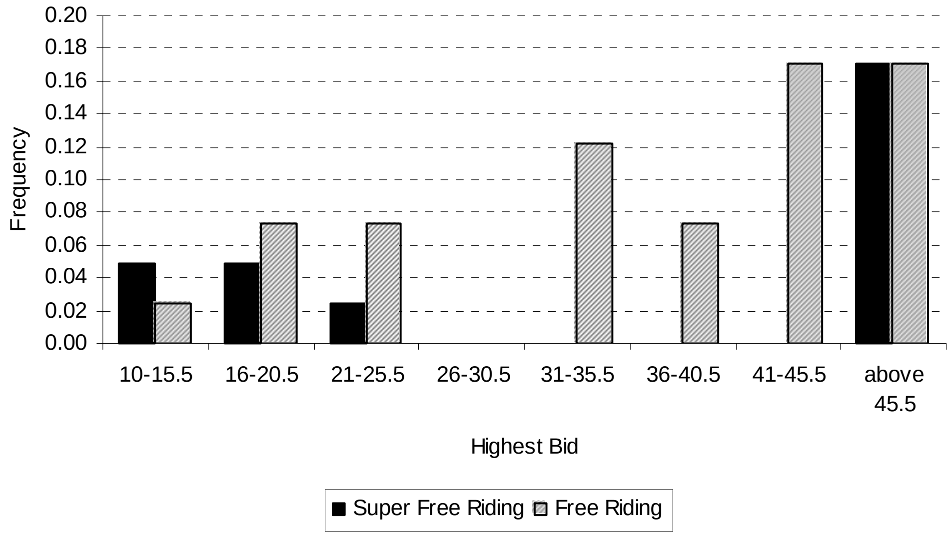


Figure 6. Small bidders with high values who lose the auction in the Ascending / Free Riding and the Ascending Super Free Riding treatments.

Of special interest is the frequency with which Small bidders stop between the values of 21 and 45.5 in the Free Riding environment. One way to interpret this behavior is that it is a deliberate attempt to "trap" the Big bidder— make him think that both small bidders are low type —and force him to win one unit and lose money. There is a cost to doing this for the Small bidder, since he foregoes positive profit, but he may be deliberately incurring this cost in order to make Big bidders more conservative in future auctions[13]—or small bidders may simply be making mistakes or being lazy. Overall, small bidders with high values stop bidding below 45.5 19% of the time in the Ascending/ Free Riding treatment, but only 4% in the Ascending/Super Free Riding treatment. The difference is weakly significant (p-value is 0.0910). Whatever the small bidders' motive, this behavior does make big bidders more conservative in the free riding environment: Big bidders bid up to 45.5 or above in 71% of the auctions in the free riding environment, and in 92% of the auctions in the super free riding environment.

In the exposure environment, low type small bidders are likely to win under equilibrium, while in the free riding environments they should never win, so we see that low types bit up to their value more often in the exposure environment than in the free riding environments. The high types, on the other hand, should always win, and we do see that overall they do not drop out early very often. In the exposure environment big bidders' optimal strategy is more complex, and only roughly 50% of the big bidders follow it[14].

In the exposure environment the equilibrium strategy predicts that 36% of the time big bidders should suffer from the exposure problem (lose money by either purchasing one or two units at a loss), and the actual amount of the exposure problem is very close to this—33%. In the free riding and the super free riding environments big bidders should never suffer from the exposure problem. In fact, 17% suffer from the exposure problem in the free riding environment, and 6% in the super free riding environment. Overall, about 24% of the auctions in the exposure environment should end in inefficient allocations if everyone were to follow optimal strategies, but the actual number is 42.5%—almost double the prediction. Auctions in free riding and super free riding environments should never end in inefficient allocation, but in fact 27% do in the free riding environment, and 7% in the super free riding environment.

6. Conclusions

The experiments we present are a first step in systematically investigating and comparing simple multi-unit auction mechanisms in environments with complementarities. We find that the Descending mechanism performs well in terms of efficiency and revenues in environments that include a danger of the exposure problem. It also performs surprisingly well in environments specifically meant to challenge it—environments with a free riding problem. The ascending price mechanism similar to the one eBay uses to sell multiple homogeneous items performs poorly in exposure environments, but well in environments without the exposure problem and with a big Free Riding problem. The Ascending mechanism can leave bidders "exposed" when

[13] That is, although we have focused our analysis on the one-time auction, some bidders in our experiment may be responding to the fact that they will play future auctions with bidders drawn from the same pool, and hence have a positive probability of meeting each big bidder again.

[14] Percentage of auctions where big bidders follow the optimal strategy in the exposure environment: 50% (35 out of 70) of the big bidders of type 20/100 stopped on or before 20; of the 40/100 type big bidders, 50% (15 out of 30) bid up to 35 and stopped there; of the 40/120 type big bidders only 58% (11 out of 19) bid up to 45 and stopped there; of the 20/120 type big bidders 50% (40 out of 80) behaved consistent with the optimal strategy.

small bidders fail to bid up to their values, but it avoids the free riding problem in environments in which it is large.

In conclusion, our results offer some insight into the properties that make the descending price Dutch auction a successful mechanism for the sale of multiple units of homogeneous goods. In terms of mechanism design, the experimental results reveal that the ascending mechanism can leave bidders "exposed" to the errors of others in a way that the descending auction does not. But the descending auction is vulnerable to the free rider problem. As Klemperer (2002) observes, "Good auction design is not 'one size fits all' and must be sensitive to the details of the context." Our results suggest that, in environments in which the exposure problem looms larger than the free rider problem, the descending price Dutch auction is likely to be a superior alternative to the uniform price ascending auction.

Overall, the descending price auction offers an attractive option for selling multiple units of a homogenous good when buyers may have increasing returns to scale.

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